LABORATORY 10: Transfer functions-Filters

Material covered:
- First order filters

Overview Notes:

Decibels: When considering the magnitude of the transfer function, a log-log plot is useful for gaining physical insight to the circuit. Typically, the vertical axis has units of Decibels (dB). The magnitude of the transfer function can be converted to dB by using the relationship 
\[
20\log\left(\frac{|V_{out}(s)|}{|V_{in}(s)|}\right) = 20\log(|H(s)|) \quad [\text{dB}].
\]
Using this formula, when we plot the transfer function of a first order circuit, we can make some observations.

Cutoff frequency: The cutoff frequency of a first order circuit is defined as the frequency at which the amplitude of the output voltage is \(\frac{1}{\sqrt{2}}\) the amplitude of the input voltage. In other words the ratio of output to input is \(\frac{|V_{out}(s)|}{|V_{in}(s)|} = \frac{1}{\sqrt{2}}\).

Substituting that value into the above dB expression, at the cutoff frequency the expression evaluates to -3dB. The cutoff frequency is often referred to as the 3dB point. In first order circuits, the cutoff frequency occurs when the magnitudes of the real part and the imaginary part of transfer function denominator are the same.

Passband: The frequency range where the output signal has approximately a constant amplitude.

Stopband: The frequency range where the output signals are attenuated relative to passband. In the stopband of a first order circuit, a 20dB difference occurs every time the frequency changes one order of magnitude. For example, in a low pass filter with a cutoff frequency of 1kHz, the dB value of \(|H(j\omega)|\) at 50kHz would have a value 20dB lower than the dB value of \(|H(j\omega)|\) at 5kHz. This attribute is called a reduction of 20dB/decade. (We will discuss these concepts in class.)
We will be performing frequency sweeps in the upcoming labs. We will need a new component, VAC, and need to set up a different simulation profile. In PSpice, build the circuit shown above. The source is the VAC component. This source is effectively sinusoidal with a possible DC offset. We will use an appropriate voltage for the sinusoidal term and zero voltage for the DC offset.

To perform frequency sweeps;

1) Select a New Simulation Profile under the PSpice tab. Under Analysis Type, select AC Sweep/Noise.
2) It should be set to a Logarithmic scale. If the Linear box is checked, change it to Logarithmic.
3) You will then need to select the frequency limits. These limits will likely depend on your circuit. For the first order circuits, we want the lower limit to be well below the cutoff frequency and the upper limit to be well above the cutoff frequency. For the above circuit, we will use a range of 1Hz-1E6Hz.
4) To generate smoother, more accurate plots, choose 100 points per decade.
5) Note, these settings will generate a logarithmic scale on the x-axis. It is likely you will want a logarithmic scale on the y-axis as well. You will change that scale after you run the simulation. On the Schematic (output) window, select the Plot tab and choose Axis Settings.
6) Select the Y-axis tab and click the Log scale in the lower left part of the window. You should now see a sloped line in the stopband region of your output plot. (There is a hotkey icon that changes the y-axis to log scale as well.)

Note: The y-axis in this log-log plot is only \( \log(|V_{out}|) \), not the decibel value. The scaling is off and there is a vertical shift that is missing. If you would like, you can use the Add a Trace option to provide the correct units for the y-axis. (This will be covered in class.)
Measuring Phase in PSpice:

Under the PSpice tab,
1. Click Markers
2. Click Advanced
3. Choose Phase of Voltage (*must have AC Sweep selected in simulation window*)
4. You will now have a marker you can place on the circuit to measure phase in degrees.
5. Important qualification regarding this marker in the following text.

PSpice has a phase marker that can be used like the voltage markers. However, it is the phase at a node relative to ground rather than the phase across a component. Therefore, in order to determine the phase across a component, one of the component legs must be connected to ground. For example, in the laboratory section, if you were to measure resistor phase, in PSpice you would need to implement the circuit such that the resistor was connected to ground instead of the inductor (as shown in the laboratory schematic). Recall, the order of components in series does not matter and won’t affect the circuit response.
Determine the transfer function for the voltage across the resistor,

\[ H_{VR}(s) = \] 

Determine the transfer function for the voltage across the inductor,

\[ H_{VL}(s) = \] 

What is the cutoff frequency for this circuit? 

As the frequency approaches zero (DC), the voltage across the inductor approaches 

As the frequency approaches infinity, the voltage across the inductor approaches 

When measuring the voltage across the inductor, is this circuit a low pass filter or a high pass filter?
**Part 1:** Build the circuit. Set your source amplitude to 2Vpp (1V amplitude) with a 0V offset and adjust the frequency as indicated in the table. Measure the output voltage amplitude and phase across the inductor for various frequencies and fill in the following table with your calculations, PSpice, Mobile Studio measurements.

<table>
<thead>
<tr>
<th>Inductor</th>
<th>Magnitude</th>
<th>Phase [Degrees]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calculated</td>
<td>PSpice</td>
</tr>
<tr>
<td>Freq. [Hz]</td>
<td>Rad. Freq. [rad/s]</td>
<td></td>
</tr>
<tr>
<td>47.7</td>
<td></td>
<td>Calculated</td>
</tr>
<tr>
<td>159</td>
<td></td>
<td></td>
</tr>
<tr>
<td>477</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_c$</td>
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<td></td>
</tr>
<tr>
<td>1590</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4770</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15.9k</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
If there are any significant differences between measured and calculated results, comment on possible reasons. (Recall the characteristics of a real inductor.)

For your above measured results, determine the magnitude in dB and the phase of the transfer function. When calculating the transfer function magnitude, remember to divide the output magnitude by the source amplitude (which in this case is 1). The phase can be obtained directly from your measured results in the previous table.

| Radial Frequency [rad/s] | log(ω) | 20log|H(s)| [dB] | Phase $\angle[H(s)]$ |
|-------------------------|--------|-----------------|------------------------|
| 300                     |        |                 |                        |
| 1E3                     |        |                 |                        |
| 3E3                     |        |                 |                        |
| $\omega_c$              |        |                 |                        |
| 1E4                     |        |                 |                        |
| 3E4                     |        |                 |                        |
| 1E5                     |        |                 |                        |
Do the following after the lecture on Thursday October 30th.

Using your measured results, generate a log-log plot (dB-log $\omega$) of the magnitude of the transfer function vs. frequency. Also, plot the angle (phase) of the transfer function against log($\omega$). For the same circuit, perform an AC Sweep in PSpice and compare your results.

Magnitude

Is the cutoff frequency a -3dB point?

Does your stopband display a 20dB/decade drop?

Phase
What is the change of phase between $\omega = 300$ [rad/s] and $\omega = 5E4$ [rad/s]? Is your result consistent with expectations from the transfer function?