Electronic Instrumentation

Experiment 3

- Part A: Making an Inductor
- Part B: Measurement of Inductance
- Part C: Simulation of a Transformer
- Part D: Making a Transformer
Inductors & Transformers

- How do transformers work?
- How to make an inductor?
- How to measure inductance?
- How to make a transformer?
Part A

- Inductors Review
- Calculating Inductance
- Calculating Resistance
Inductors-Review

- General form of I-V relationship

\[ V = L \frac{dI}{dt} \]

- For steady-state sine wave excitation

\[ Z_L = j\omega L \quad V = j\omega LI \]
Determining Inductance

- Calculate it from dimensions and material properties
- Measure using commercial bridge (expensive device)
- Infer inductance from response of a circuit. This latter approach is the cheapest and usually the simplest to apply. Most of the time, we can determine circuit parameters from circuit performance.
Making an Inductor

- For a simple cylindrical inductor (called a solenoid), we wind $N$ turns of wire around a cylindrical form. The inductance is ideally given by

$$L = \frac{(\mu_0 N^2 \pi r_c^2)}{d} \text{Henries}$$

where this expression only holds when the length $d$ is very much greater than the diameter $2r_c$. 
Making an Inductor

- Note that the constant $\mu_o = 4\pi \times 10^{-7}$ H/m is required to have inductance in Henries (named after Joseph Henry of Albany)
- For magnetic materials, we use $\mu$ instead, which can typically be $10^5$ times larger for materials like iron
- $\mu$ is called the permeability
Some Typical Permeabilities

- Air  $1.257 \times 10^{-6}$ H/m
- Ferrite U M33  $9.42 \times 10^{-4}$ H/m
- Nickel  $7.54 \times 10^{-4}$ H/m
- Iron  $6.28 \times 10^{-3}$ H/m
- Ferrite T38  $1.26 \times 10^{-2}$ H/m
- Silicon GO steel  $5.03 \times 10^{-2}$ H/m
- supermalloy  $1.26$ H/m
Making an Inductor

- If the coil length is much smaller than the diameter ($r_w$ is the wire radius)

$$L \approx \mu N^2 r_c \left\{ \ln\left(\frac{8r_c}{r_w}\right) - 2 \right\}$$

Such a coil is used in the metal detector at the right.
Calculating Resistance

- All wires have some finite resistance. Much of the time, this resistance is negligible when compared with other circuit components.
- Resistance of a wire is given by
  \[ R = \frac{l}{\sigma A} \]
  - \( l \) is the wire length
  - \( A \) is the wire cross sectional area \((\pi r_w^2)\)
  - \( \sigma \) is the wire conductivity
Some Typical Conductivities

- Silver 6.17x10^7 Siemens/m
- Copper 5.8x10^7 S/m
- Aluminum 3.72x10^7 S/m
- Iron 1x10^7 S/m
- Sea Water 5 S/m
- Fresh Water 25x10^{-6} S/m
- Teflon 1x10^{-20} S/m

Siemen = 1/ohm
Wire Resistance

- Using the Megaconverter at http://www.megaconverter.com/Mega2/ (see course website)
Part B: Measuring Inductance with a Circuit

- For this circuit, a resonance should occur for the parallel combination of the unknown inductor and the known capacitor. If we find this frequency, we can find the inductance.
Determining Inductance

- Reminder—The parallel combination of $L$ and $C$ goes to infinity at resonance. (Assuming $R_2$ is small.)

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$Z_\parallel = \frac{j\omega L \left( \frac{1}{j\omega C} \right)}{j\omega L + \left( \frac{1}{j\omega C} \right)} = \frac{j\omega L}{1 - \omega^2 LC}$$
Determining Inductance

\[ H = \frac{Z||}{R1 + Z||} \]

\[ H = \frac{j\omega L}{R1(1 - \omega^2 LC) + j\omega L} \]

\[ H_{HI} = H_{LO} = \frac{j\omega L}{R1} = small \]

at resonance, \( \omega_0 \), \( H_0 = \frac{j\omega L}{j\omega L} = 1 \)
VOFF = 0
VAMPL = 0.2
FREQ = 1kHz
AC = .2

V1

V

0

V (V1:1)  V (C1:1)
Even 1 ohm of resistance in the coil can spoil this response somewhat.
Part C

- Examples of Transformers
- Transformer Equations
Transformers

- Cylinders (solenoids)
- Toroids
Transformer Equations

\[ a = \frac{N_L}{N_S} = \frac{V_L}{V_S} = \sqrt{\frac{L_L}{L_S}} = \frac{I_S}{I_L} \]

\[ Z_{in} = \frac{R_L}{a^2} \]
Deriving Transformer Equations

- Note that a transformer has two inductors. One is the primary (source end) and one is the secondary (load end): \( L_S \) & \( L_L \)
- The inductors work as expected, but they also couple to one another through their mutual inductance: \( M^2 = k^2 L_S L_L \)
Transformers

- Assumption 1: Both Inductor Coils must have similar properties: same coil radius, same core material, and same length.

\[
\frac{L_L}{L_S} = \frac{\frac{(\mu_0 N_L^2 \pi r_c^2)}{d}}{\frac{(\mu_0 N_S^2 \pi r_c^2)}{d}} = \frac{N_L^2}{N_S^2} \quad \text{let} \quad a = \frac{N_L}{N_S} \quad \therefore \quad a = \sqrt{\frac{L_L}{L_S}}
\]
Transformers

- Let the current through the primary be $I_S$
- Let the current through the secondary be $I_L$
- The voltage across the primary inductor is $j \omega LI_S - j \omega MI_L$
- The voltage across the secondary inductor is $j \omega LI_L - j \omega MI_S$
Transformers

- Sum of primary voltages must equal the source:
  \[ V_S = R_S I_S + j\omega L_S I_S - j\omega MI_L \]

- Sum of secondary voltages must equal zero:
  \[ 0 = R_L I_L + j\omega L_L I_L - j\omega MI_S \]
Assumption 2: The transformer is designed such that the impedances \( Z = j \omega L \) are much larger than any resistance in the circuit. Then, from the second loop equation

\[
0 = R_L I_L + j \omega L_L I_L - j \omega M I_S
\]

\[
j \omega L_L I_L \approx j \omega M I_S \quad \Rightarrow \quad L_L^2 I_L^2 \approx M^2 I_S^2
\]

\[
\therefore \quad \frac{I_L}{I_S} \approx \frac{M}{L_L}
\]
Transformers

- $k$ is the coupling coefficient
  - If $k=1$, there is perfect coupling.
  - $k$ is usually a little less than 1 in a good transformer.
- Assumption 3: Assume perfect coupling ($k=1$)

We know $M^2 = k^2 \ L_S \ L_L = L_S \ L_L$ and \[ a = \sqrt{\frac{L_L}{L_S}} \]

Therefore, \[ \frac{I_L}{I_S} \approx \frac{M}{L_L} = \frac{\sqrt{L_S L_L}}{L_L} = \sqrt{\frac{L_S}{L_L}} = \frac{1}{a} \]
Transformers

- The input impedance of the primary winding reflects the load impedance. \(Z_{L_S} = Z_{in} = Z_{total} - R_S\)
- It can be determined from the loop equations
  - 1] \(V_S = R_S I_S + j \omega L_S I_S - j \omega M I_L\)
  - 2] \(0 = R_L I_L + j \omega L_L I_L - j \omega M I_S\)
- Divide by 1] \(I_S\). Substitute 2] and \(M\) into 1]

\[
Z_{IN} = \frac{V_S}{I_S} - R_S = j \omega L_S + \omega^2 \frac{L_S L_L}{(R_L + j \omega L_L)}
\]
Transformers

- Find a common denominator and simplify

\[ Z_{IN} = \frac{j \omega L_S R_L}{j \omega L_L + R_L} \]

- By Assumption 2, \( R_L \) is small compared to the impedance of the transformer, so

\[ Z_{IN} = \frac{L_S R_L}{L_L} = \frac{R_L}{\alpha^2} \]
**Transformers**

- It can also be shown that the voltages across the primary and secondary terminals of the transformer are related by

\[ N_S V_L = N_L V_S \]

Note that the coil with more turns has the larger voltage.

- Detailed derivation of transformer equations
  [http://hibp.ecse.rpi.edu/~connor/education/transformer_notes.pdf](http://hibp.ecse.rpi.edu/~connor/education/transformer_notes.pdf)
Transformer Equations

\[ a = \frac{N_L}{N_S} = \frac{V_L}{V_S} = \sqrt{\frac{L_L}{L_S}} = \frac{I_S}{I_L} \]

\[ Z_{in} = \frac{R_L}{a^2} \]
Part D

- Step-up and Step-down transformers
- Build a transformer
**Step-up and Step-down Transformers**

**Step-up Transformer**

\[ N_2 > N_1 \]
\[ V_2 > V_1 \]
\[ I_2 < I_1 \]
\[ \sqrt{L_2} > \sqrt{L_1} \]

**Step-down Transformer**

\[ N_2 < N_1 \]
\[ V_2 < V_1 \]
\[ I_2 > I_1 \]
\[ \sqrt{L_2} < \sqrt{L_1} \]

Note that power (\(P=VI\)) is conserved in both cases.
Build a Transformer

- Wind secondary coil directly over primary coil
- “Try” for half the number of turns
- At what frequencies does it work as expected with respect to voltage? When is $\omega L >> R$?

$$a = \frac{N_L}{N_S} = \frac{V_L}{V_S}$$
Some Interesting Inductors

- Induction Heating
Some Interesting Inductors

- Induction Heating in Aerospace
Some Interesting Inductors

- Induction Forming
Some Interesting Inductors

- Coin Flipper
Some Interesting Inductors

- GE Genura Light

Figure 1. Schematic of the Genura induction lamp. The power supply converts ordinary 60 or 50 hertz current into high-efficiency power that is fed into an electrical coil. The coil excites a gas plasma inside the bulb, releasing UV radiation that strikes the bulb’s phosphor coating and is converted into visible light.
Some Interesting Transformers

- A huge range in sizes
Some Interesting Transformers

- High Temperature Superconducting Transformer
Household Power

- 7200V transformed to 240V for household use
Wall Warts

Transformer