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Opportunistic Scheduling and Relaying in a Cooperative Cognitive Network

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***Abstract*—This paper considers network-layer cooperation in cognitive radio networks whereby secondary users can relay primary user's packets, in return for a more favorable spectrum access rules. Under this cooperative scheme, the paper investigates whether, and under what conditions, the primary and secondary networks can be stabilized without explicit knowledge of the packet arrival-rates. We consider a deterministic and periodic primary packet arrival process and develop a relaying and scheduling algorithm using Lyapunov drift techniques that does not require knowledge of primary and secondary packet arrival rates. The algorithm is then shown to stabilize the transmission queues in the network for all secondary packet arrival rates that lie in the interior of a certain region. The region includes all secondary arrival-rate vectors that can be supported when the secondary nodes do not cooperate. Furthermore, when the primary data arrival-rate is greater than what could have been supported without relays but less than what can be maximally supported with relays, the algorithm stabilizes the network for a non-empty set of secondary arrival-rate vectors. The significance of these results is that they show that properly designed cooperation may result in a win-win scenario for both primary and secondary users (and not just for one type of users). Finally we extend our analysis to the case of a deterministic but aperiodic primary packet arrival process.**

I. Introduction

The increase in the number of wireless devices has resulted in increasing demand for wireless spectrum. However at any given time, the licensed spectrum is often under-utilized. This observation has led to the widespread study of cognitive radio networks whereby unlicensed or secondary users can opportunistically access the spectrum when it is not being used by primary or licensed users. Typically the primary and secondary networks are thought to be non-cooperative i.e. the users in the respective networks do not assist in each other's transmissions. However if the secondary nodes somehow assist the transmission of primary users, it may reduce the amount of time the channel is occupied by primary users. This in turn may increase transmission opportunities for secondary users.

Cooperation between primary and secondary networks has been widely studied from a physical-layer perspective (e.g.- [1], [2], [3]). However interactions between primary and secondary users also affect higher-layer operations such as queuing and prioritized scheduling (since primary users enjoy higher priority of access to the channel). The above works do not address this issue. Furthermore, the fact that the actions by a cooperating secondary node can now influence the primary user-channel occupancy process has received very little attention [4]. Some works that did study the problem from a network-layer perspective are [5], [6], [7] and [8]. In addition, in [4] the authors find optimal cooperative power allocations in a network of multiple secondary users and a single primary user. However, a key assumption in their analysis is valid only for the range of primary packet arrival rates for which the primary network is stable even without any assistance from the secondary nodes. In [9] the authors extend the above work to include cases of higher primary packet arrival rates.

Consider cooperation whereby secondary users can help primary users without harming primary user traffic (otherwise cooperation would be simply disabled). In general, this type of cooperation will always be at least beneficial for primary users. However, a question remains on whether this would be beneficial also to secondary users. Intuitively, it can be seen that some secondary users may benefit while others may not. For example, some secondary users located close to a secondary relay node may obtain fewer transmission opportunities when there is cooperation. This occurs due to increased transmission activity by the secondary relays. None of the above works addressed this issue. Besides, the above works have also not considered more general network models where multiple secondary users can transmit simultaneously. In our work we address these issues.

An important network problem is to find scheduling algorithms for which the network is stable i.e. lengths of all queues in the network are bounded. Therefore, in the cooperative cognitive radio paradigm we address the following question: whether and under what conditions a general network consisting of a single primary link and multiple secondary users, few of which can act as relay for the primary user, can be stabilized without explicit knowledge of the arrival-rates. The primary packet generation rate in our model can be *greater* than what is supported by the primary network alone. We develop an algorithm that achieves this goal for networks with a *deterministic, periodic primary packet generation process*. Deterministic packet generation process has been used previously in [10] in the context of max-weight based throughput-optimal scheduling policies. We account for the *trade-off* *between extent of cooperation and throughput* of individual secondary users. We observe that the work in [9] is closest to our work. However in addition to the differences mentioned in the previous paragraph, there is the following difference between their work and ours. In [9] the authors maximize a function of throughputs of secondary users by solving a convex optimization problem with the knowledge of primary packet arrival rate. On the other hand we attempt to find a scheduling algorithm that stabilizes the network by solving a max-weight problem with knowledge of only instantaneous queue-lengths and inter-arrival time of the Head-of-Line packet (H.O.L) at primary transmitters (in this work we refer to the time difference between the arrival time of a given packet at the primary source node and that of the previous primary packet to be the *inter-arrival time* of the former packet). Thus our work is in accordance with the wide body of works on max-weight based scheduling policies for communication networks that require knowledge of only instantaneous queue-states.

The remaining of the paper is organized as follows. Section II describes our system model. In Section III we outline our objective. In Section IV we propose our scheduling algorithm that makes scheduling decisions every time-slot based only on the knowledge of the instantaneous queue lengths and inter-arrival time of packets at the primary source node. In Section V we show the stability of the network under this algorithm for all secondary arrival-rates within a well-defined region. The primary packet arrival process mentioned in Section II is periodic. Therefore the corresponding primary data arrival rate is a rational number. In Section VI we extend our analysis to a case where the primary data arrival rate can be an irrational number and the resultant primary packet arrival process is deterministic but aperiodic. Section VII concludes the paper. Due to space-constraints the proofs are omitted and can be found in the technical report [11]. A preliminary version of this work was presented at Allerton 2013 in Monticello, Illinois.

II. System model

We consider a single primary source-destination (s-d) pair in the presence of multiple secondary s-d pairs with one or more secondary node(s) that can act as relay for primary traffic. We assume there is one primary transmitter (PT) and S secondary transmitters- ST_1, $ST_2, ..., ST_S$. PR and SR_i denote the primary receiver and the secondary receiver corresponding to ST_i respectively (where $i = 1, 2, ..S$).

We assume that primary users are aware of the existence of the secondary network and can therefore request cooperation from the latter to improve latency of transmitted primary packets all the while preserving high transmission priority for primary packets. We also assume that packets can be transmitted across multiple time-slots where the length of a time-slot is defined appropriately. Both assumptions are similar to the *spectrum leasing* model of cognitive radio which has been used in works such as [12], [13] and [14]. In those works it is assumed that a time-slot used for direct transmission of data from a primary user to a primary destination can be further divided into smaller intervals in which transmissions

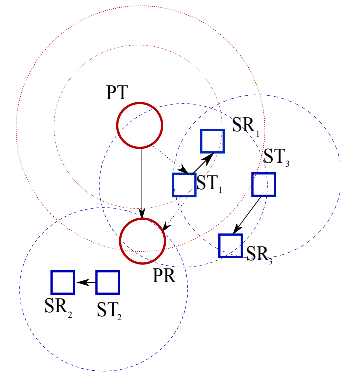

Fig. 1. A network with one primary s-d pair and three secondary s-d pairs. Each of the three blue-dashed circles with one of the ST_is (where $i = 1, 2, 3$) as center has radius $d_{s,i}$. No node within the circle drawn with ST_i as center can simultaneously receive a packet from any node except ST_i when ST_i is transmitting. The larger and smaller red, dotted circles drawn with PT as centre has radius $d_{\Upsilon_{dir}}$ and $d_{\Upsilon_{rel}}$ respectively. No node located within the larger circle, except PR, can receive a packet when PT is directly transmitting a packet. No node located within the smaller circle, except ST_1, can receive a packet when PT is relaying a packet. The dotted lines from PT to ST_1 and from ST_1 to PR represent cooperative transmission with ST_1 as a relay node.

from a primary user to a relay node, from relay node to primary destination and possible transmission of secondary network's own data takes place. We assume a similar model here. However, [12], [13] and [14] study the cooperative relaying problem from a physical-layer perspective and do not investigate the network-layer aspects. We assume ideal sensing process i.e. the sensing results are always accurate and take place in an infinitesimal time-duration.

Details of our system model is presented next.

A. Primary packet transmission model

1) Identifying the set of relay nodes: We assume that PT always transmits at fixed power Υ_{dir} to PR. PT can also transmit a packet to some secondary transmitters at fixed power Υ_{rel}, where $\Upsilon_{rel} < \Upsilon_{dir}$. The transmission range corresponding to transmission powers Υ_{dir} and Υ_{rel} are denoted by $d_{\Upsilon_{dir}}$ and $d_{\Upsilon_{rel}}$ respectively. Without loss of generality we assume that ST_1, ST_2,..., $ST_{S_{rel}}$ (where $1 \leq S_{rel} \leq S$) are the secondary transmitters that are within a distance $d_{\Upsilon_{rel}}$ from PT and can therefore receive packets from PT. We assume that a secondary node ST_i (where $1 \leq i \leq S$) uses transmission power $\Upsilon_{s,i}$ to transmit any packet and denote the corresponding transmission range by $d_{s,i}$. We assume that PR is located within distance $d_{s,j}$ from ST_j (where $1 \leq j \leq S_{rel}$) and therefore ST_1,...,ST_{rel} can act as relay for primary traffic.

2) Definition of a link and assumptions about transmissions using relay nodes: A link is defined by the ordered pair (l_1, l_2) such that, under the transmission power scheme mentioned in the previous subsection, packets can be transmitted from node l_1 to node l_2. Let L denote the set of all possible links i.e. $L = \{(PT, PR), (PT, ST_i), (ST_i, PR), (ST_j, SR_j) : 1 \leq i \leq S_{rel}, 1 \leq j \leq S\}$. We denote the set of links that are used exclusively for primary packet transmission by L_p

i.e. L_p={(PT, PR), (PT, ST_i), $(ST_i, PR) : 1 \leq i \leq S_{rel}$}.

We assume the capacity of any link is a rational number. As a result the length of a time-slot can be defined s.t. the time taken to transmit a primary packet through any link $(l_1, l_2) \in L_p$ is a multiple of the length of a time-slot. We denote by $K_{(l_1,l_2)}$ the number of time-slots required to transmit a primary packet through link (l_1, l_2). We assume that the channel quality for the direct link (PT, PR) is somewhat poor (eg- due to fading) and PT is power-limited with Υ_{dir} being the maximum power. On the other hand, the channel qualities for the links connecting PT and PR to the secondary transmitters- ST_1,.., $ST_{S_{rel}}$ are assumed to be relatively better. As a result even when PT transmits to any of those secondary nodes using lower power Υ_{rel}, the overall latency for a primary packet is still better than one ontained from using direct link. Mathematically we have,

$$K_{(PT,ST_i)} + K_{(ST_i,PR)} \leq K_{(PT,PR)} \quad \forall 1 \leq i \leq S_{rel} \quad (1)$$

In general the number of secondary nodes that can relay a primary packet with better latency than the direct link can be less than S_{rel}. In that case one can simply redefine S_{rel} to be the number of relay nodes providing better latency to primary packets without significantly affecting rest of the analysis.

3) Constraint on primary packet scheduling: PT transmits packets whenever its buffer is non-empty. If PT begins transmitting a packet to PR directly at slot t, then clearly for time-slots t, $t+1$,..,$t + K_{(PT,PR)} - 1$ it is busy transmitting the packet. Instead if at slot t the packet is scheduled to be relayed via ST_j (where $1 \leq j \leq S_{rel}$), then PT transmits the packet to ST_j during slots t, $t+1$,..,$t + K_{(PT,ST_j)} - 1$. During slots $t+K_{(PT,ST_j)}$, $t+K_{(PT,ST_j)}+1$,..,$t+K_{(PT,ST_j)} + K_{(ST_j,PR)} - 1$, node ST_j relays the packet to PR. Due to (1) cooperative relaying always reduces latency of primary packets as compared to direct transmission.

Fig. 1 shows example of a network with $S_{rel} = 1$, S=3.

B. Primary packet arrival model

We assume within every time-slot, $\lambda_p b_p$ bits, where $\lambda_p \in \mathbb{Q}$ and $\mathbb{Q}$ denotes the set of rational numbers, arrive at constant rate from the upper layers of PT to the transmission layer. Whenever the accumulated data is greater than b_p bits, those b_p bits are aggregated as a primary packet and moved to the transmission queue of PT. Let $A_p(t) \in \{0, 1\}$ denote the number of primary packet arrivals in slot t. For example, when $\lambda_p = \frac{5}{13}$ and there are 0 bits at PT initially, then $A_p(t)$ starting from $t = 1$, is the sequence: 0, 0, 1, 0, 0, 1, 0, 1, 0, 0, 1, 0, 1, 0, 0, 1,... . The process is periodic with a period of 13 slots. The inter-arrival time of the first and second packet is 3 slots and that of the third one is 2 slots.

C. Secondary packet arrival and transmission model

We assume that every time-slot with probability $\lambda_{s,i}$ $(i = 1, 2, .., S)$ a secondary packet arrives at the link layer of ST_i from the node's upper layers. For simplicity we assume all secondary transmitter-receiver links have same capacity of 1 packet per slot.

D. Interference model

Our interference model is based on the protocol model of interference whereby a node can transmit to another node within its transmission range. The transmission is successful only if the latter is not within range of another node (including itself) that is transmitting in the same slot. In any slot, a link $(l_1, l_2) \in L$ is said to be *active* if node l_1 is successfully transmitting (i.e. without facing any interference from other nodes) to node l_2; otherwise it is said to be *inactive*. Due to the interference constraints not all links in the network can be simultaneously active. We represent a set of links which can be active simultaneously by an *activation vector*. An activation vector is binary and its length is equal to the total number of possible links i.e. $S + 2S_{rel} + 1$. Without loss of generality the activation vectors are ordered such that the first $2S_{rel} + 1$ components correspond to links that are used to transmit primary packets, while the remaining components correspond to links used to transmit secondary packets. In particular, in any activation vector $\boldsymbol{E}$, the first component corresponds to the link (PT, PR); the j-th and $(j + S_{rel})$-th component (where $1 \leq j \leq S_{rel}$) of $\boldsymbol{E}$ corresponds to links (PT, ST_j) and (ST_j, PR) respectively; the $(i+2S_{rel}+1)$-th component (where $1 \leq i \leq S$) corresponds to link (ST_i, SR_i) respectively. Any component in the activation vector is set to 1 if the corresponding link is active, otherwise it is set to 0. An activation vector $\boldsymbol{E}$, feasible under protocol model of interference, is constructed by setting any of its component $\boldsymbol{E}_e$, corresponding to link $(l_{1e}, l_{2e}) \in L$, to 1 only if $\boldsymbol{E}_{e'} = 0$ for every e' such that $(l_{1e'}, l_{2e'}) \in L$ and l_{2e} is within transmission range of $l_{1e'}$.

The set consisting of all feasible activation vectors is denoted by χ. We denote the set of all feasible activation vectors, in which the component corresponding to a given link $(l_1, l_2) \in L$ is active, by $I(l_1, l_2)$ i.e. $I(l_1, l_2)$={ $\boldsymbol{E} \in \chi$: $\boldsymbol{E}_e = 1$, $1 \leq e \leq S + 2S_{rel} + 1$, $\boldsymbol{E}_e$ corresponds to link (l_1, l_2)}.

E. Queuing model

Let $U_p(t)$, $U_{s,i}(t)$ denote the queue-length of PT and ST_i $(i = 1, 2, .., S\ )$ at slot t. $U_p(t)$ evolves as

$$U_p(t + 1) = U_p(t) - C(t) + A_p(t), \quad (2)$$

where $C(t)$ is an indicator variable which is 1 if a primary packet transmission is completed at t and is 0 otherwise. The queues for ST_i evolve as:

$$U_{s,i}(t + 1) = \max[U_{s,i}(t) - \mu_{s,i}(t), 0] + A_{s,i}(t), \quad (3)$$

where $\mu_{s,i}(t) \in \{0, 1\}$ is the transmission rate offered (in secondary packets/slot) to ST_i at t for a secondary packet transmission to SR_i. $A_{s,i}(t)$ indicates the number of secondary packet arrivals to ST_i at t .

The offered secondary transmission rate to a secondary transmitter in any time-slot is a binary variable. Therefore the offered secondary transmission rate vector in any time-slot can be obtained from the binary activation vector used in that slot by simply eliminating from the latter the components corresponding to links used to transmit primary packets. The offered secondary transmission rate vector obtained from a feasible activation vector $\boldsymbol{E} \in \chi$, by eliminating its first $2S_{rel}+1$ components, is denoted by $\Pi(\boldsymbol{E})$.

If any link is used to transmit primary packets in any time-slot, it constrains the set of transmission rate vectors that can be offered to secondary users in that slot for their own transmissions. We note atmost one link in L_p can be active in any time-slot. Let $I'(l_1,l_2)$ denote the set of all transmission-rate vectors that can be offered to $ST_1,...,ST_S$ at slot t if the link (l_1,l_2), where $(l_1,l_2) \in L_p$, is active i.e. $I'(l_1,l_2)$=$\{\Pi(\boldsymbol{E}) : \boldsymbol{E} \in I(l_1,l_2)\}$.

For the particular case when no node in the network is transmitting a primary packet in some time-slot, the set of transmission-rate vectors that can be offered to secondary users in that slot is denoted by I'_0. This set can be written as,

$$I'_0 = \{\Pi(\boldsymbol{E}) : \boldsymbol{E} \in \chi, \quad \boldsymbol{E}_e = 0 \quad \forall 1 \le e \le 2S_{rel}+1\} \quad (4)$$

F. Scheduling and control model

Whenever PT is about to transmit a new packet, a decision needs to be made about whether the packet is transmitted directly to PR or it will be relayed to PR by a cooperating secondary node. Depending on that decision the scheduling for the next few slots is performed accordingly, subject to the interference constraints mentioned in Section II-D. For example, if the decision is to relay the primary packet via ST_i, then for next $K_{(PT,ST_i)}$ slots the offered secondary transmission-rate vectors belong to the set $I'(PT,ST_i)$. For the subsequent $K_{(ST_i,PR)}$ slots the offered secondary transmission-rate vectors belong to the set $I'(ST_i,PR)$.

We note that for the example in Fig. 1 with $K_{(PT,ST_1)} = K_{(ST_1,PR)} = 1$ and $K_{(PT,PR)} = 3$, ST_2 always benefits from cooperation while ST_3 always suffers due to cooperative relay.

III. Stability Objective

In this section, we observe some properties of the primary packet arrival process and use them to describe a region consisting of secondary arrival-rate vectors. Later we will propose an algorithm that guarantees network stability for arrival-rate vectors in this region.

Let f_i (where $1 \le i \le S_{rel}$) denote the maximum primary arrival-rate λ_p that can be supported if every primary packet is transmitted via relay ST_i i.e. $f_i = \frac{1}{K_{(PT,ST_i)}+K_{(ST_i,PR)}}$. Without loss of generality we assume the ST_1,...,$ST_{S_{rel}}$ are indexed such that $f_j \le f_{j+1}$ $\forall 1 \le j \le S_{rel}-1$. Let f_0 denote the maximum primary arrival-rate λ_p that can be supported if every primary packet is directly transmitted i.e. $f_0 = \frac{1}{K_{(PT,PR)}}$. By our assumption in Section II-A, $f_0 \le f_1$.

Since $\lambda_p \in \mathbb{Q}$, $A_p(t)$ is periodic. Let N denote the length of shortest period of $A_p(t)$; let M denote the number of primary packet arrivals in that period. Then λ_p can be expressed as $\lambda_p = \frac{M}{N}$ and M, N are prime to each other. We note if $\frac{1}{k_1+1} \le \lambda_p \le \frac{1}{k_1}$ (where $k_1 \in \mathbb{Z}^+$), the inter-arrival time between any two primary packets is no greater than k_1+1 slots and no lesser than k_1 slots. For such λ_p we denote by $\kappa^{(1)}(\lambda_p)$ and $\kappa^{(2)}(\lambda_p)$ the number of primary packet arrivals within any interval of length N slots with inter-arrival time of k_1+1 and k_1 slots respectively. Then we have

$$\kappa^{(1)}(\lambda_p) + \kappa^{(2)}(\lambda_p) = M \quad (5)$$

$$(k_1+1)\kappa^{(1)}(\lambda_p) + k_1\kappa^{(2)}(\lambda_p) = N \quad (6)$$

For a given primary data arrival-rate $\lambda_p \in \mathbb{Q}$ and $\frac{1}{k_1+1} \le \lambda_p < \frac{1}{k_1} \le f_{S_{rel}}$ (where $k_1 \in \mathbb{Z}^+$), define a region[1] $\Lambda(\lambda_p)$ as the set of secondary arrival rate vectors $(\lambda_{s,1}, \lambda_{s,2}, ..., \lambda_{s,S})^T$ for which there exists variables $R_{s,1}, ..., R_{s,S}$ and $\pi_0, \pi^{(i)}_{(l_1,l_2)}$ where $(l_1,l_2) \in L_p$, $i = 1,2$ such that:

$$\frac{\kappa^{(i)}(\lambda_p)}{N} = \frac{\pi^{(i)}_{(PT,PR)}}{K_{(PT,PR)}} + \sum_{1 \le j \le S_{rel}} \frac{\pi^{(i)}_{(PT,ST_j)}}{K_{(PT,ST_j)}} \quad \forall i = 1,2 \quad (7)$$

$$\pi_0, \pi^{(i)}_{(l_1,l_2)} \ge 0 \quad \forall (l_1,l_2) \in L_p, \quad i = 1,2 \quad (8)$$

$$\frac{\pi^{(i)}_{(PT,ST_j)}}{K_{(PT,ST_j)}} = \frac{\pi^{(i)}_{(ST_j,PR)}}{K_{(ST_j,PR)}} \quad \forall i = 1,2, \ 1 \le j \le S_{rel} \quad (9)$$

$$\pi^{(1)}_{(PT,PR)} = 0, \quad \text{if } K_{(PT,PR)} > k_1+1 \quad (10)$$

$$\pi^{(2)}_{(PT,PR)} = 0, \quad \text{if } K_{(PT,PR)} > k_1 \quad (11)$$

$$\pi^{(1)}_{(PT,ST_j)} = 0 \quad \forall 1 \le j \le S_{rel}, \quad \text{if } K_{(PT,ST_j)} + K_{(ST_j,PR)} > k_1+1 \quad (12)$$

$$\pi^{(2)}_{(PT,ST_j)} = 0 \quad \forall 1 \le j \le S_{rel}, \quad \text{if } K_{(PT,ST_j)} + K_{(ST_j,PR)} > k_1 \quad (13)$$

$$\pi_{(l_1,l_2)} = \pi^{(1)}_{(l_1,l_2)} + \pi^{(2)}_{(l_1,l_2)} \quad \forall (l_1,l_2) \in L_p \quad (14)$$

$$\pi_0 + \sum_{(l_1,l_2) \in L_p} \pi_{(l_1,l_2)} = 1 \quad (15)$$

$$\lambda_{s,i} \le R_{s,i} \quad \forall i = 1,2,...S \text{ for some } (R_{s,1},...,R_{s,S})^T \in \Gamma \quad (16)$$

$$\text{where } \Gamma = \pi_{(PT,PR)} \ conv(I'(PT,PR)) + \sum_{j=1}^{S_{rel}} (\pi_{(PT,ST_j)} \ conv(I'(PT,ST_j)) + \pi_{(ST_j,PR)} \ conv(I'(ST_j,PR))) + \pi_0 \ conv(I'_0) \quad (17)$$

Terms of form $\pi^{(1)}_{(x,y)}$ represents the long-term average probability of the event - "node x is directly transmitting a packet with inter-arrival time of k_1+1 slots to node y". $\pi^{(2)}_{(x,y)}$ represents long-term average probabilities for similar events for primary packets with inter-arrival time of k_1 slots.

The equality constraint (7) is a conservation constraint which indicates that arrival rate of primary packets of either type is equal to their departure rate from PT. Constraint (9) represents that the average number of primary packets of either

[1] This formulation is similar to the capacity region description in [15].

type that enter any relay node is equal to that transmitted by the relay node to PR. Additional constraints are introduced in (10)-(13) which require that primary packets with inter-arrival times of k_1+1 and k_1 slots are not transmitted directly or via a relay if such a transmission takes more than k_1+1 and k_1 slots respectively. These properties are required to use renewal-frame based techniques (to be introduced later) which in turn leads to tractable analysis. They also serve a realistic purpose by imposing a deadline constraint on relayed primary packets. Terms of form $\pi_{(x,y)}$ and π_0 represents the long-term average probabilities of the events - "x is directly transmitting a packet to y" and "no primary packet is being transmitted by any node" respectively. The inequality constraint (16) represents the stability condition for secondary transmitters. The "+" operator in (17) indicates Minkowski addition of sets i.e. a set formed by adding every element in one set to every element in another set [16]. The "*conv*" of a set of vectors is the set of all possible convex combinations of its elements. The set Γ in equation (17) characterizes the set of feasible secondary transmission-rate vectors subject to the scheduling and interference constraints mentioned in Section II.

When $\lambda_p \in \mathbb{Q}$ and $\lambda_p < f_{S_{rel}}$, let $\Lambda_0(\lambda_p)$ denote the set of secondary arrival-rate vectors for which the network is stable under any non-cooperative algorithm. This set can be obtained by setting $\pi^{(i)}_{(PT,ST_j)}, \pi^{(i)}_{(ST_j,PR)} = 0$ for every $1 \le j \le S_{rel}$, $i = 1,2$ in (7)- (17). Clearly the set is empty for $\lambda_p > f_0$.

Our main contribution in this work is to develop a scheduling algorithm that guarantees stability of the network for all secondary arrival rates in the interior of $\Lambda(\lambda_p)$ when $\lambda_p < f_{S_{rel}}$, $\lambda_p \in \mathbb{Q}$. The guaranteed stability region of the algorithm includes the capacity region corresponding to the non-cooperative case (ignoring the secondary arrival-rate vectors that form the boundary of $\Lambda_0(\lambda_p)$ for any $\lambda_p \le f_0$). When $f_0 < \lambda_p < f_{S_{rel}}$ and $\lambda_p \in \mathbb{Q}$, the interior of $\Lambda(\lambda_p)$ can include a non-empty set of secondary arrival-rate vectors whose j-th component is non-zero. The network may not even be stabilizable for these arrival-rate vectors without cooperation when λ_p was f_0. The proposed algorithm therefore results in a win-win scenario for such an ST_j and PT. For the example in Fig. 1, if $K_{(PT,ST_1)} = K_{(ST_1,PR)} = 1$ and $K_{(PT,PR)} = 3$, cooperation can result in win-win scenario for ST_2 and PT. For every $\lambda_p \in \mathbb{Q}$, ignoring the set of arrival-rate vectors that are at the boundary of $\Lambda_0(\lambda_p)$ whenever it is non-empty, the set of secondary arrival-rate vectors that can be stabilized is therefore expanded under this cooperative scheduling algorithm.

IV. Dynamic relaying and scheduling policy

In this section we develop a dynamic Scheduling and Cooperative Relay Policy (SCRP) that, for all $\lambda_p \in \mathbb{Q}$, $\lambda_p < f_{S_{rel}}$, satisfies the stability objective mentioned in the previous section. If in any slot the transmission queue at PT is non-empty and no primary packet is being transmitted by any node in the network, the network controller schedules transmission of a primary packet from PT either directly or via some secondary relay node by solving a max-weight problem. The algorithm uses information about instantaneous queue-lengths at secondary transmitters and the inter-arrival time of the H.O.L primary packets at PT. The H.O.L packet is transmitted via a secondary relay or directly such that the overall transmission time is less than or equal to inter-arrival time of the packet. The offered secondary transmission rate vectors are then obtained by solving a related max-weight problem.

If at the current slot, there is no primary packet at PT, it is considered an *idle slot*. All the slots when the transmission of j-th primary packet ($j = 1,2,....$) takes place is said to constitute the j-th *busy period*. Such a busy period always consists of contiguous time-slots because according to our primary transmission model, every time a secondary relay node receives a primary packet it begins transmitting the same in the very next slot. Any time-slot when the network is in a busy period is called a *busy slot*.

Every time-slot the network controller observes the queue-length of PT, ST_i (where $i = 1,2,...,S$) and the inter-arrival time of primary packets present at PT. Let $(U_{s,i}(t))_{i=1}^S$ and $(\mu_{s,i}(t))_{i=1}^S$ denote the queue-length vector $(U_{s,1}(t),...,U_{s,S}(t))^T$ and offered secondary transmission rate-vector $(\mu_{s,1}(t),...,\mu_{s,S}(t))^T$ at slot t. Based on the above knowledge, the algorithm makes the following scheduling and relay decisions:

1) *Scheduling decision in idle slots:* At any idle slot t, the network assigns a secondary transmission-rate vector $(\mu_{s,i}(t))_{i=1}^S$ according to a max-weight scheduling policy:

$$(\mu_{s,i}(t))_{i=1}^S \in \underset{v\in I_0'}{\operatorname{argmax}}((U_{s,i}(t))_{i=1}^S)^T v. \tag{18}$$

2) *Cooperative relaying decisions in busy slots:* If the transmission queue of PT is non-empty and the H.O.L packet in its queue is not being served currently at slot t, then its service begins at t in the following manner:
(i) For each possible link $(l_1,l_2) \in L_p$ that can be used to send a primary packet find the secondary transmission-rate vector that maximizes the following:

$$v^*_{(l_1,l_2)}(t) = \underset{v\in I'(l_1,l_2)}{\operatorname{argmax}}((U_{s,i}(t))_{i=1}^S)^T v. \tag{19}$$

We also find the transmission-rate vector that maximizes the following max-weight expression over all transmission-rate vectors in set I_0',

$$v^*_0(t) = \underset{v\in I_0'}{\operatorname{argmax}}((U_{s,i}(t))_{i=1}^S)^T v. \tag{20}$$

(ii) If the inter-arrival time of the H.O.L primary packet is greater than or equal to $\frac{1}{f_0}$ slots, then solve the following max-weight problem:

$$\max(K_{(PT,PR)}((U_{s,i}(t))_{i=1}^S)^T v^*_{(PT,PR)}(t), ((U_{s,i}(t))_{i=1}^S)^T$$
$$\{K_{(PT,ST_1)}v^*_{(PT,ST_1)}(t) + K_{(ST_1,PR)}v^*_{(ST_1,PR)}(t)+$$
$$(K_{(PT,PR)} - K_{(PT,ST_1)} - K_{(ST_1,PR)})v^*_0(t)\}, ...,$$

$$((U_{s,i}(t))_{i=1}^{S})^T\{K_{(PT,ST_{S_{rel}})}v^*_{(PT,ST_{S_{rel}})}(t)$$
$$+K_{(ST_{S_{rel}},PR)}v^*_{(ST_{S_{rel}},PR)}(t)+(K_{(PT,PR)}$$
$$-K_{(PT,ST_{S_{rel}})}-K_{(ST_{S_{rel}},PR)})v^*_0(t)\}). \qquad (21)$$

(iii) Otherwise if the inter-arrival time of the H.O.L primary packet is greater than or equal to $\frac{1}{f_{S_{rel}}}$ slots but less than $\frac{1}{f_0}$ slots, then solve the following max-weight problem:

$$\max(((U_{s,i}(t))_{i=1}^{S})^T\{K_{(PT,ST_k)}v^*_{(PT,ST_k)}(t)$$
$$+K_{(ST_k,PR)}v^*_{(ST_k,PR)}(t)\},((U_{s,i}(t))_{i=1}^{S})^T\{$$
$$K_{(PT,ST_{k+1})}v^*_{(PT,ST_{k+1})}(t)+K_{(ST_{k+1},PR)}v^*_{(ST_{k+1},PR)}(t)$$
$$+(K_{(PT,ST_k)}+K_{(ST_k,PR)}-K_{(PT,ST_{k+1})}-K_{(ST_{k+1},PR)})$$
$$v^*_0(t)\},..,((U_{s,i}(t))_{i=1}^{S})^T\{K_{(PT,ST_{S_{rel}})}v^*_{(PT,ST_{S_{rel}})}(t)$$
$$+K_{(ST_{S_{rel}},PR)}v^*_{(ST_{S_{rel}},PR)}(t)+(K_{(PT,ST_k)}+K_{(ST_k,PR)}$$
$$-K_{(PT,ST_{S_{rel}})}-K_{(ST_{S_{rel}},PR)})v^*_0(t)\}), \qquad (22)$$

where ST_k ($1 \leq k \leq S_{rel}$) is such that inter-arrival time of the primary packet is less than $\frac{1}{f_{k-1}}$ slots but greater than or equal to $\frac{1}{f_k}$ slots.

(iv) If there is some ST_{i^*} that maximizes (21) or (22) (depending on the inter-arrival time of the H.O.L primary packet) then use that particular ST_{i^*} as relay (in case of multiple solutions pick an ST_{i^*} arbitrarily). Transmit the H.O.L primary packet from PT to ST_{i^*} in slots t, $t+1$,..,$t+K_{(PT,ST_{i^*})}-1$ and from ST_{i^*} to PR in slots $t+K_{(PT,ST_{i^*})}$, $t+K_{(PT,ST_{i^*})}+1$,..,$t+K_{(PT,ST_{i^*})}+K_{(ST_{i^*},PR)}-1$. If no such ST_{i^*} is the solution of (21), then directly transmit the primary packet to PR in slots t, $t+1$,..,$t+K_{(PT,PR)}-1$.

3) *Secondary scheduling decisions in busy slots:* Suppose the decision about transmitting the primary packet in the previous step was to relay the same via ST_{i^*}. Then the secondary transmission rate-vector to be offered in slots t, $t+1$,.., $t+K_{(PT,ST_{i^*})}+K_{(ST_{i^*},PR)}-1$ are obtained as follows:

(i) For slots $\tau \in [t,t+K_{(PT,ST_{i^*})}-1]$ assign transmission rate vector $(\mu^*_{s,1}(\tau),...,\mu^*_{s,S}(\tau))^T$ which is obtained as

$$(\mu^*_{s,i}(\tau))_{i=1}^{S} \in \underset{v\in I'(PT,ST_{i^*})}{\text{argmax}}((U_{s,i}(\tau))_{i=1}^{S})^T v. \qquad (23)$$

(ii) For slots $\tau \in [t+K_{(PT,ST_{i^*})},t+K_{(PT,ST_{i^*})}+K_{(ST_{i^*},PR)}-1]$ assign transmission rate vector $(\mu^*_{s,1}(\tau),...,\mu^*_{s,S}(\tau))^T$ which is obtained as

$$(\mu^*_{s,i}(\tau))_{i=1}^{S} \in \underset{v\in I'(ST_{i^*},PR)}{\text{argmax}}((U_{s,i}(\tau))_{i=1}^{S})^T v. \qquad (24)$$

If the decision about transmission of primary packet was to directly transmit the same, then the secondary transmission rate-vector $(\mu^*_{s,1}(\tau),...,\mu^*_{s,S}(\tau))^T$to be offered in slots $\tau \in [t,t+K_{(PT,PR)}-1]$ are obtained as:

$$(\mu^*_{s,i}(\tau))_{i=1}^{S} \in \underset{v\in I'(PT,PR)}{\text{argmax}}((U_{s,i}(\tau))_{i=1}^{S})^T v. \qquad (25)$$

4) *Transmission and queue-update:* For $i = 1,2,...,S$ transmit $\min(\mu^*_{s,i}(t),U_{s,i}(t))$ secondary packets from ST_i in slot t. If t is the last slot in j-th busy period, remove the j-th primary packet from PT's transmission queue at the end of t.

The SCRP algorithm takes into account both backlog across the secondary users and also the restriction that once a primary packet is scheduled to be transmitted via a certain relay (or directly), it prevents some secondary users from transmitting their own packets for the duration of the packet transmission. We assume all secondary users provide their queue-length information to a centralized controller which selects a transmission-rate vector by searching from a combinatorial set of transmission-rate vectors. Thus in terms of complexity the algorithm suffers from similar drawbacks as backpressure-type algorithms. One may find simpler suboptimal solutions to the max-weight problem similar to the work in [17]. However it is beyond the scope of this paper.

We show that under SCRP, for secondary packet arrival rates in the interior of the region described in Section III, the queue-length processes in the network are strongly stable.

Theorem 1: For all $\lambda_p \in \mathbb{Q}$, $\lambda_p < f_{S_{rel}}$, under the SCRP policy, $U_p(t)$ and $U_{s,i}(t)$ ($i = 1,2,...S$) are strongly stable for all secondary arrival-rates in the interior of $\Lambda(\lambda_p)$.

When $\lambda_p = 0$ the algorithm reduces to traditional Back-pressure theorem with capacity region $\Lambda(0)$ whose proof can be found in [18]. For the case when $\lambda_p \neq 0$ the theorem is proven in next section.

V. Stability Analysis

In this section we prove Theorem 1. The proof uses the concept of renewal frames and relies on comparing SCRP against some other policies that are developed using the renewal frame structure. We first describe the construction of renewal frames for our system model through appropriate partitioning of the time-line. Renewal frame based techniques typically use policies for which the system state is refreshed at the beginning of every frame. We identify a class of such policies in the context of our problem and present a Stationary Scheduling Policy (SSP) and three alternate policies ALT1, ALT2 and ALT3 that belongs to this class. SSP is defined for $\lambda_p \leq f_{S_{rel}}$ and performs scheduling independent of the length of queues corresponding to secondary packets. ALT1 and ALT2 are defined for $\lambda_p \leq f_0$ and $\lambda_p \in \mathbb{Q}$. Throughout every frame ALT1 makes scheduling decisions based on secondary queue-lengths at the beginning of the frame. ALT2 performs scheduling in idle slots like ALT1 while in other slots it performs scheduling like SCRP. We also present Lemmas 1-5. SSP, ALT1, ALT2 along with Lemmas 1-4 will be used to prove stability of SCRP when $\lambda_p \leq f_0$ and $\lambda_p \in \mathbb{Q}$. ALT3 is defined for $f_0 < \lambda_p < f_{S_{rel}}$, $\lambda_p \in \mathbb{Q}$ and performs scheduling throughout every frame based on secondary queue-lengths at the beginning of the frame. SSP and ALT3 along with Lemmas 1, 2 and 5 will be used to prove stability of SCRP when $f_0 < \lambda_p < f_{S_{rel}}$ and $\lambda_p \in \mathbb{Q}$. Details about the policies are provided in [11].

A. Partitioning time-line into frames

For every $\lambda_p \in \mathbb{Q}$, assuming the network was initialized at $t = 0$ when all the queues in the network were empty, the time-line can be partitioned into a finite interval $[0, T]$ and successive non-overlapping frames of length N slots each as: $[T+1, T+N]$, $[T+N+1, T+2N]$,..... By setting T to different $\zeta(j)$, the arrival time of j-th primary packet where $j \in \{1, 2, ...N\}$, we obtain different partitions of the time-line. Fig. 2 shows two partitions of time-lines when $\lambda_p = \frac{3}{8}$ by choosing T to be $\zeta(1)$ and $\zeta(2)$ respectively. Given a partition of time-lines, we denote the k-th indexed slot (where $k = 1, 2, .., N$) in r-th frame by $t_{r,k}$ i.e. $t_{r,k} \triangleq T + k + (r-1)N$ for $r = 1, 2, ...$.

B. Relevant classes of high priority scheduling policies for primary packets

In our analysis we use renewal frame based optimization techniques as described in (Chapter 7, [19]). The frame sizes are constant in our analysis and therefore our problem is a special case of the variable frame-based optimization problems described there. If we think of the $S_{rel}+1$-dimensional vector consisting of primary packet queue-lengths at PT and ST_i ($1 \leq i \leq S_{rel}$) as "state" of the network, then in order to apply renewal frame based techniques we need to make sure that the system state is refreshed at the beginning of every frame. In this work we use a class of policies referred to as "non-idling and clearing for primary ($n.i.c.p$)", described below, which satisfies that requirement.

We call a scheduling and relaying policy to be "non-idling for primary ($n.i.p$)" if the transmission process of some primary packet is on-going at every slot t when $U_p(t) > 0$. $n.i.p$ policies thus ensure high priority for primary packet transmissions in the network. For a given partition of time-lines into frames, we call a policy to be $n.i.c.p$ if it is n.i.p and M primary packets are transmitted every frame. Since when $\lambda_p \leq f_0$, either directly transmitting or relaying any primary packet result in transmission of M primary packets in every frame, every n.i.p policy is n.i.c.p when $\lambda_p \leq f_0$. This is not true when $f_0 < \lambda_p < f_{S_{rel}}$.

For any n.i.c.p policy ϕ, $\lambda_p \in \mathbb{Q}$ and a given partition of time-line we define the function $\psi^{\phi}(t_{r,1})$ ($r = 1, 2, ...$) as

$$\psi^{\phi}(t_{r,1}) \triangleq \sum_{i=1}^{S} U_{s,i}(t_{r,1}) \mathbb{E}[\sum_{t=t_{r,1}}^{t_{r,1}+N-1} \mu_{s,i}^{\phi}(t) | (U_{s,i}(t_{r,1}))_{i=1}^{S}]. \tag{26}$$

where $\mu_{s,i}^{\phi}(t)$ denotes the offered transmission rate to ST_i at t.

C. Stationary randomized policy

For any $(\lambda_{s,1}, \lambda_{s,2}, .., \lambda_{s,S})^T \in$ Interior $(\Lambda(\lambda_p)$ (where $\lambda_p \in \mathbb{Q}$) there exists $\epsilon > 0$ such that the arrival-rate vector $(\lambda_{s,1}+\epsilon, \lambda_{s,2}+\epsilon, ..., \lambda_{s,S}+\epsilon)^T \in$ Interior $(\Lambda(\lambda_p))$. Using a similar approach as in [15], we can show the following:

Lemma 1: If $\lambda_p \in \mathbb{Q}$ and $\frac{1}{k_1+1} < \lambda_p \leq \frac{1}{k_1} \leq f_0$ for some $k_1 \in \mathbb{Z}^+$, partition the time-line by setting T, as mentioned in Section V-A, to be $\zeta(1)$. Otherwise if $\lambda_p \in \mathbb{Q}$ and $f_0 \leq \frac{1}{k_1+1} \leq \lambda_p < \frac{1}{k_1} \leq f_{S_{rel}}$ for some $k_1 \in \mathbb{Z}^+$, set T to be $\zeta(e-1)$ where e is the smallest non-negative integer such that inter-arrival time of e-th primary packet is k_1 slots and that of $(e-1)$-th primary packet is k_1+1 slots. Such a variable e exists because $\zeta(1)$ is always k_1+1 slots. (For example, for the arrival process in Fig. 2, if $f_0 = \frac{1}{3}$ and $f_1 = f_2 = ... = f_{S_{rel}} = \frac{1}{2}$, e is 3)

Then for all arrival-rate vector $(\lambda_{s,1}+\epsilon, \lambda_{s,2}+\epsilon, ..., \lambda_{s,S}+\epsilon)^T \in$ Interior $(\Lambda(\lambda_p))$, where $\lambda_p < f_{S_{rel}}$ and $\lambda_p \in \mathbb{Q}$, there exists an n.i.c.p stationary scheduling policy SSP that makes scheduling and relaying decisions based on knowledge of the primary and secondary arrival rates but independent of the queue-lengths of secondary transmitters and under which for all $r = 1, 2, ...$,

$$\mathbb{E}[\sum_{\tau=T+1+(r-1)N}^{\tau=T+rN} \mu_{s,i}^{SSP}(\tau)] \geq (\lambda_{s,i}+\epsilon)N \quad \forall i = 1, 2, .., S \tag{27}$$

Proof: Proof can be found in [11]. ■

Lemma 2: For any $\lambda_p \in \mathbb{Q}$ and given secondary arrival-rate vector $(\lambda_{s,i})_{i=1}^{S} \in$ Interior$(\Lambda(\lambda_p))$, define a policy SSP, using the procedure in proof of Lemma 1 in [11], for arrival-rate vector $(\lambda_{s,i}+\epsilon)_{i=1}^{S} \in$ Interior$(\Lambda(\lambda_p))$ where $\epsilon > 0$.

1) If $\lambda_p \leq f_0$, partition the time-line similarly as in ALT1. Then for every $r = 1, 2, ...$

$$\psi^{ALT1}(t_{r,1}) \geq \psi^{SSP}(t_{r,1}). \tag{28}$$

2) If $f_{S_{rel}} > \lambda_p > f_0$, partition the time-line similarly as in ALT3. Then for every $r = 1, 2, ...$

$$\psi^{ALT3}(t_{r,1}) \geq \psi^{SSP}(t_{r,1}). \tag{29}$$

Proof: Proof can be found in [11]. ■

Lemma 3: For every $\lambda_p \leq f_0$, $\lambda_p \in \mathbb{Q}$, partition the time-line similarly as in ALT1. Then for every $r = 1, 2, ...$

$$\psi^{ALT2}(t_{r,1}) \geq \psi^{ALT1}(t_{r,1}) - B_1, \tag{30}$$

where $B_1 > 0$ is a finite constant.

Proof: Proof is provided in [11]. ■

Lemma 4: For every $\lambda_p \leq f_0$, $\lambda_p \in \mathbb{Q}$, partition the time-line similarly as in ALT2. Then for every $r = 1, 2, ...$

$$\psi^{SCRP}(t_{r,1}) \geq \psi^{ALT2}(t_{r,1}) - B_2, \tag{31}$$

where $B_2 \geq 0$ is a finite constant.

Proof: Proof provided in [11]. ■

Lemma 5: If $f_0 < \lambda_p < f_{s_{rel}}$, $\lambda_p \in \mathbb{Q}$, $r = 1, 2, ..$, and the partition of the time-line is done in same way as in ALT3,

$$\psi^{SCRP}(t_{r,1}) \geq \psi^{ALT3}(t_{r,1}) - B_3, \tag{32}$$

where $B_3 > 0$ is a finite constant.

Proof: Proof provided in [11]. ■

Proof of Theorem 1: We prove Theorem 1 by using the Lyapunov drift technique and Lemmas 1-5. The proof can be found in [11]. ■

(a) $T = \zeta(1) = 3$ (b) $T = \zeta(2) = 6$

Fig. 2. Partition of time-line into frames when $\lambda_p = \frac{3}{8}$. Each small rectangle represents a time-slot. The arrival of a primary packet at the transmission queue of PT during any slot is indicated by a vertical arrow at the boundary between the slot and the one immediately after it.

VI. General primary data arrival rate

In this section we extend the analysis in previous sections to a case where λ_p is not restricted to the set of rational numbers. We consider the particular case where $K_{(PT,PR)} = 3$, $S_{rel} = 1$, $K_{(PT,ST_1)} = K_{(ST_1,PR)} = 1$ and show the following:

Lemma 6: Let the region $\Lambda(\lambda_p)$ be defined for any $\lambda_p \in \mathbb{R}$ s.t. $\frac{1}{k_1+1} \leq \lambda_p \leq \frac{1}{k_1} \leq f_0$ (for some $k_1 \in \mathbb{Z}^+$), by using this value of λ_p in (8)- (17) and the following equation:

$$\lambda_p = \frac{\pi_{(PT,PR)}}{K_{(PT,PR)}} + \sum_{1 \leq j \leq S_{rel}} \frac{\pi_{(PT,ST_j)}}{K_{(PT,ST_j)}}. \tag{33}$$

When $K_{(PT,PR)} = 3$, $S_{rel} = 1$ and $K_{(PT,ST_1)} = K_{(ST_1,PR)} = 1$, SCRP stabilizes the network for any $(\lambda_{s,1}, \lambda_{s,2}, .., \lambda_{s,S})^T \in$ Interior($\Lambda(\lambda_p)$) where $0 \leq \lambda_p \leq \frac{1}{3}$.

Proof: Proof can be found in [11]. ∎

Lemma 6 can be extended to networks with general values of S_{rel} and $K_{(l_1,l_2)}$ (where $(l_1, l_2) \in L_p$). However for simplicity of analysis, in this paper we limit our discussion to the particular case considered above.

VII. Conclusion

In this work we studied the problem of opportunistic cooperation in a cognitive network where some nodes may benefit from cooperative relaying while others may suffer loss of transmission of opportunities. Assuming a deterministic periodic primary packet arrival process, a scheduling and relaying algorithm is developed for this network using Lyapunov drift techniques. The set of primary arrival-rate and secondary arrival-rate vectors for which the network can be stabilized is shown to be greater under this cooperative scheduling algorithm than without cooperation. In future we seek to extend this analysis to a more general network where the service-time of packets in different links are stochastic and cases involving multiple primary source-destination pairs.

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