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A Reserve Price Auction for Spectrum Sharing with Heterogeneous Channels

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***Abstract*—Cognitive radio is a novel communication paradigm that can significantly improve spectrum utilization by allowing the cognitive radio users to dynamically utilize the licensed spectrum. To achieve this, studying efficient spectrum allocation mechanisms is imperative. In this paper, we consider a cognitive radio network consisting of a primary spectrum owner (PO), multiple primary users (PU) and multiple secondary users (SU). We propose a reserve price auction mechanism for spectrum sharing in cognitive radio networks where the SUs bid to buy spectrum bands from the PO who acts as the auctioneer, selling idle spectrum bands to make a profit. Unlike most existing auction mechanisms that assume identical channels, we consider a more general and more realistic case where channels have different qualities. Also, SUs are allowed to express their preferences for each channel separately. That is, each SU submits a vector of bids, one for each channel. In addition, reservation prices that are proportional to channel qualities are imposed by the PO. The proposed auction mechanism results in efficient allocation that maximizes SUs' valuations subject to reserve price constraints, and it has desired economic properties that we formally prove in the analysis. Numerical results show performance improvements compared to the case of reserve price auction with identical channels and the case of having no reservation prices.**

Keywords*—Cognitive Radio Networks, Spectrum Sharing, Heterogeneous channels, Game Theory, Reserve Price Auction.

I. INTRODUCTION

With the ever-increasing demand for wireless communications, spectrum scarcity and efficient use of wireless spectrum is becoming a major challenge. The Federal Communications Commission (FCC) has reported that the conventional fixed spectrum assignment is no longer capable of meeting today's wireless spectrum requirements. Also, according to the spectrum usage measurements by the FCC's Spectrum Policy Task Force, many of the allocated spectrum bands are idle most of the times or not used in some areas [1]. That calls for better spectrum management techniques and policies.

A promising approach to improve spectrum utilization is dynamic spectrum sharing which is realized by cognitive radio networks [2]. In dynamic spectrum sharing, unlicensed secondary users (SU) are allowed to utilize the radio spectrum owned by a primary owner (PO). For this purpose, designing a spectrum sharing mechanism that can efficiently allocate the spectrum bands to SUs, seems imperative. It is necessary for the mechanism to provide sufficient incentives for both PO and SUs to participate in spectrum sharing.

In a simple spectrum auction scenario, the POs act as auctioneers and sell their idle spectrum bands to SUs to make a profit, and the SUs act as bidders who want to buy spectrum bands. In such a setting, auction-based mechanisms appear to be the most appropriate approach because they can capture many of the key features of the spectrum sharing problem. First, in an auction, it is possible to consider situations where the seller is not assumed to know any prior information about the valuation of items to the buyers. This aspect can not be easily taken into account in pricing-based or other conventional market-based mechanisms. Second, auctions can be designed to allocate items to the buyers with highest valuations, thus making an efficient allocation. Third, auctions require minimum interactions between seller and buyers, because the buyers just need to submit their bids for the items. This makes the implementation of the mechanisms easier and more practical compared to the other market mechanisms.

In this paper, we consider a cognitive radio network consisting of a PO, multiple primary users (PUs) and multiple (SUs). The PO acts as the auctioneer, selling idle spectrum bands to SUs. Unlike most existing auction mechanisms that assume identical channels, we consider a more general case where channels have different qualities. Also, SUs are allowed to express their preferences for each channel separately. That is, each SU submits a vector of bids, one for each channel. This model provides much more flexibility for SUs compared to the existing spectrum auctions. Reservation prices that are proportional to channel qualities are imposed by the PO and they show minimum prices that the PO is willing to sell channels. Simple auction mechanisms with no reservation prices may result in small revenues. For example, a second-price auction was used in 1990 in New Zealand for selling spectrum licenses. The winner bid \$100,000 but paid only \$6; in another case, the winner bid \$7,000,000 but paid only \$5,000 [3]. Reserve pricing is a simple and effective way to avoid such situations. Here, we design a reserve price auction to allocate the channels efficiently, maximizing SUs' valuations subject to price reservation constraints. Also, the proposed auction has some desired economic properties.

Technically, this problem can be modeled by a non-identical multiple item reserve price auction where each bidder has a different view of the available items. An auction is described by a pair of functions, namely the allocation function and the payment function.

In order to determine the allocation rule of the auction, we transform the problem into the problem of finding a maximum weight matching in a bipartite graph. This matching provides us with the efficient allocation that maximizes SUs' valuations subject to reserve price constraints. For the payment function, we use a modified form of the Vickrey Clarke Groves (VCG) mechanism [4]. This auction mechanism runs in polynomial time and satisfies economic properties of Incentive Compatibility, Individual Rationality, and No Positive Transfers.

The main contributions of this paper are as follows. We consider a cognitive radio network with heterogeneous channel qualities which is more realistic compared to prior work. Also, SUs are given the flexibility to express their preferences for channels separately. In this way, an SU can submit a different bid for each available channel. In our model, the PO can impose reservation prices on channels to secure minimum trading prices. We then propose a reserve price auction for spectrum sharing that results in efficient spectrum allocation subject to reserve price constraints. The proposed auction has desired economic properties (Incentive Compatibility, Individual Rationality, and No Positive Transfers) that we formally prove in the analysis. In addition, we provide numerical results that show performance improvements compared to the case of reserve price auction with identical channels and the case of having no reservation prices.

The rest of this paper is organized as follows. In Section II we review and discuss related work. Section III presents the system model used in this paper. In Section IV, we present the reserve price auction mechanism and prove its properties. Numerical results are presented in Section V. Finally, Section VI concludes the paper and outlines possible future work.

II. Related Work

Game theory and auction design have been recently used for wireless spectrum allocation and management [5]–[17]. Here we summarize some of the most relevant results.

In [5], an auction-based spectrum management scheme for cognitive radio networks has been presented. The network consists of a primary base station and several primary and secondary users. The service provider determines the number of channels to be sold and holds the auction among the secondary users. Since the channels are assumed to be identical, the Vickrey auction determines the winners and payments. A similar network topology has been considered in [6], however channels are assumed to be different. The model is based on the contract theory in which the PO acts like a monopolist and determines the qualities and prices for spectrum bands with the objective of maximizing his own revenue. However, in this approach SUs cannot submit bids and the PO needs some prior information about SUs' valuations.

In [7], the idea of having multiple auctioneers, i.e. multiple POs, has been presented. In this setting, each PO gradually raises the trading price and each SU chooses one auctioneer for bidding. After several bidding/asking rounds, the mechanism converges to an equilibrium where no PO and SU would like to change his decision. Also, [8] considers two wireless service providers, and the authors study the optimal pricing for service providers and optimal service provider selection for SUs. They show that the equilibrium price and its uniqueness depend on the SUs' geographical density and spectrum propagation characteristics. In [9], the authors study the dynamics of spectrum sharing and pricing in a competitive environment where multiple POs try to sell spectrum bands to multiple SUs. They use evolutionary game theory to model the evolution and the dynamic behavior of SUs. The competition among POs has been modeled as a noncooperative game, and an iterative algorithm has been presented to find the Nash equilibrium.

In [10], Zhou et al. proposed TRUST, a general framework for truthful double spectrum auctions. This framework aims to provide spectrum reuse while achieving truthfulness and other desired economic properties. TRUST takes any reusability-driven spectrum allocation method as an input, and applies its own winner determination and pricing policy. There is an external auctioneer with complete information that holds the auction between POs and SUs.

The authors in [11] consider a setting in which SUs have flexibility to bid for a bundle of frequencies at different times. In fact, the spectrum opportunity is divided by frequency and time, so that SUs can bid for a combination of them. This flexibility, however, brings computational complexity. Since the general problem falls into the combinatorial auctions category, obtaining the efficient allocation is NP-hard, and only approximate solutions can be achieved. In [12], the authors study the effect of interference created among different agents who may obtain the right to use the same spectrum at nearby locations. This interference results in complementarities among the traded spectrum bands, which brings computational complexity to the design of efficient mechanisms. Since finding the efficient allocation is NP-hard, some constant factor approximations have been discussed.

Recently, a group of researchers considered two-tier market models for dynamic spectrum access. In tier-1, SUs buy the spectrum from the POs in a large time scale, and in tier-2, SUs trade the obtained spectrum among themselves in a small time scale. In [13], for example, the authors use Nash bargain games to derive the equilibrium prices for each tier. However, each tier is studied independently and the connection of tiers has not been explored yet.

In our previous work [17], we proposed a simple auction for spectrum sharing with heterogeneous channels. In this paper, we present a reserve price auction for a similar setting and we compare performance of these two models to see the effect of reservation prices. Despite all the previous work, the problem of designing a reserve price auction for spectrum sharing with heterogeneous channels and expressive bidding capability for SUs has not been addressed. Here, we tackle this problem.

III. System Model

In this paper, we consider a cognitive radio network consisting of a primary network and a secondary network. There is a primary spectrum owner (PO) (a base station or an access point) and a set of primary users (PU) in the primary network. The PO has some idle spectrum bands (or channels) that are not used by PUs. The secondary network consists of a set of secondary users (SUs), where each SU refers to a pair of secondary transmitter and receiver. The PO is willing to sell his idle channels to the SUs to obtain some profit, and SUs are willing to buy channels for their services. An example of cognitive radio network is depicted in Fig. 1.

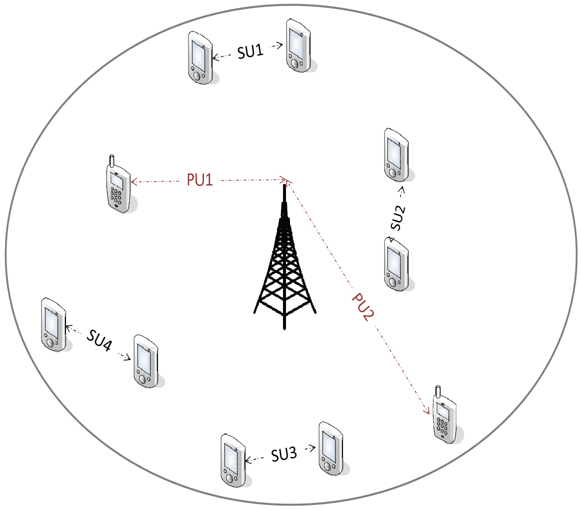

Fig. 1. A cognitive radio network with one primary owner, two primary users, and four secondary users.

We model the spectrum trading process as an auction in which the PO is the auctioneer and the SUs are the bidders. In our model, we consider heterogeneous channels, that is, channels have different qualities. The quality of channel j is defined as the maximum allowable transmission power on it, and is denoted by q_j. Also, there is a reservation price on each channel imposed by the PO. The reservation price on channel j is proportional to the quality of channel q_j and is defined as $r_j = \alpha q_j$ where $\alpha \geq 0$.

In our setting, each SU has a different view of the available channels. We allow SUs to express their preferences over each channel separately. Thus, each SU submits a vector of bids; one for each channel. Let m denote the number of available channels and n denote the number of SUs. Then, $V_i = (v_{i1}, v_{i2}, \ldots, v_{im})$ is the vector of bids submitted by SU i, consisting of m values for the available channels. The valuation matrix submitted to the PO will be of the following form:

$$W = \begin{pmatrix} V_1 \\ V_2 \\ \ldots \\ V_n \end{pmatrix}$$

For an SU, the valuation for a channel is defined as the benefit of obtaining that channel. We assume that SUs prefer channels with higher capacities. Therefore, SUs' valuation for a channel is related to the channel capacity, which is a function of channel quality, the interference coming from PUs, and the channel gain between the secondary transmitter and receiver. We define SU i's valuation for channel j as:

$$v_{ij} = B \log_2 \big(1 + q_j \frac{G_i}{I_i + \sigma^2}\big) \tag{1}$$

where B is the channel bandwidth, σ^2 is the noise variance, G_i is the channel gain between the SU i's transmitter and receiver, I_i is the interference coming from the PO and PUs. Without loss of generality, we assume that σ^2 is the same for all SUs.

It should be noted that valuations are private information of SUs, and it is not reasonable to assume that this information is known by other SUs or the PO. In fact, this is one reason that we use auction mechanisms. In this way, SUs declare their valuations, and by designing an Incentive Compatible (IC) auction we make sure that SUs do not have incentives to lie about their valuations.

We assume that each channel can only be used by one SU at a time. Also, each SU can only use one channel at a time. Let p_i denote the payment that SU i has to make if he gets a channel. Then, the utility of SU i, denoted by u_i, is defined as the difference between his valuation for the obtained channel, say channel j, and the price he has to pay, i.e. $u_i = v_{ij} - p_i$. Also, $u_i = 0$ if SU i does not get any channel. Another essential assumption in designing a truthful (or IC) auction is the rationality of bidders. That means that they want to maximize their own utilities. Therefore, an SU tries to obtain a channel with a price lower than his valuation for the channel.

IV. The Reserve Price Auction Mechanism

In this section, we derive a reserve price auction mechanism for spectrum sharing with certain guaranteed properties that we prove in this section. From the bidding perspective, the proposed auction is a one-shot auction where SUs submit their bids to the PO simultaneously, and the PO holds the auction, taking into account the bids and reservation prices collectively. SUs compute their valuations according to (1) after the PO announces the qualities of the available channels.

The reserve price auction takes the valuation matrix, W, and reservation prices $r = (r_1, r_2, \ldots, r_m)$ as inputs and determines two outputs; the channel allocation and the payments. The channel allocation is represented by an $n \times m$ matrix, denoted by X. Each element of the allocation matrix $x_{ij} \in \{0, 1\}$ indicates whether the channel j is allocated to SU i or not. That is, $x_{ij} = 1$ shows that the SU i has obtained the right to access channel j and $x_{ij} = 0$ otherwise. As mentioned in the system model, we assume that each channel can only be used by one SU at a time, and each SU can only use one channel at a time. Therefore, we impose the following constraints for a feasible allocation: $\sum_j x_{ij} \leq 1$, and $\sum_i x_{ij} \leq 1$. There is also another feasibility constraint because of reservation prices: $x_{ij} \cdot (v_{ij} - r_j) \geq 0$ that ensures channels are not allocated to SUs with valuations lower than the reservation prices. The auction should also determine the payments for each SU. We represent the payments by a payment vector $P = (p_1, p_2, \ldots, p_n)$ where p_i denotes the price that SU i has to pay.

The objective of the proposed reserve price auction is to maximize the *social income*. If net profit is defined as the difference of SU's valuation and the reservation price, the social income of an allocation $X = \{x_{ij}\}_{n \times m}$ can be defined as the aggregate net profits of this allocation. Formally, it can be written as:

$$S = \sum_i \sum_j x_{ij} \cdot (v_{ij} - r_j) \tag{2}$$

The allocation that maximizes users' valuations is referred to as an efficient allocation. The proposed mechanism allocates the channels efficiently subject to the reserve price constraints. The problem can formally be written as:

$$X^* = \arg\max_X S = \arg\max_X \sum_i \sum_j x_{ij} \cdot (v_{ij} - r_j) \quad (3)$$

$s.t.$

$$\sum_j x_{ij} \leq 1, \forall i$$
$$\sum_i x_{ij} \leq 1, \forall j$$
$$x_{ij} \cdot (v_{ij} - r_j) \geq 0, \forall i, \forall j$$
$$x_{ij} \in \{0, 1\}, \forall i, \forall j$$

where the constraints in the above formulation are the feasibility constraints that we discussed earlier. In the next subsection, we present a method to achieve an efficient allocation.

A. Efficient Channel Allocation with Reserve Price Constraints

Now we transform the problem of channel allocation, i.e. (3), into a maximum weight matching problem in graph theory [18]. Then, the problem can be solved using Kuhn-Munkres algorithm (also known as Hungarian algorithm) [19]. We first review some basic concepts from graph theory and the matching problem.

A *bipartite graph* is a graph whose vertices can be divided into two disjoint sets V_1 and V_2, such that every edge in the graph connects a vertex in V_1 to one in V_2. A *weighted graph* is a graph whose edges are associated with weights, usually a real number. In a bipartite graph, a *matching* is a subset of edges such that they do not share an endpoint. In other words, a matching is a subset of edges such that for each vertex, there is at most one edge in the matching that is incident upon this vertex.

Now, given a weighted bipartite graph, the problem of maximum weight matching is to find a matching with maximum weight. This is a well-studied problem in graph theory and it can be solved by the Kuhn-Munkres algorithm (or Hungarian algorithm) in polynomial time [19]. We do not present the details of the Kuhn-Munkres algorithm in this paper. Instead, we transform the original channel allocation problem, i.e. (3), into a maximum weight matching problem, and we show that these two problems are equivalent.

We can easily build a bipartite graph $G(V_1, V_2)$ by letting V_1 be the set of SUs and V_2 be the set of available channels. We draw an edge between SU i and channel j if SU i bids at least r_j. The weight of the edge ij is defined as the net profit of SU i getting channel j, i.e. $v_{ij} - r_j$.

We claim that X is a social income maximizing channel allocation if and only if M is a maximum weight matching in the constructed graph G. First, suppose there is a channel allocation X that maximizes the social income. Then each nonzero element of X corresponds to an edge in the maximum weight matching M. For example, $x_{ij} = 1$ means that channel j is allocated to SU i, so the edge ij will be in the matching. It should be noted that this set of edges form a matching, because each channel can only be allocated to one SU and each SU can only use one channel at a time (feasibility constraints for the allocation). Also, this is a maximum weight matching since the allocation X maximizes summation of net profits that correspond to edge weights in the graph.

Conversely, suppose that we have a maximum weight matching M in graph G, then the channel allocation matrix $X = \{x_{ij}\}_{n \times m}$ can be formed easily. For each edge ij in M, set its corresponding element in X to 1, i.e. $x_{ij} = 1$, and set all the other elements to zero. This results in a social income maximizing channel allocation. First, according to the definition of a matching, the resulting allocation matrix satisfies the feasibility constraints. Second, since edge weights in the graph represent net profits and M is a maximum weight matching, the resulting allocation maximizes the social income. It should be noted that reserve price constraints have already been taken into account when building the graph G.

In addition to the allocation rule, the proposed auction should specify the payment rule, i.e. the price each SU has to pay. In the next subsection, we provide details on the payment rule of the proposed auction.

B. The Payment Rule

The goal is to find a payment rule for the social income maximizing allocation that satisfies some desired economic properties (Incentive Compatibility, Individual Rationality and No Positive Transfers). We present the payment rule in this subsection and we discuss the economic properties in the next subsection.

We use a modified form of Vickrey Clarke Groves (VCG) mechanism with Clarke pivot payments [4]. VCG payments are applied to net profits (i.e. $v_{ij} - r_j$). Based on the VCG payment rule, SU i pays the externality he causes. In other words, SU i pays the difference between the social income of the others with and without his participation. We then add reservation prices to the VCG results. Let $X = \{x_{ij}\}_{n \times m}$ and $Y = \{y_{ij}\}_{n \times m}$ be social income maximizing allocations with and without SU i's participation, respectively. Then, the payment for SU i is calculated by the following formula:

$$p_i = \begin{cases} \sum_{j \neq i} \sum_k y_{jk} \cdot (v_{jk} - r_k) - \sum_{j \neq i} \sum_k x_{jk} \cdot (v_{jk} - r_k) + r_j \\ \qquad \text{If channel } j \text{ is obtained} \\ 0 \qquad \text{If no channel is obtained} \end{cases} \quad (4)$$

It is worth noting that in (4), valuations of SU i are excluded in the summations and SU i does not control his payment. This makes the mechanism robust against SUs' strategic behaviors. In the next subsection, we discuss the economic properties of the proposed auction.

C. Desired Economic Properties

It is desired for an auction to have certain economic properties. First, we formally define these properties, then we show that the proposed auction satisfies the desired economic properties.

- **Incentive Compatibility**; Let V_i be user i's true valuation vector and V_{-i} be the valuation vectors of all other users (excluding i). Let the utility of i be $u_i = \sum_j x_{ij} \cdot v_{ij} - p_i$ when V_i and V_{-i} are declared, and be $u'_i = \sum_j x'_{ij} \cdot v_{ij} - p'_i$ when V'_i and V_{-i} are declared. An auction is called incentive compatible if for every user i, every V_i and every V'_i we have $u_i \geq u'_i$. This is sometimes referred to as truthfulness, and states that the dominant strategy for users is to declare their true valuations regardless of what other users do.
- **Individual Rationality**; An auction is individually rational if for every user i, we have $u_i \geq 0$. That means, users do not suffer as a result of participating in the auction.
- **No Positive Transfers**; In an auction with no positive transfers we have $p_i \geq 0$, for every user i. This property prevents the auctioneer from having to pay agents.

Theorem 1: The proposed reserve price auction is incentive compatible, individually rational and has no positive transfers.

Proof: We first prove incentive compatibility. Using the payment rule, i.e. (4), utility of user i, when declaring V_i and V_{-i}, is $u_i = \sum_j x_{ij} \cdot (v_{ij} - r_j) + \sum_{j \neq i} \sum_k x_{jk} \cdot (v_{jk} - r_k) - \sum_{j \neq i} \sum_k y_{jk} \cdot (v_{jk} - r_k)$, but when declaring V'_i and V_{-i}, is $u'_i = \sum_j x'_{ij} \cdot (v_{ij} - r_j) + \sum_{j \neq i} \sum_k x'_{jk} \cdot (v_{jk} - r_k) - \sum_{j \neq i} \sum_k y_{jk} \cdot (v_{jk} - r_k)$. Since X maximizes social welfare among all the possible allocations, we have this inequality: $\sum_j x_{ij} \cdot (v_{ij} - r_j) + \sum_{j \neq i} \sum_k x_{jk} \cdot (v_{jk} - r_k) \geq \sum_j x'_{ij} \cdot (v_{ij} - r_j) + \sum_{j \neq i} \sum_k x'_{jk} \cdot (v_{jk} - r_k)$. Now, by subtracting the term $\sum_{j \neq i} \sum_k y_{jk} \cdot (v_{jk} - r_k)$ from both sides of the inequality, we get $u_i \geq u'_i$. Which is the incentive compatibility property.

Let $X = \{x_{ij}\}_{n \times m}$ and $Y = \{y_{ij}\}_{n \times m}$ be social income maximizing allocations with and without SU i's participation, respectively. To show individual rationality, consider the utility of user i:

$$
\begin{aligned}
u_i &= \sum_j x_{ij} \cdot (v_{ij} - r_j) + \sum_{j \neq i} \sum_k x_{jk} \cdot (v_{jk} - r_k) - \sum_{j \neq i} \sum_k y_{jk} \cdot (v_{jk} - r_k) \\
&\geq \sum_j \sum_k x_{jk} \cdot (v_{jk} - r_k) - \sum_j \sum_k y_{jk} \cdot (v_{jk} - r_k) \\
&\geq 0
\end{aligned}
$$

The first inequality holds since $\sum_j y_{ij} \cdot (v_{ij} - r_j) \geq 0$. The second inequality holds because $X = \{x_{ij}\}_{n \times m}$ is the allocation that maximizes the social income, $\sum_j \sum_k x_{jk} \cdot (v_{jk} - r_k)$.

Showing no positive transfers is quite easy. Using the payment rule, (4), we have $p_i = \sum_{j \neq i} \sum_k y_{jk} \cdot (v_{jk} - r_k) - \sum_{j \neq i} \sum_k x_{jk} \cdot (v_{jk} - r_k) \geq 0$, since $Y = \{y_{ij}\}_{n \times m}$ maximizes the social income without i's participation, $\sum_{j \neq i} \sum_k y_{jk} \cdot (v_{jk} - r_k)$.

■

V. Numerical Results

In this section, we evaluate the performance of the proposed auction mechanism in different network scenarios. To determine channel gain between a secondary transmitter and receiver, we use a simple signal propagation model $G_i = (\frac{c}{f})^a \cdot d_i^{-a}$, where c is the speed of light, f is the center frequency of the channel, d_i is the distance between SU i's transmitter and receiver, and a is the path-loss exponent. f is set to 1 GHz, a is set to 2 and d_is are randomly chosen from Uniform distribution U[25,50].

We assume unit bandwidth demand, i.e. $B = 1$, noise variance is chosen to be $\sigma^2 = 10^{-6}$ and I_i, the interference coming from the PO and PUs is randomly drawn from $U[10^{-7}, 10^{-6}]$. Also, channel qualities (maximum allowable transmission powers) are randomly drawn from Uniform distribution U[0.01,1]. α, the factor in reservation prices is set to be 3. We change number of SUs and number of channels and we run each setting 500 times in MATLAB. At first, SUs compute their valuations according to the (1). Then, a bipartite graph is formed considering reserve price constraints, the Hungarian algorithm is used to determine channel allocations. knowing the allocations, we determine payments using (4).

The performance of the proposed reserve price auction is compared with the case of identical channels where all the channel qualities are set to a mean value. Social income, average payment of SUs, average utility of SUs, and revenue of the PO are considered as performance metrics, where revenue of the PO is defined as the sum of SU payments $\sum_i p_i$.

In Fig. 2 social income is depicted versus number of SUs when there are 3 channels available. As can be seen, social income increases with number of SUs. With more SUs participating in the auction, we have wider range of valuations, and since the auction favors SUs with high valuations, the winners have higher valuations that results in higher social income.

The average payment of SUs is depicted in Fig. 3. We observe that as the number of SUs increases and channel access becomes more competitive, payments increase. This is because with more competitors, winning SUs cause more externality, and consequently they have to pay more. This competitive environment is not favorable for SUs. Fig. 4 shows that the average utility of SUs decreases with the number of SUs. That happens because of the increase in SU payments, resulting in lower utilities.

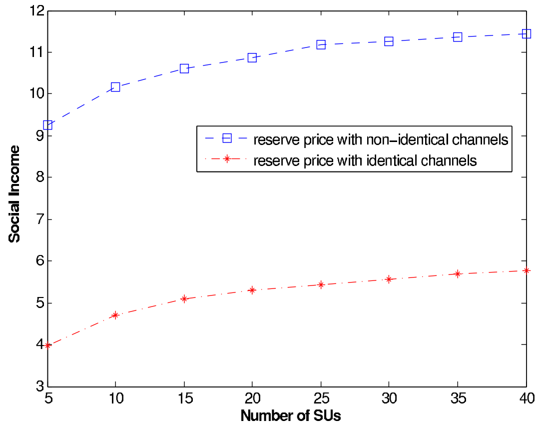

Fig. 2. Social Income (equation 2) versus the number of SUs, with fixed number of channels m=3.

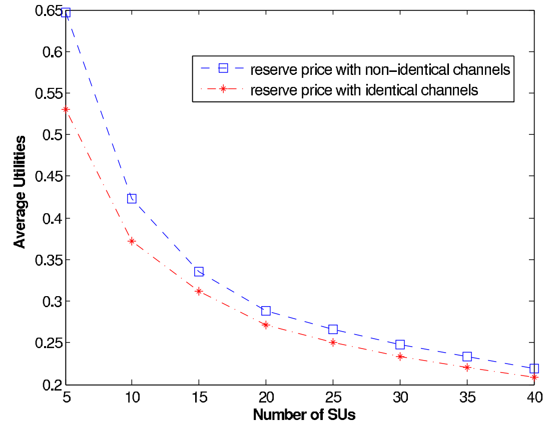

Fig. 4. Average utilities versus the number of SUs, with fixed number of channels m=3.

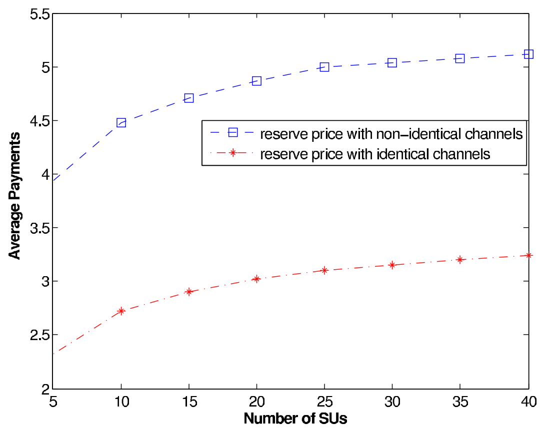

Fig. 3. Average payments versus the number of SUs, with fixed number of channels m=3.

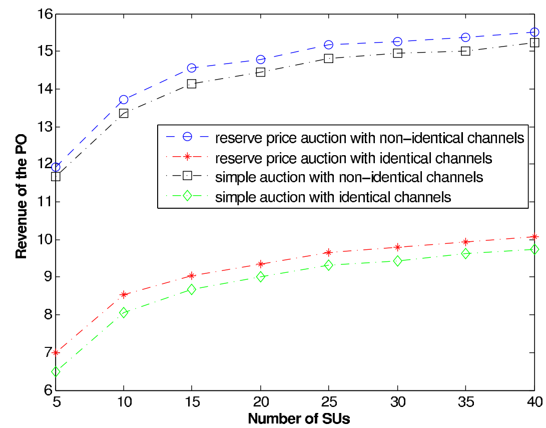

Fig. 5. Revenue of the PO versus the number of SUs, with fixed number of channels m=3.

Fig. 5 shows the revenue of the PO versus number of SUs using four different types of auctions; reserve price auction with non-identical channels, reserve price auction with identical channels, simple auction (with no reservation prices) with non-identical channels and simple auction with identical channels. As can be seen, revenue of the PO increases with number of SUs. This is because with more SUs channel access becomes more competitive, consequently winning SUs cause more externality and they have to pay more to the PO. Also, from Fig. 5 we observe that reserve price auction with non-identical channels results in the highest revenue among the four types of auctions. When reservation prices are set properly, reserve price auction brings high revenue for the auctioneer, especially when bidders are asymmetric and we have a wide range of valuations. In these cases, VCG like payments may be too low, but in reserve price auction the auctioneer is assured of getting at least the reservation price.

Fig. 6 also shows the revenue of the PO using the same four types of auctions. Here, the number of SUs are fixed at 25 and number of channels changes from 3 to 18. This figure shows that revenue of the PO increases with number of channels. That is because the PO sells more items (channels) and gets higher revenue, even though each item's price is slightly lowered (because of less competition). Similar to Fig. 5 reserve price auction with non-identical channels results in the highest revenue compared to the other auctions.

It is worth noting that in all the above diagrams, auctions designed for non-identical channels (i.e. channels with different qualities) considerably perform better than the case of identical channels. With non-identical channels, SUs can better express their needs and we get a wide range of valuations. Since the auction favors SUs with higher valuations, winners in the non-identical channels case have higher valuations compared to the winners in the identical channels case. Therefore, we get higher social income and higher revenue for the PO, in addition to the improved utilities for SUs.

An important problem for the auctioneer is choosing reservation prices. If reservation prices are too low, the reserve price auction reduces to the simple auction case, and if reservation prices are set to be high, then a large group of bidders will be excluded since their valuations are below the reservation prices. Fig. 7 shows how revenue of the PO changes when α (the factor in reservation prices $r_j = \alpha q_j$) increases. We observe that revenue slightly increases up to some point and drops afterwards. This point is like a threshold above which many bidders are excluded because their valuations are lower than the reservation prices. An interesting point is that revenue from the auction with non-identical channels drops at a higher α compared to the identical channels case. This happens

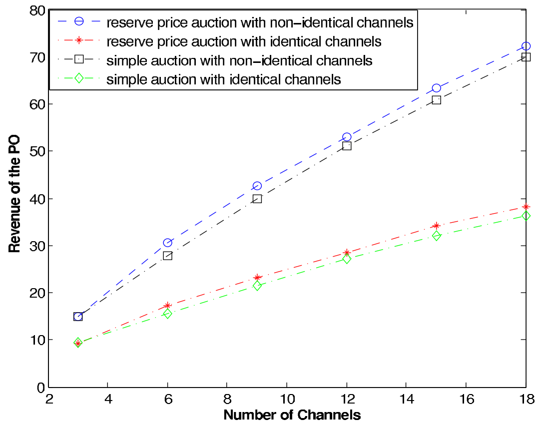

Fig. 6. Revenue of the PO versus the number of channels, with fixed number of SUs n=25.

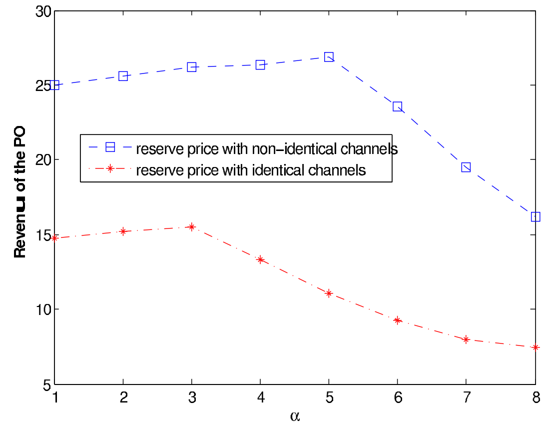

Fig. 7. Revenue of the PO versus α, with 15 SUs and 6 channels.

because of more diverse valuations in non-identical channels auction and the fact that winners have higher valuations.

VI. Conclusion

In this paper, we studied the problem of spectrum sharing with heterogeneous channels. A reserve price auction has been proposed where the SUs bid to buy spectrum bands from the PO who acts as the auctioneer. Unlike most existing auction mechanisms that assume identical channels, we have considered a more general case where channels have different qualities. Also, in our setting, SUs are allowed to express their preferences for each channel separately. In addition, reservation prices that are proportional to channel qualities are imposed by the PO. The proposed reserve price auction results in efficient allocation that maximizes SUs' valuations subject to reserve price constraints, and it has desired proven economic properties. Numerical results have shown performance improvements compared to the case of reserve price auction with identical channels and the case of having no reservation prices. As discussed in numerical analysis section, revenue of the PO can be increased by properly choosing the reservation prices. In this paper, we considered a simple form for reservation prices that was based on channel qualities. A possible direction for future work is to find optimal reservation prices that yield the maximum revenue among all the social income maximizing allocations.

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