LECTURE 27.1

- 3 Phase AC Power
- 3 Phase Generators
  - Wye
  - Delta
  - Conversion

3 PHASE AC POWER

- Most Commercial Power to Large Users (e.g. RPI) is 3 Phase
- Most Residential Power is 1 Phase
- Brief Overview of 3 Phase Circuits
- Electric Power Engineering Courses If You Want More Detail

3 PHASE POWER

- Why Use 3 Phase Power?
  - Easy to Generate
  - Provides Constant Instantaneous Power to Load
  - Uses Fewest Wires to Deliver Constant Instantaneous Power

\[ p(t) = \frac{\frac{V_A}{2} + \frac{V_B}{2}}{a} \cdot \left(1 + \cos(3\omega t)\right) \cdot \frac{V_A}{2} \]
3 PHASE GENERATORS

- **Wye Generator:**
  - Produces "Phase Voltages"
  - $v_a(t), v_b(t), v_c(t)$
  - These Create "Line Voltages"
    - (from the difference between the phases)
    - $v_{ab}(t), v_{bc}(t), v_{ca}(t)$

Wye Generator

\[ v_a(t) = \sqrt{3}V \cos(\alpha + \phi) \]
\[ v_b(t) = \sqrt{3}V \cos(\alpha + \phi - 120^\circ) \]
\[ v_c(t) = \sqrt{2}V \cos(\alpha + \phi + 120^\circ) \]

Positive (or abc) Phase Sequence

Wye Generator

"Line Voltages": $v_{ab}(t), v_{bc}(t), v_{ca}(t)$ are "Line Voltages"
3 PHASE GENERATORS

**Delta Generator:**
- Produces "Line Voltages":
  - $v_{ab}(t)$, $v_{bc}(t)$, $v_{ca}(t)$
- These Create "Phase Voltages"
  - $v_a(t)$, $v_b(t)$, $v_c(t)$

**3 Phase Generators**

Frequency Domain:

- $v_a(t)$ Voltage at "a" Relative to Ground
  - $v_a(t) = 2V \cos(\omega t + \phi)$ Volts
- $v_b(t)$ Voltage at "b" Relative to Ground
  - $v_b(t) = 2V \cos(\omega t + \phi - 120^\circ)$ Volts
- $v_c(t)$ Voltage at "c" Relative to Ground
  - $v_c(t) = 2V \cos(\omega t + \phi + 120^\circ)$ Volts
- $V_p$ = Phase Voltage (RMS Value)
Let $V_a = \sqrt{3}V_p / 0^\circ$

Line Voltages:
- $v_{ab}(t) =$ Voltage Between Line "a" and Line "b"
- $v_{ba}(t) =$ Voltage Between Line "b" and Line "c"
- $v_{cb}(t) =$ Voltage Between Line "c" and Line "a"
- $v_{ca}(t) =$ $\sqrt{2}V_p \cos (\omega t + \phi - 90^\circ)$ Volts
- $v_{bc}(t) =$ $\sqrt{2}V_p \cos (\omega t + \phi + 150^\circ)$ Volts
- $V_l =$ Line Voltage (RMS Value) = $\sqrt{3}V_p$

Phase Voltages
- $V_a = \sqrt{2}V_p / 0^\circ$ Volts
- $V_b = \sqrt{3}V_p / \phi - 120^\circ$ Volts
- $V_c = \sqrt{2}V_p / \phi + 120^\circ$ Volts
- $V_l / \sqrt{3} =$ Phase Voltage (RMS)
**3 PHASE GENERATORS**

**Line Voltages**

\[ V_{ab} = \sqrt{2}V \phi + 30^\circ \text{ Volts} \]
\[ V_{bc} = \sqrt{2}V \phi - 90^\circ \text{ Volts} \]
\[ V_{ca} = \sqrt{2}V \phi + 150^\circ \text{ Volts} \]

\[ V_l = \sqrt{3}V \phi = \text{Line Voltage (RMS)} \]

**3 PHASE GENERATORS**

**RMS Line Voltage**

\[ V_l = \sqrt{3}V \phi \]

**Line Voltages Lead Phase Voltages by +30°**

\[ /V_{ab} = /V_c + 30^\circ \]

If Know Any 1 of the 6 Phase and Line Voltages Can Write Down All the Others by Inspection

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**3 PHASE LOADS**

Balanced Wye Load

RMS Voltage Across \( Z_y = V_y \)

RMS Current Thru \( Z_y = I_l \)

\( I_a, I_b, I_c = \text{Line Currents} \)

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**3 PHASE LOADS**

Balanced Delta Load

RMS Voltage Across \( Z_\Delta = V \)

Line Currents Do NOT Flow Thru \( Z_\Delta \)

\( I_a, I_b, I_c = \text{Line Currents} \)

No Neutral
3 PHASE LOADS

- Can Convert any Balanced Delta Load to an Equivalent Wye Load:
  - $Z_{\text{eq}} = Z_{\text{Delta}} / 3$
  - Easier to Analyze Circuits using Balanced Wye Loads
  - Can Use a 1 Phase Equivalent Circuit Model for Balanced Wye Loads

CONVERT DELTA TO WYE

Balanced Delta Load

No Neutral

$\sum_{a,b,c} I = \sum_{a,b,c} I$

Line Currents Do NOT Flow Thru $Z_{\text{Delta}}$

$V = V_{\text{rms}}$

RMS Voltage Across $Z_{\text{Delta}} = V_{\text{rms}}$

RMS Current Thru $Z_{\text{Delta}} = I_{\text{rms}}$

$Z_{\text{eq}} = Z_{\text{Delta}} / 3$

Write down all other Line Currents by Inspection

CONVERT DELTA TO WYE

1. $Z = \frac{Z_{\text{Delta}}}{3}$
2. $I_a, I_b, I_c = \text{Line Currents}$
3. $V_{\text{rms}} = V$
4. $I_{\text{rms}} = \frac{V}{Z_{\text{Delta}}}$
5. $N$ and $N'$ are not connected in the Wye Load

EQUIVALENT 1 PHASE MODEL

$a'$

$a$

$V_a$

$I_a$

$Z_{\text{eq}}$ or $Z_{\text{Delta}} / 3$

Write down all other Line Currents by Inspection
**INSTANTANEOUS POWER**

**Instantaneous Power in a 3φ Balanced Circuit:**

\[ Z_Y = \left| Z_V \right| \theta \]

\[ p(t) = p_a(t) + p_b(t) + p_c(t) \]

\[ p_a(t) = (v_a(t))(i_a(t)) = \left( \sqrt{2} V \cos(\omega t) \right) \left( \sqrt{2} I \cos(\omega t - \theta) \right) \]

\[ p_b(t) = \left( \sqrt{2} V \cos(\omega t - 120^\circ) \right) \left( \sqrt{2} I \cos(\omega t - 120^\circ - \theta) \right) \]

\[ p_c(t) = \left( \sqrt{2} V \cos(\omega t + 120^\circ) \right) \left( \sqrt{2} I \cos(\omega t + 120^\circ - \theta) \right) \]

Use \( \cos x \cos y = \frac{1}{2} \cos(x - y) + \frac{1}{2} \cos(x + y) \)

\[ p_a(t) = V_a I \cos \theta + V_b I \cos(2\omega t - \theta) \]

\[ p_b(t) = V_b I \cos \theta + V_c I \cos(2\omega t + 120^\circ - \theta) \]

\[ p_c(t) = V_c I \cos \theta + V_a I \cos(2\omega t - 120^\circ - \theta) \]

\[ p(t) = 3V_a I \cos \theta = \text{Constant} \]

3φ Delivers Constant Instantaneous Power to Load