ELECTRIC CIRCUITS
ECSE-2010
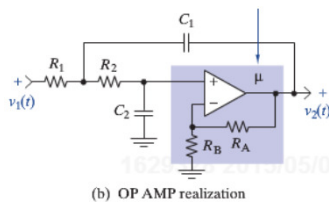
Lecture 28: Course Wrap Up

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AGENDA

- Active Filter Building Blocks: Sallen-Key Configuration
- FINAL EXAM DETAILS
- Hand back EXAM 3
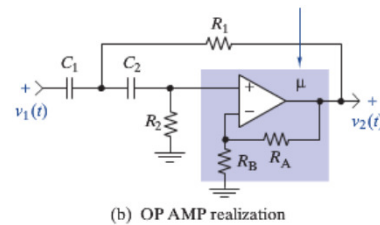- Preliminary Exam 3 regrade
- Brief Overview of Topics (on your own)

SALLEN-KEY LOW PASS

(b) OP AMP realization

From Textbook, Thomas, Rosa, and Toussaint, pg. 739

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SALLEN-KEY HIGH PASS

(b) OP AMP realization

From Textbook, Thomas, Rosa, and Toussaint, pg. 744
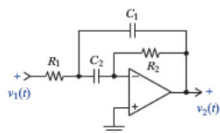
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SALLEN-KEY BANDPASS

FIGURE 14-14 Second-order bandpass circuit.

From Textbook, Thomas, Rosa, and Toussaint, pg. 749

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SALLEN-KEY LOW PASS

$$H(s) = \frac{K\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

$$K = \mu = 1 + \frac{R_A}{R_B} \qquad \omega_0 = \frac{1}{\sqrt{R_1 C_1 R_2 C_2}}$$

$$2\zeta = \sqrt{\frac{R_2 C_2}{R_1 C_1}} + \sqrt{\frac{R_1 C_2}{R_2 C_1}} + (1-\mu)\sqrt{\frac{R_1 C_1}{R_2 C_2}}$$

2nd Order Low Pass Filter

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1

SALLEN-KEY LOW PASS: EQUAL ELEMENT

If Choose $R_1 = R_2 = R$; $C_1 = C_2 = C$

$$\omega_0 = \frac{1}{RC} \qquad \zeta = \frac{3-\mu}{2} = 1 - \frac{R_A}{2R_B}$$

Note: Cannot Make $\zeta > 1$

For $R_A = 0 \Rightarrow \zeta = 1 \Rightarrow$ Critically Damped

For $R_A < 2R_B \Rightarrow 0 < \zeta < 1 \Rightarrow$ Underdamped

For $R_A = 2R_B \Rightarrow \zeta = 0 \Rightarrow$ Oscillator

For $R_A > 2R_B \Rightarrow \zeta < 0 \Rightarrow$ Unstable

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EXAM DETAILS

- Monday, May 18th, **Sage 3303**, 11:30-2:30 pm
- Bring a calculator (no wireless, no cell phones please)
- One new crib sheet + 3 previous crib sheets, front and back!
- Folks with *approved extra time*, meet in my office *before test* .

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FINAL EXAM STRUCTURE

1) Short Answer (25 points) *Any conceptual question from any unit!*
2) Unit 1: Basic Circuit Analysis (25 points)
3) Unit 2: Transient Response (25 points)
   1) First order transient
   2) Laplace second order
4) Unit 3: AC Steady State and Power (25 points)
   1) Complex Power
   2) Transformer
5) Unit 4: AC Steady State Frequency Response (25 points)
   1) First/Second Order Bode plots with corrections and SLA
   2) Cascading filters
6) Filter Design Problem (25 points)

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ADVICE ON FINAL EXAM AND BEYOND....

1) Final Exam
   a) Do homework 10 all questions as a review
   b) Try to take some time to match process with theory
2) After Circuits....
   1) Stay ahead of the professor (read book/videos/go online....anything)
   2) It takes practice to match theory to analytical (give yourself enough time!)
   3) Always check your exams for missed concepts
   4) Stay engaged and ask questions during lecture...after lecture

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Congratulations, you are officially
electrical engineering students!

Now you must become electrical
engineers/computer systems
engineers/dual major
engineers.......may the
(electromagnetic) Force be with you...

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UNIT 1

DC Circuit Analysis and OP Amps

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REVIEW: LECTURE 1

- Passive Sign Convention
- Voltage Reference Point – Ground
- Linear Resistor – Ohm's Law
- Open/Short Circuits – Ideal Switches
- Ideal Voltage and Current Sources
- Reference Marks
- Kirkoff's Laws

CHECK SLIDES...

- *If you don't have Lecture 1 down, you're in trouble!!*

I'll skip.

REVIEW: LECTURE 2

- Series and Parallel Connections
- Equivalent Circuits
  - Series and Parallel Resistors
  - Voltage and Current Dividers

CHECK SLIDES...

- *If you don't have Lecture 2 down, you're still in trouble!!*

I'll skip.

REVIEW: LECTURE 3

- Node Analysis
- Mesh Analysis

SUMMARY

- **Node Voltage Analysis:**
  - **Label All Node Voltages, Known and Unknown, Identifying Variables (v_1, v_2, etc.)**
  - **# of Unknown Node Voltages = # of Nodes - # of Voltage Sources - 1 (Reference)**
  - **Write a KCL at Each Unknown Node Voltage**
    - **Best to Use: Sum of Currents Out of Node = 0**
    - **Express i's in terms of Node Voltages**
  - **Solve Algebraic Equations for Node Voltages**
    - **Use MAPLE, MATLAB, Cramer's Rule, etc.**
  - **Solve for Currents Using Ohm's Law**

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NODE ANALYSIS –SOLVING PROCEDURE

1. **Identify all nodes**
2. **Choose a reference node (ground)**
3. **Label the unknown nodes**
4. **Locate all voltage sources**
   1. **Determine either absolute or relative voltages based on the voltage sources**
5. **Write a KCL equation for each node**
6. **Use Ohm's Law to rewrite the currents as voltage differences over resistance**
   1. **If a voltage source is on one of the current paths, 'follow' it to the next node to get an expression for current.**
7. **Set up the linear system**
8. **Solve the matrix**

NODE ANALYSIS-WITH VOLTAGE SOURCES SEE PG 84

Adding voltage sources to circuits modifies node analysis procedure because the current through a voltage source is not directly related to the voltage across it.

Actually simplifies node analysis by reducing the number of equation required.

Method 1: Use source transformation to replace the voltage source and series resistance with an equivalent current source and parallel resistance

Method 2: Strategically select reference node and write node equations at the remaining N-2 non-reference nodes in the usual way (can be used whether or not there is a resistance in series with voltage source)

Method 3: Combine nodes to make a supernode

SUMMARY

- **Mesh Current Analysis:**
  - **Label and Define ALL Mesh Currents**
    - **Unknown Mesh Currents <u>and</u> Currents from Current Sources**
  - **# of Unknown Mesh Currents = # of Meshes - # of Current Sources;**
  - **Write a KVL around Each Unknown Mesh Current**
    - **Sum of Voltages due to All Mesh Currents = 0**
    - **Best to Go Backwards Around Current Arrow**
  - **Solve Algebraic Equations for Mesh Currents (Maple, Cramer's Rule, etc.)**
  - **Solve for Voltages Using Ohms Law**

MESH ANALYSIS –SOLVING PROCEDURE

1. **Identify all loops**
2. **Locate all current sources**
3. **If possble, simplify the problem by redrawing the circuit with current sources on the 'outside'**
4. **Label the currents in each loop**
5. **Assign the current directly if a current source is on the 'outside'**
6. **Assign a relative current expression if the current source is shared by two loops.**
7. **Write a KVL expression for each loop**
   1. **If a current source is shared by two loops, combine them to form a larger loop.**
8. **Use Ohm's Law to write the KVL in terms of currents**
9. **Set up the linear system**
10. **Solve the matrix**

MESH ANALYSIS-WITH CURRENT SOURCES SEE PG 97

Method 1: Use source transformation to replace current source and parallel resistance the with an equivalent voltage source and series resistance

Method 2: If current source is contained in only one mesh, then that mesh current is determined by the source current and is no longer an unknown. Write mesh equations around the remaining meshes in the usual way and move known mesh current to the source side of the equations in the final step.

Method 3: Create a supermesh by excluding the current source and any elements connected in series with it.

REVIEW: LECTURE 4

- Linearity
- Superposition Principle
- Superposition Example
- Dependent Sources

LINEARITY

- If have multiple inputs
- Input = $x_1 + x_2 + x_3$
- Output must be additive
- $y = k_1x_1 + k_2x_2 + k_3x_3$
- Leads to Superposition Principle
  - Can use only for multiple inputs to a linear circuit

SUPERPOSITION

- Find Output due to <u>each independent source with all other independent sources set = 0;</u> then Add to find Total Output:
  - Source of 0 is called a "dead source"
  - "Dead" voltage source = 0 V = Short Circuit
  - "Dead" current source = 0 A = Open Circuit

SUPERPOSITION

- Total Output = Sum of all Outputs due to <u>each independent source with all other independent sources "dead"</u>:
  - Simply Add them
  - Works only for Linear Circuits; Only kind we will consider

DEPENDENT SOURCES

- Symbol:
  - Diamond = Symbol for Dependent Source
  - Circle = Symbol for Independent Source
- 4 Types of Dependent Sources
  - Voltage Controlled Voltage Source (VCVS), E
  - Current Controlled Current Source (CCCS), F
  - Voltage Controlled Current Source (VCCS), G
  - Current Controlled Voltage Source (CCVS), H
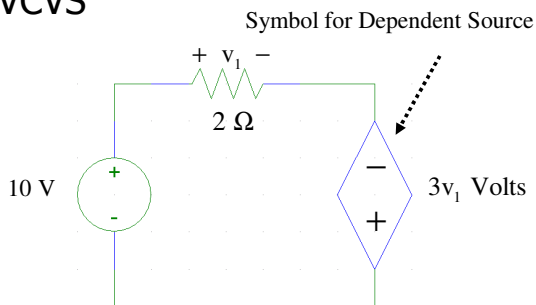
VCVS

Symbol for Dependent Source

$+ \ v_1 \ -$

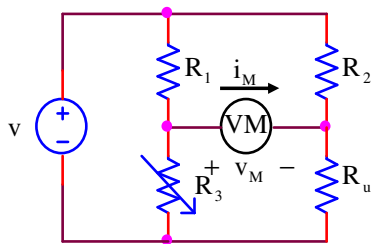2 Ω

10 V

$3v_1$ Volts

Voltage Controlled Voltage Source (VCVS)

REVIEW: LECTURE 5

- Wheatstone bridge
- Norton/Thevinin equivalent circuits

5

WHEATSTONE BRIDGE
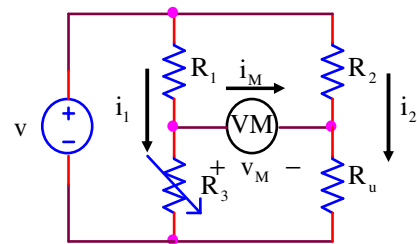

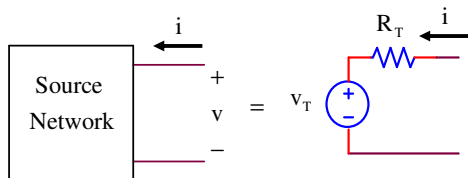When Bridge is Balanced, $i_M = 0$; $v_M = 0$

Meter Draws No Current

WHEATSTONE BRIDGE

Bridge Balanced

Solve for R_u $R_u = \dfrac{R_2 R_3}{R_1}$

THEVENIN'S THEOREM

Any Source Network
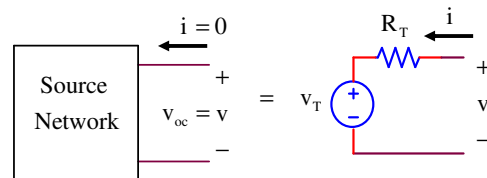
May be Replaced with its

Thevenin Equivalent Circuit

$v_T = $ Thevenin Voltage

$R_T = $ Thevenin Resistance

THEVENIN'S THEOREM

$v_T = v_{oc} = $ Open Circuit Voltage

$v_T = v_{oc} = v$ when $i = 0$

$R_T = $ Thevenin Resistance
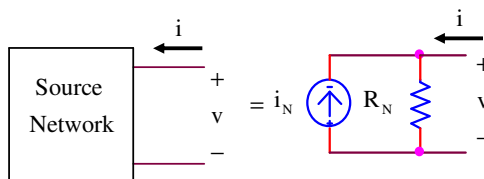
NORTON'S THEOREM

Any Source Network

May be Replaced with its

Norton Equivalent Circuit

$i_N = $ Norton Current

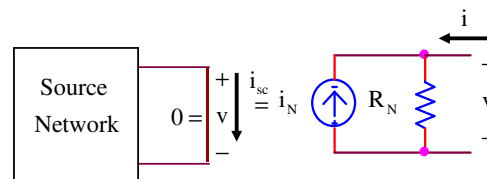$R_N = $ Norton Resistance

NORTON'S THEOREM
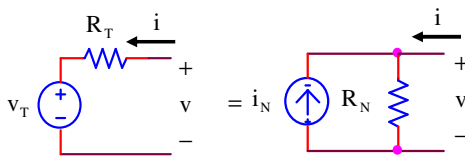

$i_N = i_{sc} = $ Short Circuit Current

$i_N = i_{sc} = $ Current Flowing from $+$ to $-$ when $v = 0$

$R_N = $ Norton Resistance

EQUIVALENT CIRCUITS

Thevenin Equivalent Circuit   Norton Equivalent Circuit

From Source Conversions: $i_N = \dfrac{v_T}{R_T}$ and $R_N = R_T$

$v_T = v_{oc} = $ Open Circuit Voltage

$i_N = i_{sc} = $ Short Circuit Current


THEVENIN/NORTON SOURCES-SOLVING PROCEDURE

1. **Thevenin – Remove the load**
   1. Find the voltage ($V_{oc} = V_{TH}$) between the two nodes where the load was connected, using any method
2. **Norton – Remove the load and connect a short circuit (wire) between the two nodes where the load was attached**
   1. Find the current ($I_{sc} = I_N$ through that short circuit (wire), using any method
   2. Note: the short circuit may 'combine' nodes. Recognize that you can do KCL at a node to find current through an individual wire connecting components.
3. **Resistance – Remove the load**
   1. Apply a test voltage source, V_{test}, at the nodes where the load was attached
   2. Short circuit all other independent voltage sources and open circuit all other independent current sources.
   3. Find the current through that source, I_{test}
   4. $R_{EQ} = R_N = R_{TH} = V_{test}/I_{test}$
4. Note: only two of these are needed since $V_{TH} = (R_{TH})(I_N)$

MAXIMUM SIGNAL TRANSFER
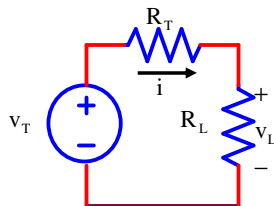
For Maximum Power Transfer; Choose $R_L = R_T$

Best You Can Do

REVIEW: LECTURE 6

- Amplifier circuit model
- Ideal Operational Amplifiers (Op Amps)

OPERATIONAL AMPLIFIERS

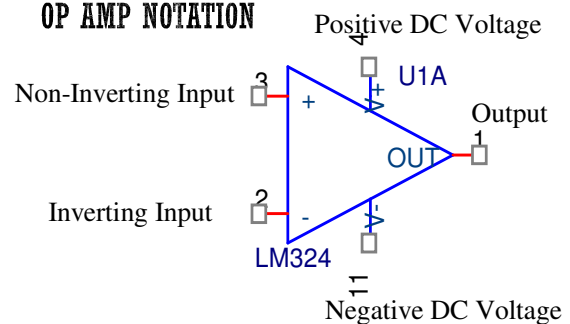- An Operational Amplifier is a *High Gain Voltage Amplifier that can be used to perform Mathematical Operations:*
  - Addition and Subtraction
  - Differentiation and Integration
  - Other Functions as Well
- Op Amps are the building blocks for many, many electronic circuits

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OP AMP NOTATION

Positive DC Voltage

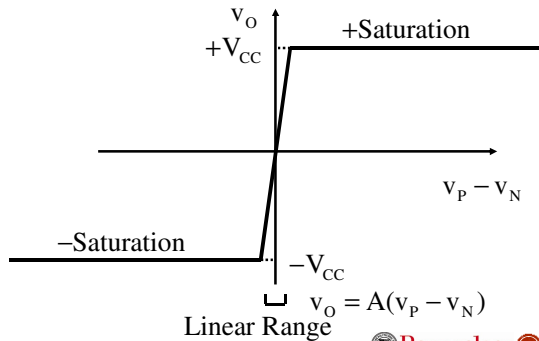Non-Inverting Input   3   U1A

Output   1

OUT

Inverting Input   2

LM324

Negative DC Voltage

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TRANSFER CHARACTERISTIC

v_O
$+V_{CC}$ +Saturation
$v_P - v_N$
−Saturation
$-V_{CC}$
$v_O = A(v_P - v_N)$
Linear Range

IDEAL OP AMP

$i_P = 0$ $|v_O|$ Can Never be greater than $|V_{CC}|$
v_P
$i_N = 0$
v_O
v_N
OPAMP
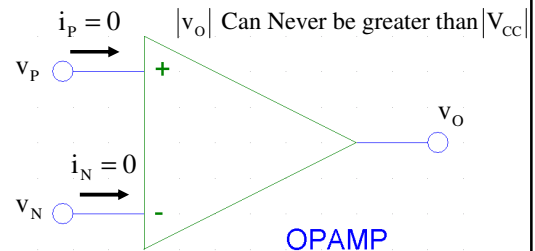
Since $R_1 \to \infty$, Ideal Op Amp Draws No Current!

Since $A \to \infty$, $v_P = v_N$ in Linear Range
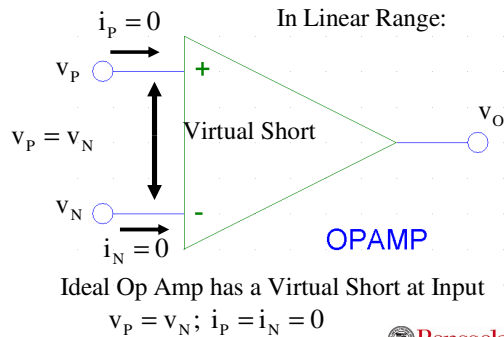
IDEAL OP AMP

$i_P = 0$ In Linear Range:
v_P
$v_P = v_N$ Virtual Short
v_O
v_N
$i_N = 0$
OPAMP

Ideal Op Amp has a Virtual Short at Input

$v_P = v_N$; $i_P = i_N = 0$

ZERO CROSSING DETECTOR

No Negative Feedback

v_1
v_O
OPAMP

If $v_1 > 0$, $v_O = +V_{CC}$ +Saturation

If $v_1 < 0$, $v_O = -V_{CC}$ −Saturation

COMPARATOR

No Negative Feeback

v_1
v_O
OPAMP
v_2

If $v_1 > v_2 \Rightarrow v_O = +V_{CC}$

If $v_1 < v_2 \Rightarrow v_O = -V_{CC}$
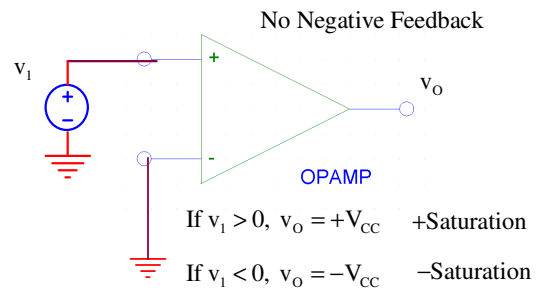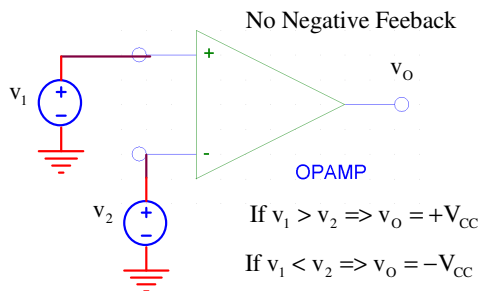
OP AMP CIRCUITS

- For most Op Amp circuits, we add negative feedback:
  - Circuit connection between v_O and v_N
  - Helps to keep Op Amp in Linear Range
  - This will help keep $v_P = v_N$
  - Output, $v_O = A(v_p - v_N)$, will be finite, as long as its magnitude is less than V_{CC}
  - Output can never be greater than $\pm V_{CC}$

VOLTAGE FOLLOWER

$v_P = v_S$ Apply v_S to v_P

v_S $v_N = v_S$

v_O

v_S

OPAMP

$v_P = v_N$ Negative Feedback

Draws No Current from Source $v_O = v_S$
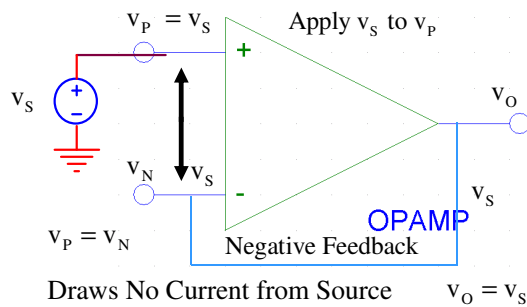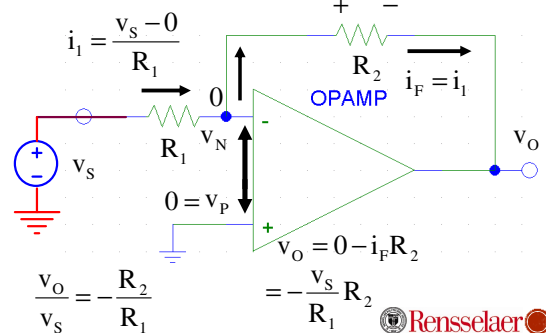
Buffer, or Isolation Amplifier

INVERTING AMPLIFIER

$i_1 = \dfrac{v_S - 0}{R_1}$

$+$ $-$

R_2 $i_F = i_1$

0

v_N OPAMP

R_1

v_O

v_S

$0 = v_P$

$v_O = 0 - i_F R_2$

$\dfrac{v_O}{v_S} = -\dfrac{R_2}{R_1}$ $= -\dfrac{v_S}{R_1} R_2$

NON-INVERTING AMPLIFIER

$v_P = v_S$

v_S

v_N

v_O

OPAMP

v_S $-$ $+$

$v_O = v_S + i_F R_1$

$i_2 = \dfrac{v_S}{R_2}$ R_2 R_1 $i_F = i_2$

$= v_S + \dfrac{v_S}{R_2} R_1$

$\dfrac{v_O}{v_S} = \left(1 + \dfrac{R_1}{R_2}\right)$
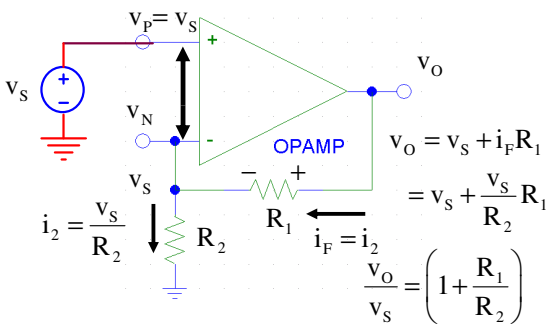
SUMMING AMPLIFIER

R_1

v_1

R_F

OPAMP

R_2 v_N

v_O

v_2

v_P

$v_O = -\left(\dfrac{R_F}{R_1} v_1 + \dfrac{R_F}{R_2} v_2\right)$

DIFFERENTIAL AMP

R_2

R_1 OPAMP

v_O

v_1

R_3

v_2 R_4

Let's Use Superposition
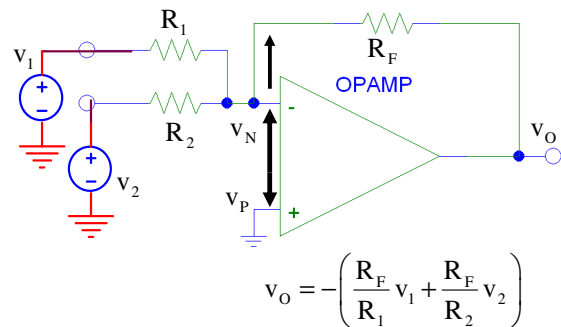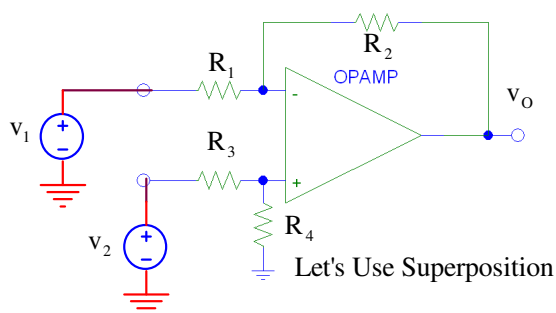
OP AMP CAD ILM

- Go to WebCT Site, Click on Modules
- Click on Op Amp CAD Module
- Move top slider to choose type of circuit
- Inverting, Non-Inverting Amplifier
- Differential Amplifier, Comparator
- Integrator, Differentiator (Later in Course)

http://www.academy.rpi.edu/projects/ccli/module_display.php?ModulesID=11

UNIT 2

Transient Response: First Order/Second Order Circuits

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REVIEW: LECTURE 8

- Signals and waveforms
  - DC waveforms
  - Unit step functions
  - Ramp functions
  - Exponential functions
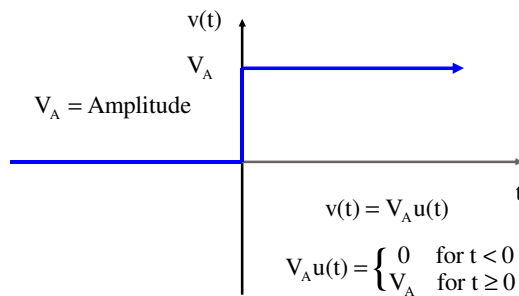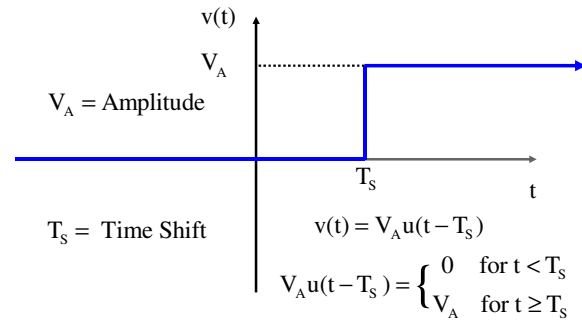 - Sinusoidal functions

sawyes@rpi.edu   www.rpi.edu/~sawyes

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STEP FUNCTION

$v(t)$

V_A

V_A = Amplitude

t

$$v(t) = V_A u(t)$$

$$V_A u(t) = \begin{cases} 0 & \text{for } t < 0 \\ V_A & \text{for } t \geq 0 \end{cases}$$

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DELAYED STEP FUNCTION

$v(t)$

V_A

V_A = Amplitude

T_S

t

T_S = Time Shift

$$v(t) = V_A u(t - T_S)$$

$$V_A u(t - T_S) = \begin{cases} 0 & \text{for } t < T_S \\ V_A & \text{for } t \geq T_S \end{cases}$$

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EXPONENTIAL FUNCTION

$v(t)$

V_A

$.368 V_A$

T_c

t

$$v(t) = [V_A e^{-t/T_C}] u(t)$$

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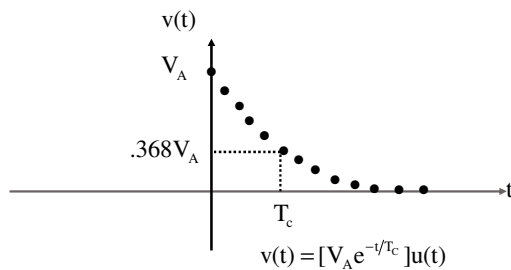
SINUSOIDAL WAVEFORM

See Pages 219 - 226 Thomas and Rosa

$$v(t) = V_A \cos(\omega t + \phi)$$

https://www.youtube.com/watch?v=QFi16s4RXXY

Amplitude = V_A; volts

Angular Frequency = $\omega = 2\pi f$; radians/sec

f = Frequency = $\dfrac{1}{T_0}$; hertz

http://www.analyzemath.com/unitcircle/unit_circle_applet.html

T_0 = Period; seconds

ϕ = Phase Angle; degrees

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REVIEW: LECTURE 9

- First order RC and RL circuits
- Already a solved problem!
- Get RC and RL into the form of the solved problem
  - Find Thevenin Equivalent circuit
  - Find τ
- Find coefficients
  - Need $t \to \infty$ and initial condition t=0+

DYNAMIC CIRCUITS

$$y(t) = y_H + y_P$$

Homogeneous Response + Particular Response

$$y(t) = y_N + y_F$$

Natural Response + Forced Response

$$y_N = y_H; \quad y_F = y_P$$

$$y(t) = y_{ZI} + y_{ZS}$$

Zero-Input Response + Zero-State Response

RC CIRCUITS

Solution to Any Current or Voltage in Any Circuit Containing 1 C plus R's, Independent Sources and Dependent Sources, with a Switched DC Input:

$$y(t) = y_{SS} + (y_0 - y_{SS})e^{-(t-t_0)/\tau} \quad \text{for } t \geq t_0$$

$$\tau = R_{eq}C$$

Can Find y_0, y_{SS}, τ Directly From Circuit

R_{eq} = Equivalent Resistance Seen at Terminals of C

RL CIRCUITS

Solution to Any Current or Voltage in Any Circuit Containing 1 L plus R's, Independent Sources and Dependent Sources, with a Switched DC Input:

$$y(t) = y_{SS} + (y_0 - y_{SS})e^{-(t-t_0)/\tau} \quad \text{for } t \geq t_0$$

$$\tau = \frac{L}{R_{eq}}$$

Can Find y_0, y_{SS}, τ Directly From Circuit

R_{eq} = Equivalent Resistance Seen at Terminals of L

REVIEW: LECTURE 10

- Second order Series RLC and Parallel RLC
- Already solved problems!
- Get into standard from and find α, ω_0 and β (if needed)
- Compare α, ω_0 to find form of solution
- Find coefficients
- Need $t \to \infty$ and initial conditions both Vc(0+) and dVc(0+)/dt for example

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SERIES RLC CIRCUITS
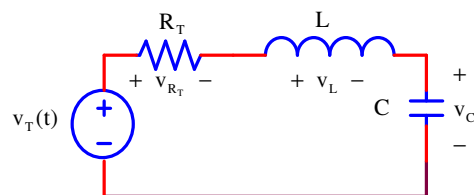
$$LC\frac{d^2v_C}{dt^2} + R_T C\frac{dv_C}{dt} + v_C = v_T$$

$$\frac{d^2v_C}{dt^2} + \frac{R_T}{L}\frac{dv_C}{dt} + \frac{1}{LC}v_C = \frac{1}{LC}v_T$$

SERIES RLC CIRCUITS
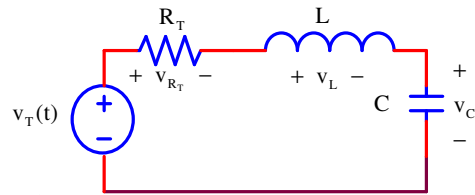
$$\frac{d^2 v_C}{dt^2} + \frac{R_T}{L}\frac{dv_C}{dt} + \frac{1}{LC}v_C = \frac{1}{LC}v_T$$

$$\left[\frac{1}{LC}\right] = \frac{1}{(\text{seconds})^2} = \omega_0^2 \qquad \left[\frac{R_T}{L}\right] = \frac{1}{\text{seconds}} = 2\alpha$$

$$\frac{d^2 v_C}{dt^2} + 2\alpha\frac{dv_C}{dt} + \omega_0^2 v_C = \omega_0^2 v_T$$

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SERIES RLC CIRCUITS

$$\frac{d^2 v_C}{dt^2} + 2\alpha\frac{dv_C}{dt} + \omega_0^2 v_C = \omega_0^2 v_T \quad \text{Need Initial Conditions}$$

$$v_C(0^+) \text{ and } \frac{dv_C}{dt}(0^+) \qquad \frac{dv_C}{dt}(0^+) = \frac{1}{C}i_L(0^+)$$

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SERIES RLC CIRCUITS

$$\frac{d^2 v_{CN}}{dt^2} + 2\alpha\frac{dv_{CN}}{dt} + \omega_0^2 v_{CN} = 0$$

Assume $v_{CN}(t) = Ke^{st}$

$$s^2 + 2\alpha s + \omega_0^2 = 0$$

Characteristic Equation

Roots are $s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$

$$\boxed{v_{CN}(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t}}$$

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SERIES RLC CIRCUITS

$$s^2 + 2\alpha s + \omega_0^2 = 0$$

Roots are $s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$

3 Possible Cases:

Case 1: $\alpha^2 > \omega_0^2$: 2 Real, Unequal Roots

Case 2: $\alpha^2 = \omega_0^2$: 2 Real, Equal Roots

Case 3: $\alpha^2 < \omega_0^2$: 2 Complex Conjugate Roots

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SERIES RLC CIRCUITS

Case 1: $\alpha^2 > \omega_0^2$: 2 Real, Unequal Roots

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} \qquad \alpha = \frac{R_T}{2L}$$
$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} \qquad \omega_0^2 = \frac{1}{LC}$$

$$\boxed{v_{CN} = K_1 e^{s_1 t} + K_2 e^{s_2 t}}$$

2 Decaying Exponentials

Circuit is Overdamped

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SERIES RLC CIRCUITS

Case 2: $\alpha^2 = \omega_0^2$: 2 Real, Equal Roots

$$s_1 = -\alpha \qquad \alpha = \frac{R_T}{2L} \qquad \omega_0^2 = \frac{1}{LC}$$
$$s_2 = -\alpha$$

$$\boxed{v_{CN} = K_1 e^{-\alpha t} + K_2 t e^{-\alpha t}}$$

Decaying Exponential + Exponentially Damped Ramp

Circuit is Critically Damped

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SERIES RLC CIRCUITS

Case 3: $\alpha^2 < \omega_0^2$: 2 Complex Conjugate Roots

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -\alpha + j\sqrt{\omega_0^2 - \alpha^2} = -\alpha + j\beta$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -\alpha - j\sqrt{\omega_0^2 - \alpha^2} = -\alpha - j\beta$$

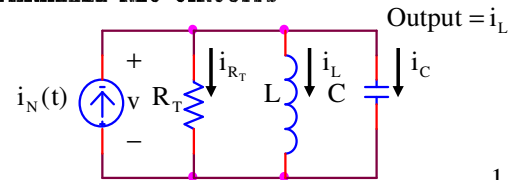$$v_{CN} = K_1 e^{(-\alpha + j\beta)t} + K_2 e^{(-\alpha - j\beta)t}$$
$$v_{CN} = A e^{-\alpha t} \cos(\beta t + \phi)$$

Exponentially Damped Sinusoid

Circuit is Underdamped

PARALLEL RLC CIRCUITS

Output $= i_L$

$$\frac{d^2 i_L}{dt^2} + 2\alpha \frac{di_L}{dt} + \omega_0^2 i_L = \omega_0^2 i_N$$

$$\alpha = \frac{1}{2R_T C}$$

Same Form of Equation as for Series RLC

$$\omega_0^2 = \frac{1}{LC}$$

Slightly Different α

PARALLEL RLC CIRCUITS

Parallel RLC Circuits

LHS of Differential Equation is Same for Any Output

Natural Response for Any Output

$$\frac{d^2 y_N}{dt^2} + 2\alpha \frac{dy_N}{dt} + \omega_0^2 y_N = 0$$

Same as for Series RLC Circuits

PARALLEL RLC CIRCUITS

Natural Response

$$\frac{d^2 y_N}{dt^2} + 2\alpha \frac{dy_N}{dt} + \omega_0^2 y_N = 0$$

Characteristic Equation

$$s^2 + 2\alpha s + \omega_0^2 = 0$$

Same Roots as for Series RLC

Overdamping, Critical Damping, Underdamping

REVIEW: LECTURE 12 AND 13

- Laplace transforms
- Finding poles and zeros
- Partial Fraction Expansion
  - Simple real poles
  - Complex conjugate poles
  - Double poles
- Relationship to differential equations
- S-domain impedances (zero and non-zero initial conditions)

LAPLACE TRANSFORMS

Signal	f(t)	F(s)
Impulse	$\delta(t)$	1
Step	u(t)	$\dfrac{1}{s}$
Constant	Au(t)	$\dfrac{A}{s}$
Ramp	tu(t)	$\dfrac{1}{s^2}$

LAPLACE TRANSFORMS

Signal	f(t)	F(s)
Exponential	$e^{-\alpha t}u(t)$	$\dfrac{1}{s+\alpha}$
Damped Ramp	$[te^{-\alpha t}]u(t)$	$\dfrac{1}{(s+\alpha)^2}$
Cosine Wave	$[\cos\beta t]u(t)$	$\dfrac{s}{s^2+\beta^2}$
Damped Cosine	$[e^{-\alpha t}\cos\beta t]u(t)$	$\dfrac{s+\alpha}{(s+\alpha)^2+\beta^2}$

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LAPLACE TRANSFORMS

Time Domain	s-Domain
$Af_1(t)+Bf_2(t)$	$AF_1(s)+BF_2(s)$
$\displaystyle\int_0^t f(\tau)\,d\tau$	$\dfrac{F(s)}{s}$
$\dfrac{df(t)}{dt}$	$sF(s)-f(0^-)$
$e^{-\alpha t}f(t)$	$F(s+\alpha)$
$t\,f(t)$	$-dF(s)/ds$
$f(t-a)u(t-a)$	$e^{-as}F(s)$

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POLES AND ZEROS

$$F(s)=\frac{b_m s^m + \ ......\ + b_1 s + b_0}{a_n s^n + ...\ + a_1 s + a_0}$$

Factor F(s):

$$F(s)=K\,\frac{(s-z_1)\,(s-z_2)\,(.....)\,(s-z_m)}{(s-p_1)\,(s-p_2)\,(.....)\,(s-p_n)}$$

$$K=\frac{b_m}{a_n}=\text{Scale Factor}$$

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POLES AND ZEROS

$$F(s)=K\,\frac{(s-z_1)\,(s-z_2)\,(.....)\,(s-z_m)}{(s-p_1)\,(s-p_2)\,(.....)\,(s-p_n)}$$

At $s=z_i\Rightarrow F(s)\to 0\ \Rightarrow$ Zeros of F(s)

At $s=p_j\Rightarrow F(s)\to\infty\Rightarrow$ Poles of F(s)
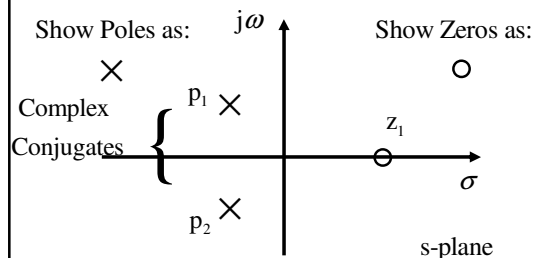
Poles and Zeros are "Critical Frequencies" of F(s)

Useful to Plot "Pole-Zero Diagram" in s-plane

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POLE-ZERO DIAGRAMS

Show Poles as: $\times$

Show Zeros as: $\bigcirc$

Complex Conjugates

s-plane

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PARTIAL FRACTION EXPANSION

There are only 3 Types of Poles:

Simple, Real Poles: $(s-4),\ \Rightarrow p_1=4$

Real, Equal Poles: $(s+3)^2,\ \Rightarrow p_1=p_2=-3$

Complex Conjugate Poles: $(s^2+8s+25)$
$$\Rightarrow p_1,p_2=-4\pm j3$$

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PARTIAL FRACTION EXPANSION

For m < n:

- Simple Real Poles

In General:

Expand: $F(s) = \dfrac{A_1}{s-p_1} + \dfrac{A_2}{s-p_2} + \dfrac{A_3}{s-p_3} + \dots$

$A_n = [(s-p_n)F(s)]\big|_{s=p_n}$; Cover-Up Rule

$\Rightarrow f(t) =)A_1 e^{p_1 t} + A_2 e^{p_2 t} + A_3 e^{p_3 t} + \dots)$ $t \geq 0$

PARTIAL FRACTION EXPANSION

- Complex Conjugate Poles

In General:

Expand $F(s) = \dfrac{A_1}{s-p_1} + \dots + \dfrac{A}{s+\alpha - j\beta} + \dfrac{A^*}{s+\alpha + j\beta}$

Find A_1 and $A = |A|\underline{/\phi}$ from Cover-Up Rule

$\Rightarrow f(t) = A_1 e^{p_1 t} + \dots + 2|A|e^{-\alpha t}\cos(\beta t + \phi)$ $t \geq 0$

   Simple Poles   Complex Poles

PARTIAL FRACTION EXPANSION

- Real, Equal Poles – Double Pole:

Expand $F(s) = \dfrac{A_1}{s-p_1} + \dots + [\dfrac{A_{n1}}{s-p_n} + \dfrac{A_{n2}}{(s-p_n)^2}]$

$A_{n2} = \left[(s-p_n)^2 F(s)\right]\big|_{s=p_n}$; Cover-Up Rule

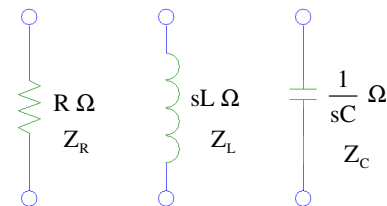Usually Find A_{n1} from evaluating $F(0)$ or $F(1)$

$\Rightarrow f(t) = (A_1 e^{p_1 t} + \dots + A_{n1}e^{p_n t} + A_{n2}te^{p_n t})$ $t \geq 0$

   Simple Poles   Repeated Poles
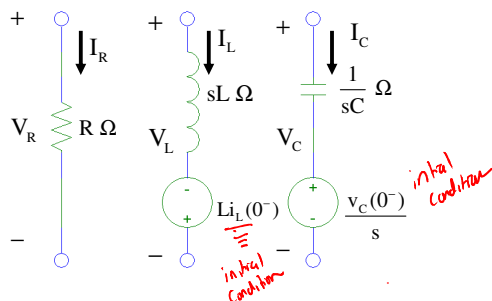

IMPEDANCE

Zero initial conditions

$R\ \Omega$ $sL\ \Omega$ $\dfrac{1}{sC}\ \Omega$

Z_R Z_L Z_C

$Z = \text{Impedance} = \dfrac{V(s)}{I(s)}$

NON-ZERO INITIAL CONDITIONS

$+\ I_R$ $+\ I_L$ $+\ I_C$

$V_R\ R\ \Omega$ $V_L\ sL\ \Omega$ $V_C\ \dfrac{1}{sC}\ \Omega$

$Li_L(0^-)$ *initial condition*

$\dfrac{v_C(0^-)}{s}$ *initial condition*

initial condition

REVIEW: LECTURE 14 CIRCUIT ANALYSIS

- Essentially Unit 1 + Unit 2 in one problem

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GENERAL PROCESS

s-Domain Circuit Analysis

Time domain (t domain)		Complex frequency domain (s domain)
Linear Circuit	Laplace Transform $\mathcal{L}$	Transformed Circuit
Differential equation	Laplace Transform $\mathcal{L}$	Algebraic equation
Classical techniques		Algebraic techniques
Response waveform	Inverse Transform $\mathcal{L}^{-1}$	Response transform

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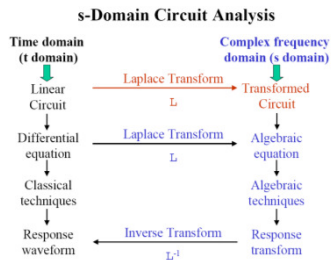
CIRCUITS WITH LAPLACE

1. Find Initial Conditions
2. Determine Laplace Equivalent circuit
3. Use Unit 1 concepts (node/mesh/voltage dividers etc.) to find an expression for the parameter of interest (impedances)
   a. "Clean up" expression to have $\dfrac{N(s)}{D(s)}$
4. Find poles (zeros, Unit 3)
5. Partial fraction expansion
   a. Cover up rule for coefficients or $F(0), F(1)$
6. Inverse Laplace gives time domain response

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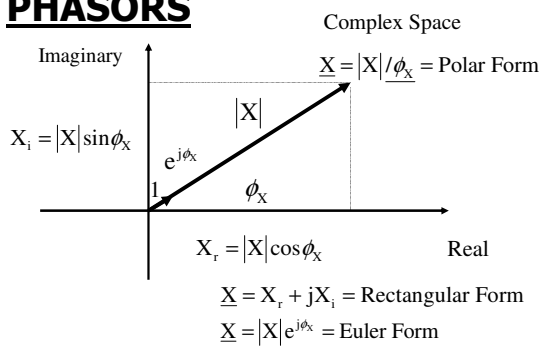
UNIT 3

AC Steady State: Phasor Analysis and Power Circuits

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LECTURE 18.1 REVIEW

- Transfer Functions
- Phasors
- Phasor Math

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PHASORS

Complex Space

Imaginary

$\underline{X} = |X| / \underline{\phi_X} = $ Polar Form

$|X|$

$X_i = |X| \sin \phi_X$

$e^{j\phi_X}$

1 ϕ_X

$X_r = |X| \cos \phi_X$ Real

$\underline{X} = X_r + jX_i = $ Rectangular Form

$\underline{X} = |X| e^{j\phi_X} = $ Euler Form

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PHASORS

- 3 Ways to Express Phasors

  Rectangular Form; $\underline{X} = X_r + jX_i$

  Polar Form; $\underline{X} = |X| / \underline{\phi_X}$

  Euler Form; $\underline{X} = |X| e^{j\phi_X}$

- Will Need to Be Able to Easily Convert Between the 3 Different Forms

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LECTURE 19.1 AGENDA

- Kirkoff's laws for phasors
- AC steady state impedence

K'S LAWS FOR PHASORS

- KCL:
  - If $i_1 + i_2 = i;$ $\Rightarrow \underline{I}_1 + \underline{I}_2 = \underline{I}$
- KVL:
  - If $v_1 + v_2 = v;$ $\Rightarrow \underline{V}_1 + \underline{V}_2 = \underline{V}$
- K's Laws Work for Phasors!
  - Complex Addition, not Simple Addition

AC STEADY STATE IMPEDANCE

$$Z_R = R \ \Omega$$
$$Z_L = j\omega L \ \Omega$$
$$Z_C = -\frac{j}{\omega C} = \frac{1}{j\omega C} \ \Omega$$

AC STEADY STATE IMPEDANCE

- In General, $\underline{V} = Z \ \underline{I}$ in AC Steady State:
  - Z = AC SS Impedance
  - Units of Ohms
  - Ohm's Law for AC Steady State
- Y = AC Steady State Admittance
  - = 1/Z   (Units of mhos)

AC STEADY STATE IMPEDANCE

$\underline{V} = Z\underline{I};$ Ohm's Law for AC Steady State

$Z = R(\omega) + jX(\omega) = $ AC Steady State Impedance

   $R(\omega) = $ AC Steady State Resistance

   $X(\omega) = $ AC Steady State Reactance

$Y = G(\omega) + jB(\omega) = $ AC Steady State Admittance

   $G(\omega) = $ AC Steady State Conductance

   $B(\omega) = $ AC Steady State Susceptance
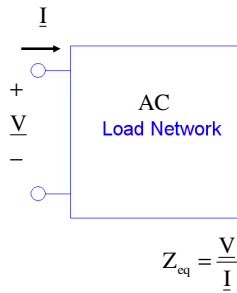
LECTURE 20.1 AGENDA

- AC Thevenin/Norton circuits
- AC node equations
- ~~AC mesh equations~~ (not on the test)
- ~~AC bridge circuits~~ (not on the test)

EQUIVALENT IMPEDANCE

$$Z_{eq} = \frac{V}{I}$$

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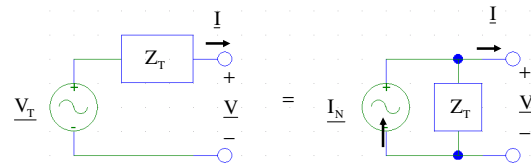
AC THEVENIN/NORTON

AC Thevenin Circuit   AC Norton Circuit

$$\underline{V}_T = \underline{I}_N\, Z_T$$

$Z_T = Z_{eq}$ of Dead Source Network

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AC SOURCE CONVERSIONS

$$\underline{I}_s = \frac{\underline{V}_s}{Z_s}$$

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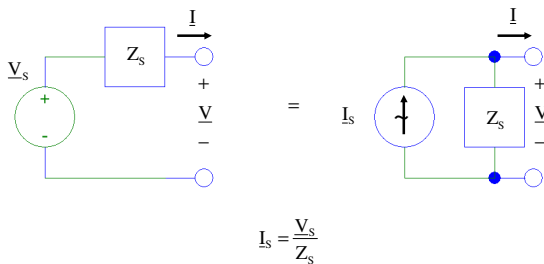
IMPEDANCE BRIDGES

Parallel voltage dividers

$$V_{.M} = V_{.A} - V_{.B} = \left(\frac{Z_{.2}}{Z_{.1} + Z_{.2}}\right)\cdot V_{.s} - \left(\frac{Z_{.u}}{Z_{.3} + Z_{.u}}\right)\cdot V_{.s}$$

$$V_{.M} = \left[\frac{Z_{.2}\cdot Z_{.3} - Z_{.1}\cdot Z_{.u}}{\left(Z_{.1} + Z_{.2}\right)\cdot\left(Z_{.3} + Z_{.u}\right)}\right]\cdot V_{.s}$$

VM is zero when
$$Z_2 Z_3 = Z_1 Z_u$$

$$Z_{.u} = \frac{Z_{.2}\cdot Z_{.3}}{Z_{.1}} = R_{.X} + jX_{.X}$$

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LECTURE 21.1

- Review AC Power
  - Complex Power
  - Real Power
  - Reactive Power
  - Apparent Power
- Power Factor

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REACTIVE POWER

Define P = "Real Power" = $V_{RMS}I_{RMS}\cos\theta$

P is Measured in Watts

Define Q = "Reactive Power" = $V_{RMS}I_{RMS}\sin\theta$

Q is Measured in VAR's

(Volt-Amperes-Reactive)

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REACTIVE POWER

- **Q is a Measure of the Rate of Change of Energy Stored in the Reactive Elements (L, C):**
  - **Power companies must worry about Q since they supplied this energy**
  - **Supplied Q over their Lines => Real Cost**
  - **Power companies want customers to have Low Q**

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REACTIVE POWER

$$P = I^2_{RMS} |Z| \cos\theta$$
$$= I^2_{RMS} \ R(\omega)$$
$$= V_{RMS}I_{RMS}\cos\theta$$

$\left.\begin{array}{l}\text{Equivalent ways of}\\ \text{expressing Real Power}\end{array}\right.$
[Watts]

$$Q = I^2_{RMS} |Z| \sin\theta$$
$$= I^2_{RMS} \ X(\omega)$$
$$= V_{RMS}I_{RMS}\sin\theta$$

$\left.\begin{array}{l}\text{Equivalent ways of}\\ \text{expressing Reactive Power}\end{array}\right.$
[VAR's]

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REACTIVE POWER

- **Notes on Reactive Power:**
  - **Real Power = P is always $\geq$ 0**
  - **Reactive Power = Q can be $\geq$ 0 or $\leq$ 0**
  - **For Inductive Load, X > 0 => Q > 0**
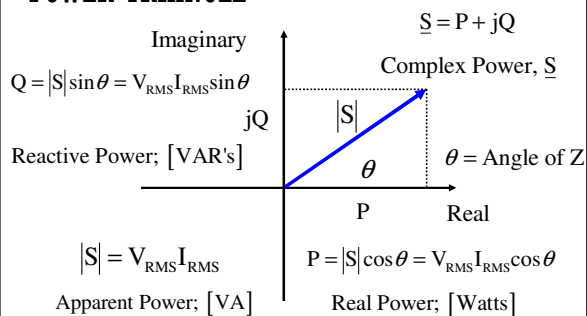 - **For Capacitive Load, X < 0 => Q < 0**

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APPARENT POWER

$$\text{Magnitude of } \underline{S} = |S| = \sqrt{P^2 + Q^2} = V_{RMS}I_{RMS}$$

$$|S| = \text{"Apparent Power"} => [\text{Volt-Amperes}]$$

$$|S| = \text{Product of } V_{RMS} \text{ x } I_{RMS} \text{ at Terminals}$$

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POWER TRIANGLE

$$\underline{S} = P + jQ$$
Imaginary
Complex Power, $\underline{S}$
$$Q = |S|\sin\theta = V_{RMS}I_{RMS}\sin\theta$$
jQ $|S|$
Reactive Power; [VAR's] θ $\theta = $ Angle of Z
P   Real
$$|S| = V_{RMS}I_{RMS}$$ $$P = |S|\cos\theta = V_{RMS}I_{RMS}\cos\theta$$
Apparent Power; [VA]   Real Power; [Watts]

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POWER FACTOR

Imaginary $\underline{S}$
jQ $|S|$
θ
P   Real

$$\cos\theta = \frac{P}{|S|} = \frac{\text{Real Power}}{\text{Apparent Power}} = \text{Power Factor} = pf$$

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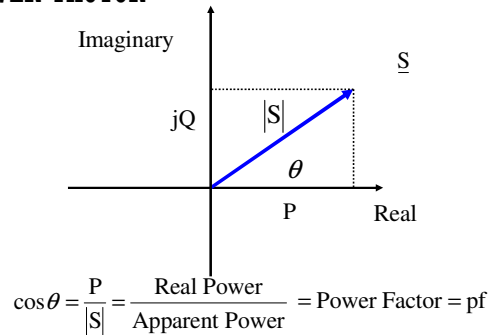
POWER FACTOR

For Inductive Loads, $\theta > 0$; $\cos\theta > 0$

For Capacitive Loads, $\theta < 0$; $\cos\theta > 0$

Need a Way to Distinguish

$$\underline{I} = \frac{\underline{V}}{\underline{Z}} = \frac{|V|\underline{/\phi}}{|Z|\underline{/\theta}} = \frac{|V|}{|Z|}\underline{/\phi - \theta}$$

If $\theta > 0$; $\Rightarrow$ Lagging Power Factor ($\underline{I}$ lags $\underline{V}$)

If $\theta < 0$; $\Rightarrow$ Leading Power Factor ($\underline{I}$ leads $\underline{V}$)

POWER FACTOR

Power Factor:

Define pf $= \cos\theta$; $\quad 0 \le$ pf ≤ 1

Must distinguish between $\theta \ge 0$, $\theta \le 0$:

$\theta \ge 0$; $X \ge 0$; $Q \ge 0$; $\underline{I}$ lags $\underline{V}$; $\quad$ lagging pf

$\theta \le 0$; $X \le 0$; $Q \le 0$; $\underline{I}$ leads $\underline{V}$; $\;$ leading pf

e.g: pf $= .8$ lagging $\Rightarrow$ Inductive Load
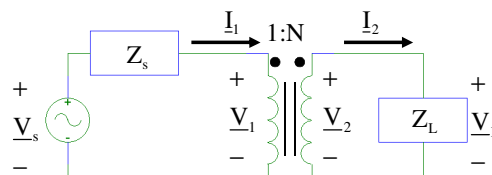$\qquad$ pf $= .8$ leading $\Rightarrow$ Capacitive Load

---

LECTURE 22.1

- Coupled Inductors
- Ideal Transformer
- Transformer Circuit
- Power Transfer
- Impedance Matching
- Mutual Inductance (Tee Model)

TRANSFORMER CIRCUIT

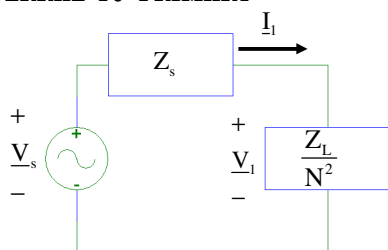

2 Choices for the Equivalent Circuit

Refer Secondary Circuit to the Primary

OR

Refer Primary Circuit to the Secondary

REFERRAL TO PRIMARY

Equivalent to Basic Transformer Circuit
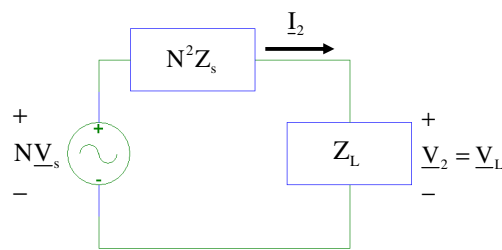
Can Now Do AC Steady State Circuit Analysis

REFERRAL TO SECONDARY

Equivalent to Basic Transformer Circuit

Can Now Do AC Steady State Circuit Analysis
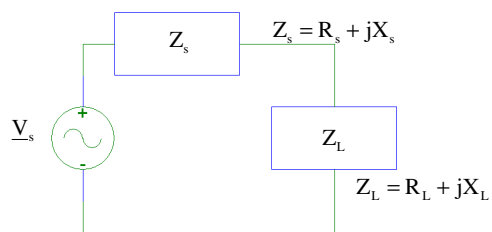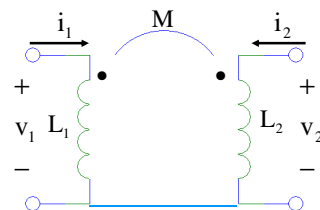
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POWER TRANSFER

$Z_s = R_s + jX_s$

$\underline{V}_s$

Z_s

Z_L

$Z_L = R_L + jX_L$

For Maximum Power to Z_L Choose $Z_L = Z_s^*$

$\Rightarrow R_L = R_s$ and $X_L = -X_s$

MUTUAL INDUCTANCE

i_1 M i_2

$+$ v_1 L_1 L_2 v_2 $+$

$-$ $-$
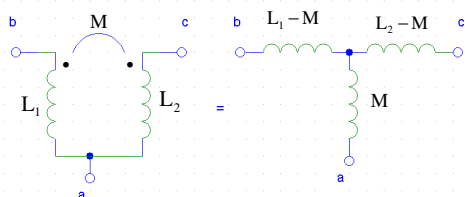
$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

TEE MODEL

M $L_1 - M$ $L_2 - M$

L_1 L_2 $=$ M

Transformer-like Model   Tee Model

If Dots on Opposite Sides $\Rightarrow M \rightarrow -M$
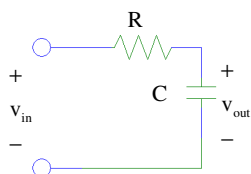
Some Inductors in Tee Model May Be Negative!

UNIT 4

AC Steady State: Frequency Response

1st ORDER LOW PASS FILTER

R

$+$ v_{in} C $+$ v_{out} $-$
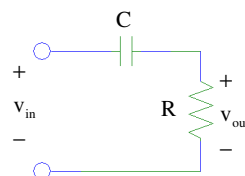
$-$

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{1/RC}{s + 1/RC} = \frac{\omega_c}{s + \omega_c} \quad \Rightarrow \text{Low Pass Filter}$$

Low Frequencies "Pass"; High Frequencies "Stopped"

1st ORDER HIGH PASS FILTER

C

$+$ v_{in} R $+$ v_{out} $-$

$-$

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{s}{s + 1/RC} = \frac{s}{s + \omega_c} \quad \Rightarrow \text{High Pass Filter}$$

High Frequencies "Pass"; Low Frequencies "Stopped"

LOW PASS + HIGH PASS

$$H(s) = K\left(\frac{s}{s+\omega_{cH}}\right)\left(\frac{\omega_{cL}}{s+\omega_{cL}}\right)$$

$$|H(j\omega)| = |K|\left(\frac{\omega}{\sqrt{\omega^2+\omega_{cH}^2}}\right)\left(\frac{\omega_{cL}}{\sqrt{\omega^2+\omega_{cL}^2}}\right)$$
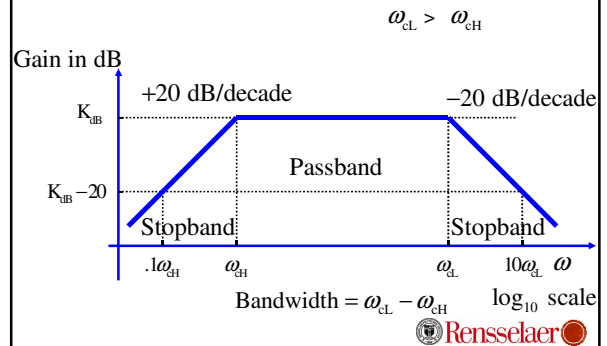
High Pass   Low Pass

Let's Design Such that $\omega_{cL} > \omega_{cH}$

$$\Rightarrow R_H C_H > R_L C_L$$

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BANDPASS FILTER

$\omega_{cL} > \omega_{cH}$

Bandwidth $= \omega_{cL} - \omega_{cH}$ $\log_{10}$ scale

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LOW PASS + HIGH PASS

$$H(s) = H_L(s) + H_H(s) = \left(1+\frac{R_A}{R_B}\right)\left(\frac{\omega_{cL}}{s+\omega_{cL}} + \frac{s}{s+\omega_{cH}}\right)$$

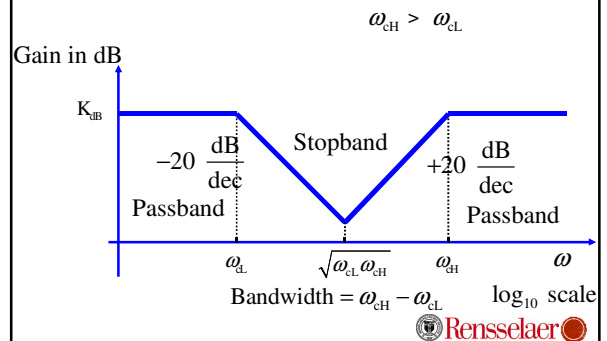For Low Frequencies $\Rightarrow$ Looks Like a 1st Order Low Pass

For High Frequencies $\Rightarrow$ Looks Like a 1st Order High Pass

Let's Design Such that $\omega_{cH} > \omega_{cL}$

$$\Rightarrow R_L C_L > R_H C_H$$

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BANDGAP OR NOTCH FILTER

$\omega_{cH} > \omega_{cL}$

Bandwidth $= \omega_{cH} - \omega_{cL}$ $\log_{10}$ scale

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2ND ORDER PROCESS SUMMARY

Overdamped
1) Find Poles
2) Identify Regions
3) Build Straight Line Approximations
4) Add corrections (-3db)

Rensselaer 131

2ND ORDER PROCESS SUMMARY

Critically Damped
1) Find Poles
2) Identify Regions
3) Build Straight Line Approximations
4) Add corrections (-6db)

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2ND ORDER PROCESS SUMMARY
Underdamped LPF, HPF
1) Start with critically damped case $\omega_c = \omega_o$
2) Sketch Straight Line Approximations away from ω_o
3) At ω_o 20 log abs $H(j\,\omega_o)$=20 log$(1/(2\,\zeta))$ > -6 dB relative to passband

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2ND ORDER PROCESS SUMMARY
Underdamped BPF
1. Asymptotes take the form of inverted V
2. Each side of V has 20 dB rolloff
3. At ω_o 20 log abs $H(j\,\omega_o)$=0 dB ALWAYS
4. The point of the inverted V is 20 log abs $H(j\,\omega_o)$ away from 0dB
5. Use 20 log$(2\,\zeta)$ to find this point pulling V up or down relative to 0dB making it narrow or wide

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FILTER TYPE SUMMARY (FROM H(S))

- *First order filters*
  - Low pass: (no zeros), 1 pole
  - High pass: 1 zero at origin, 1 pole

- *Second order filters*
  - Low pass: 2 poles
  - High pass: 2 zeros at origin, 2 pole
  - Bandpass filter: 1 zero, 2 poles
  - Notch filter:

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DAMPING RATIO

$$\zeta = \alpha/\omega$$

- $\zeta > 1$ Overdamped
- $\zeta = 1$ Critically damped
- $\zeta < 1$ Underdamped
  - $1 > \zeta > 0.5$ Correction is a –db of some value
  - $\zeta = 0.5$ Correction is 0db
  - $\zeta < 0.5$ Correction is +db (Strongly underdamped which means there is a peak!)

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Congratulations, you are officially
electrical engineering students!

Now you must become electrical engineers/computer systems engineers/dual major engineers…….may the (electromagnetic) Force be with you…

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