ENGR-4300  
Spring 2009  
Test 1

Name ______ SOLUTION ____________

Section:  1(MR 8:00am)  2(TF 2:00pm)  3(MR 6:00pm)  
(circle one)

Question I (20 points) ____________

Question II (20 points) ____________

Question III (20 points) ____________

Question IV (20 points) ____________

Question V (20 points) ____________

Total (100 points) ________________

On all questions: SHOW ALL WORK. BEGIN WITH FORMULAS, THEN SUBSTITUTE VALUES AND UNITS. No credit will be given for numbers that appear without justification.
Question I – Circuit Analysis (20 points)

1) (2 pts) What is the value of R23, the parallel combination of R2 and R3?

\[ R_{23} = \frac{R_2 \times R_3}{R_2 + R_3} = \frac{7k \times 42k}{7k + 42k} = 6k \]

2) (3 pts) What is the value of R456, the parallel combination of R4, R5 and R6?

\[ \frac{1}{R_{456}} = \frac{1}{R_4} + \frac{1}{R_5} + \frac{1}{R_6} = \frac{1}{20k} + \frac{1}{6k} + \frac{1}{30k} = 0.00025 \]
\[ R_{456} = \frac{1}{0.00025} = 4k \]

3) (3 pts) Find the total resistance attached to the 50V supply.

\[ R_{total} = \frac{R_1 \times (R_{23} + R_{456})}{R_1 + (R_{23} + R_{456})} = \frac{10k \times 10k}{10k + 10k} = 5k \]

4) (3 pts) Find the total current from the 50V supply into the circuit.

\[ I_{total} = \frac{V}{R_{total}} = \frac{50}{5k} = 10mA \]
5) (4 pts) Find voltage point A using the voltage divider equation.

\[ V_A = 50 \frac{4k}{4k + 6k} = 50(0.4) = 20\text{volts} \]

6) (4 pts) How much current is flowing through R4, the 20k resistor?

\[ I_{R4} = \frac{20V}{20k} = 1\text{mA} \]

7) (1 pt) What would the voltage at A be if the 10k resistor R1 were removed from the circuit?

The voltage would not change since the voltage at the top of the voltage divider would still be 50V

\[ V_A = 20V \]
Question II – Filters (20 points)

The following circuits consist of a sinusoidal source, resistors, capacitors, and inductors. V1 and V2 are the sinusoidal sources and R1 and R3 are internal impedances. Analyze all with appropriate simplification for the impedances.

1) (3 pts) Redraw circuit A at low frequencies

H=0 (not needed in answer)
2) (3 pts) Redraw circuit B at low frequencies

3) (3 pts) Redraw the circuit A at high frequencies
4) (3 pts) Redraw circuit B at high frequencies

5) (4 pts) Create a rough sketch of the magnitude of the transfer function of Circuit A as a function of frequency (Hz). You need only to show the general shape, not the phase. **Indicate on the graph where the resonant frequency is located and show its numerical value.**

\[
\begin{align*}
L_1 &:= 100 \, \text{mH} \\
C_4 &:= 10 \, \mu \text{F} \\
C_5 &:= 5 \, \mu \text{F} \\
\frac{1}{2 \pi \sqrt{L_1 (C_4 + C_5)}} \, f_{0A} &= 129.949 \, \text{Hz}
\end{align*}
\]

-1 if did not remember or mention the possibility of resonance for both graphs
6) (2 pts) Create a rough sketch of the magnitude of the transfer function of Circuit B as a function of frequency (Hz). You need only to show the general shape, not the phase. *Indicate on the graph where the resonant frequency is located and show its numerical value.*

\[
L_3 := 100 \text{ mH} \\
L_4 := 100 \text{ mH} \\
C_6 := 10 \mu \text{F} \\
L_T := \frac{L_3 \cdot L_4}{L_3 + L_4} \\
f_{0B} := \frac{1}{2 \pi \sqrt{L_T C_6}} \\
f_{0B} = 225.079 \text{Hz}
\]

did not remember resonance; point is already taken above
7) (2 pts) The following graph is the source voltage for either V1 (Circuit A) or V2 (Circuit B). Given the knowledge of corner and/or resonant frequencies, which circuit (A or B) would cause $V_{out}$ to be of much less amplitude than V1 or V2 (input voltage)?

No points for continuation of errors above!

Frequency of graph is $1/100\text{us} = 10K$. According to resonant frequency $V_{out}$ would be low for a low pass filter. Therefore, Circuit B would cause $V_{out}$ to be low.
Question III – Transfer Functions (20 points)

1) (6 pts) What is the transfer function $H(j\omega)$ for the circuit above in terms of $R$, $L$, and $C$? You must reduce it to a simple ratio of polynomials in $\omega$.

$$H(j\omega) = \frac{j\omega L}{R + j\omega L + \frac{1}{j\omega C}} = \frac{j\omega L}{R + j(\omega L - \frac{1}{\omega C})} = \frac{j\omega L}{\omega RC + j(\omega^2 LC - 1)}$$

2) (4 pts) Find the simplified transfer function for small (not zero) and large $\omega$.

Small: $H(j\omega) = \frac{j\omega^2 LC}{\omega RC + j(\omega^2 LC - 1)} \Rightarrow \frac{j\omega^2 LC}{j(-1)} = \frac{\omega^2 LC}{(-1)} = -\omega^2 LC$

Large: $H(j\omega) = \frac{j\omega^2 LC}{\omega RC + j(\omega^2 LC - 1)} \Rightarrow \frac{j\omega^2 LC}{j(\omega^2 LC)} = 1$

3) (4 pts) Substitute the values $R = 1$, $C = 0.25$, and $L = 4$ into $H(j\omega)$ in 1) and simplify.

$$H(j\omega) = \frac{j\omega^2 LC}{\omega RC + j(\omega^2 LC - 1)} = \frac{j\omega^2(4)(0.25)}{(0.25)(1) + j(\omega^2(4)(0.25) - 1)} = \frac{j\omega^2}{(0.25)\omega + j(\omega^2 - 1)}$$
4) (2 pts) Find the magnitude of $H(j\omega)$ for $\omega = 1$ radian/s.

$$|H(j1)| = \frac{j\omega^2}{(0.25\omega + j(\omega^2 - 1))} = \frac{|j1|}{|0.25 + j0|} = \frac{1}{0.25} = 4$$

5) (2 pts) For $\omega = 1$ radian/s, if the input has an amplitude of 2 volts [$Vin = 2\sin(t)$], what is the amplitude of $V_{out}$?

$$|V_{out}| = |H(j1)|\times|Vin| = 4\times2 = 8V$$

6) (2 pts) At what frequency would you expect the voltage $V_{out}$ to go to zero?

From 2), as $\omega$ approaches 0, the output approaches 0, so

$V_{out} = 0$ for $\omega = 0$
Question IV - Transformers and Inductors (20 points)

1) (7 pts) In the circuit above, R1 = 10KΩ. If the input voltage has an amplitude of 5V and the voltage at point A is 600mV, what is the value of R2?

\[ R_1 : = 10K\Omega \]
\[ N_2 : = 6 \]
\[ N_1 : = 1 \]
\[ a : = \frac{N_2}{N_1} \]
\[ Z_{AB} : = \frac{R_1}{a^2} \]
\[ Z_{AB} = 277.778\Omega \]

\[ V_{in} : = 5V \]
\[ V_A : = 600mV \]
\[ V_A = \frac{Z_{AB}}{R_2 + Z_{AB}} \cdot V_{in} \]
\[ R_2 = \frac{Z_{AB} \cdot V_{in} - Z_{AB} \cdot V_A}{V_A} \]
\[ R_2 = 2.037 \times 10^3 \Omega \]
You have found an inductor and wish to determine its inductance. Here is a picture:

You find that it has a wire gauge diameter of 0.51 mm (24 gauge), a length of 10.5 mm, a core diameter of 7.0 mm and 47 turns. You assume that it has an air core ($\mu=1.257 \times 10^{-6}$ Henries/meter).

2) (3 pts) Calculate the theoretical inductance

\[
\mu := 1.257 \times 10^{-6} \frac{H}{m} \\
N_{\text{turns}} := 47 \\
r_c := \frac{7.0}{2} \times 10^{-3} m \\
d := 10.5 \times 10^{-3} m \\
L_{\text{ind}} := \left( \frac{\mu \cdot N_{\text{turns}}^2 \cdot \pi \cdot r_c^2}{d} \right) \\
L_{\text{ind}} = 1.018 \times 10^{-5} H \\
L_{\text{ind}} = 10.18 \mu H
\]
3) (2 pts) You place the inductor you found in the following circuit

If the value of the resistor is 50 ohms and the value of the capacitor is 100\(\mu\)F, what is the resonant frequency of the circuit?

\[
f_0 := \frac{1}{2\pi \sqrt{10.18\mu H \cdot 100\mu F}} \quad f_0 = 4.988 \times 10^3 \text{ Hz}
\]

( note: one point for correct equation, if value of inductor is incorrect)

4) (3 pts) Based on simple analysis of what occurs at high and low frequencies (redraw if needed), draw the sketch of the magnitude of the transfer function at point B. *Mark the resonant frequency on the sketch.*

5) (2 pts) What kind of filter is this?

*Band reject*

6) (3 pts) If Vout is placed between the inductor and capacitor instead of point B, what happens to the magnitude of the transfer function from high to low frequencies?

*Becomes a low pass filter, magnitude is 1 at low and 0 at high*
Question V - PSpice (20 points)

The following circuit is setup in PSpice

1) (5 pts) Setup a transient analysis in the simulation settings window below that will show 10 cycles of the signal, (the "start saving data after:" box can be neglected)

1kHz => 1ms for 1 complete cycle, 10ms => 10 cycles.
Max step size is 10us or something close to this.
2) (2 pts) Your friend in the course is trying to create an AC sweep for the above circuit. He or she is frustrated and has spent hours trying to get the simulation to run. You decide to help and find that the simulation settings are correct and the components in the circuit are connected correctly. You still receive an error, “No AC sources- AC sweep ignored”. What is the easiest and fastest solution to this error?

   a) Call over Professor Sawyer and/or Professor Kraft
   b) Call over a TA
   c) Close the program and restart
   d) Double click the AC source and add the amplitude parameter in the “AC” box
   e) Use “VAC/Source” component in your library

   *(d if they have a, b, and d, take off 1 point!)*

3) (3 pts) The AC sweep problem is solved. The simulation output looks like:

![Simulation Output](image.png)

You would like to see the transient output plot at 1.0MHz. What would you change in the circuit and in the settings to clearly see the change in voltage with time? Circle all that apply.

   a) The values of the capacitor and inductor
   b) The analysis type in the simulation settings to Time Domain (Transient)
   c) The position of the voltage probes
   d) The value of the AC source amplitude to 1V
   e) The value of the AC source amplitude to .1V
   f) The value of the AC source frequency to 1M
   g) The value of the AC source frequency to 1Meg
   h) The run time to 40ms and maximum step size to 40us
   i) The run time to .004 ms and maximum step size to .004us

   *b, g, i*
4) (4 pts) Refer to the AC sweep diagram above. Choose and label the plots of the transient output at 1.0MHz and 200Hz (label Vin and Vout).

- 1 Meg, Vout is red, Vin is green
- Determined from plot above, the transfer function is nearly 1

- 200 Hz, Vout is red, Vin is green
- Determined from plot above, the transfer function is nearly 1
5) (6 pts) You begin to build the circuit on the protoboard. You would like to compare the input and output of the circuit as shown in the diagram as probes. What options can you use on the oscilloscope with the corresponding IOBoard connections? (Mark all that apply)

- Wire to circuit at voltage point at point 1 using A1+ and Ground
- Wire to circuit at voltage point at point 2 using A2+ and Ground
- Wire to circuit at voltage point at probe 1 using A1+ and Ground
- Wire to circuit at voltage point at probe 2 using A2+ and Ground

Measuring from the outside using wires A1 and A2 single ended is correct

Measuring the output from the outside using wires A2 single ended and the input at probe 1 from the inside AWG1 is correct