Questions:
What do I need to know about this class?
What is voltage? What is a voltage difference/voltage drop?
What is current?
What is polarity?
How do we define power?
What is a circuit model?
What is a ground?
What is the IV characteristic of an open circuit? a short circuit? an ideal voltage source?
an ideal current source? a resistor? a diode?
What is Ohm's Law?
What is a node in a circuit?
What is a closed loop in a circuit?
How do we apply Kirchhoff's Voltage Law (KVL)? Kirchhoff's Current Law (KCL)?


Finding I (left)

Current goes to the right

$$
\begin{aligned}
& \mathrm{V}_{\mathrm{R} 2 \mathrm{a}}:=\mathrm{V}_{1 \mathrm{a}}-\mathrm{V}_{2 \mathrm{a}} \\
& \mathrm{~V}_{\mathrm{R} 2 \mathrm{a}}=1 \mathrm{~V} \\
& \mathrm{I}_{\mathrm{R} 2}:=\frac{\mathrm{V}_{\mathrm{R} 2 \mathrm{a}}}{\mathrm{R}_{2 \mathrm{a}}}=250 \cdot \mathrm{~mA}
\end{aligned}
$$

Find V (right) (KVL clockwise loop)

$$
-v_{2 a}+v_{3 a}+v_{n 3}=0
$$

$$
\mathrm{V}_{\mathrm{n} 3}:=\mathrm{V}_{2 \mathrm{a}}-\mathrm{V}_{3 \mathrm{a}}
$$

$$
\mathrm{V}_{\mathrm{n} 3}=-2 \mathrm{~V}
$$

Finding I (right) (KCL) Finding V ( top) KCL
Node 3

$$
\begin{array}{lc}
\frac{-\mathrm{V}_{\mathrm{n} 3}}{\mathrm{R}_{1 \mathrm{a}}}-\mathrm{I}_{\mathrm{n} 3}+\mathrm{I}_{2 \mathrm{a}}=0 & \mathrm{I}_{1}-\mathrm{I}_{2}-\mathrm{I}_{\mathrm{n} 2}=0 \\
\mathrm{I}_{\mathrm{n} 3}:=-\left(-\mathrm{I}_{2 \mathrm{a} 2}: \frac{\mathrm{V}_{\mathrm{n} 3}}{\mathrm{R}_{1 \mathrm{a}}}\right) & \mathrm{I}_{\mathrm{n} 2}=-\left(-\mathrm{I}_{1 \mathrm{a}}+\mathrm{I}_{2 \mathrm{a}}\right) \\
\mathrm{I}_{\mathrm{n} 3}=3 \mathrm{~A} & \mathrm{I}_{\mathrm{n} 2} \cdot \mathrm{R}_{4 \mathrm{a}}+\mathrm{V}_{2 \mathrm{a}}=3 \mathrm{~V}
\end{array}
$$

b)


Assign polarities to each resistor and apply KCL or KVL to obtain three independent expression for the voltage across each resistor

$$
\mathrm{VR} 1=\mathrm{VR} 2=\mathrm{VR} 3=6 \mathrm{~V}
$$

c)


1k

Node 1


Ask about polarity

Assign polarities to each resistor and apply KCL or KVL to obtain three independent expressions for the voltage across each resistor

$$
\begin{aligned}
& \mathrm{R}_{1 \mathrm{c}}:=3 \mathrm{k} \Omega \\
& \mathrm{R}_{2 \mathrm{c}}:=2 \mathrm{k} \Omega \\
& \mathrm{R}_{3 \mathrm{c}}:=1 \mathrm{k} \Omega
\end{aligned}
$$

Do KVL

$$
-6+\mathrm{V}_{\mathrm{R} 1 \mathrm{c}}+\mathrm{V}_{\mathrm{R} 2 \mathrm{c}}+\mathrm{V}_{\mathrm{R} 3 \mathrm{c}}=0
$$

Then do KCL at
Nodes,

$$
\mathrm{i}_{1 \mathrm{c}}-\mathrm{i}_{2 \mathrm{c}}=0
$$

substitute ohms law

$$
\frac{\mathrm{V}_{\mathrm{R} 1 \mathrm{c}}}{\mathrm{R}_{1 \mathrm{c}}}-\frac{\mathrm{V}_{\mathrm{R} 2 \mathrm{c}}}{\mathrm{R}_{2 \mathrm{c}}}=0
$$

Node 2

$$
\begin{aligned}
& \mathrm{i}_{2 \mathrm{c}}-\mathrm{i}_{3 \mathrm{c}}=0 \\
& \frac{\mathrm{~V}_{\mathrm{R} 2 \mathrm{c}}}{\mathrm{R}_{2 \mathrm{c}}}-\frac{\mathrm{V}_{\mathrm{R} 3 \mathrm{c}}}{\mathrm{R}_{3 \mathrm{c}}}=0
\end{aligned}
$$

solve for VR1c and VR3c (get in terms of VR2c

$$
\begin{aligned}
\mathrm{V}_{\mathrm{R} 1 \mathrm{c}} & =\frac{\mathrm{V}_{\mathrm{R} 2 \mathrm{c}}}{\mathrm{R}_{2 \mathrm{c}}} \cdot \mathrm{R}_{1 \mathrm{c}} \\
\mathrm{~V}_{\mathrm{R} 3 \mathrm{c}} & =\frac{\mathrm{R}_{3 \mathrm{c}} \cdot \mathrm{~V}_{\mathrm{R} 2 \mathrm{c}}}{\mathrm{R}_{2 \mathrm{c}}}
\end{aligned}
$$

Put in KVL
equatinn

$$
\begin{aligned}
& -6+\frac{\mathrm{R}_{1 \mathrm{c}} \cdot \mathrm{~V}_{\mathrm{R} 2}}{\mathrm{R}_{2 \mathrm{c}}}+\frac{\mathrm{R}_{3 \mathrm{c}} \cdot \mathrm{~V}_{\mathrm{R} 2}}{\mathrm{R}_{2 \mathrm{c}}}+\mathrm{V}_{\mathrm{R} 2}=0 \\
& \frac{6 \cdot \mathrm{R}_{2 \mathrm{c}}}{\mathrm{R}_{1 \mathrm{c}}+\mathrm{R}_{2 \mathrm{c}}+\mathrm{R}_{3 \mathrm{c}}}=2 \quad \mathrm{~V}_{\mathrm{R} 2 \mathrm{c}}:=2
\end{aligned}
$$

Do the matrix!

$$
\begin{gathered}
\mathrm{V}_{\mathrm{R} 1 \mathrm{c}}+\mathrm{V}_{\mathrm{R} 2 \mathrm{c}}+\mathrm{V}_{\mathrm{R} 3 \mathrm{c}}=6 \\
\frac{1}{\mathrm{R}_{1 \mathrm{c}}} \cdot \mathrm{~V}_{\mathrm{R} 1 \mathrm{c}}-\frac{1}{\mathrm{R}_{2 \mathrm{c}}} \cdot \mathrm{~V}_{\mathrm{R} 2 \mathrm{c}}+0=0 \\
0+\frac{1}{\mathrm{R}_{2 \mathrm{c}}} \cdot \mathrm{~V}_{\mathrm{R} 2 \mathrm{c}}-\frac{1}{\mathrm{R}_{3 \mathrm{c}}} \cdot \mathrm{~V}_{\mathrm{R} 3 \mathrm{c}}=0 \\
\mathrm{M}_{1}:=\left(\begin{array}{ccc}
1 & 1 & 1 \\
\frac{1}{3 \cdot 10^{3}} \frac{-1}{2 \cdot 10^{3}} & 0 \\
0 & \frac{1}{2 \cdot 10^{3}} & \frac{-1}{1 \cdot 10^{3}}
\end{array}\right) \\
\mathrm{C}_{1}:=\left(\begin{array}{l}
6 \\
0 \\
0
\end{array}\right) \\
\mathrm{X}_{1}:=\mathrm{M}_{1}^{-1} \cdot \mathrm{C}_{1} \\
\mathrm{X}_{1}=\left(\begin{array}{l}
3 \\
2 \\
1
\end{array}\right) \quad \begin{array}{l}
\mathrm{V}_{\mathrm{R} 1 \mathrm{c}}=3 \\
\mathrm{~V}_{\mathrm{R} 2 \mathrm{c}}=2
\end{array} \\
\mathrm{~V}_{\mathrm{R} 3 \mathrm{c}}=1
\end{gathered}
$$

To find VR3

$$
\begin{aligned}
& \mathrm{V}_{\mathrm{R} 3 \mathrm{c}}:=\frac{\mathrm{R}_{3 \mathrm{c}}}{\mathrm{R}_{2 \mathrm{c}}} \cdot \mathrm{~V}_{\mathrm{R} 2 \mathrm{c}} \\
& \mathrm{~V}_{\mathrm{R} 3 \mathrm{c}}=1
\end{aligned}
$$

To find VR1

$$
\begin{aligned}
\mathrm{V}_{\mathrm{R} 1 \mathrm{c}} & :=\frac{\mathrm{R}_{1 \mathrm{c}} \cdot \mathrm{~V}_{\mathrm{R} 2 \mathrm{c}}}{\mathrm{R}_{2 \mathrm{c}}} \\
\mathrm{~V}_{\mathrm{R} 1 \mathrm{c}} & =3
\end{aligned}
$$



Assign polarities to each resistor and apply KCL and/or KVL to obtain three independent expression for the voltage across each resistor


At node 1

$$
\begin{array}{ll}
\mathrm{R}_{1 \mathrm{~d}}:=4 \mathrm{k} \Omega \quad \mathrm{R}_{2 \mathrm{~d}}:=2 \mathrm{k} \Omega \quad \mathrm{R}_{3 \mathrm{~d}}:=1 \mathrm{k} \Omega \quad & \mathrm{I}_{1 \mathrm{~d}}:=2 \mathrm{~mA} \\
& \mathrm{I}_{2 \mathrm{~d}}:=1 \mathrm{~mA}
\end{array}
$$

(1) $\quad-I_{R 1 d}+I_{1 d}-I_{R 2 d}=0$
or $\quad-\frac{\mathrm{V}_{\mathrm{R} 1 \mathrm{~d}}}{\mathrm{R}_{1 \mathrm{~d}}}+\mathrm{I}_{1 \mathrm{~d}}-\frac{\mathrm{V}_{\mathrm{R} 1 \mathrm{~d}}-\mathrm{V}_{\mathrm{R} 3 \mathrm{~d}}}{\mathrm{R}_{2 \mathrm{~d}}}=0$
AT node 2
(2)

$$
\mathrm{I}_{\mathrm{R} 2 \mathrm{~d}}-\mathrm{I}_{2 \mathrm{~d}}-\mathrm{I}_{\mathrm{R} 3 \mathrm{~d}}=0
$$

or $\quad \frac{\mathrm{V}_{\mathrm{R} 1 \mathrm{~d}}-\mathrm{V}_{\mathrm{R} 3 \mathrm{~d}}}{\mathrm{R}_{2 \mathrm{~d}}}-\mathrm{I}_{2 \mathrm{~d}} \cdot-\frac{\mathrm{V}_{\mathrm{R} 3 \mathrm{~d}}}{\mathrm{R}_{3 \mathrm{~d}}}=0$

Solve first
equation
for VR1d

$$
\mathrm{V}_{\mathrm{R} 1 \mathrm{~d}}=\frac{\mathrm{R}_{1 \mathrm{~d}} \cdot \mathrm{~V}_{\mathrm{R} 3 \mathrm{~d}}+\mathrm{I}_{1 \mathrm{~d}} \cdot \mathrm{R}_{1 \mathrm{~d}} \cdot \mathrm{R}_{2 \mathrm{~d}}}{\mathrm{R}_{1 \mathrm{~d}}+\mathrm{R}_{2 \mathrm{~d}}}
$$

Put in (2) equation
$\frac{-V_{R 3 d}+\frac{R_{1 d} \cdot V_{R 3 d}+I_{1 d} \cdot R_{1 d} \cdot R_{2 d}}{R_{1 d}+R_{2 d}}}{R_{2 d}}-I_{2 d}-\frac{V_{R 3 d}}{R_{3 d}}=0$
Solve for VR3
$-\frac{I_{2 d} \cdot R_{1 d} \cdot R_{3 d}-I_{1 d} \cdot R_{1 d} \cdot R_{3 d}+I_{2 d} \cdot R_{2 d} \cdot R_{3 d}}{R_{1 d}+R_{2 d}+R_{3 d}}=285.714 \cdot m V$

$$
\mathrm{V}_{\mathrm{R} 3 \mathrm{~d}}:=285.7 \mathrm{mV}
$$

Put VR3d in equation (3)

$$
\begin{aligned}
& \mathrm{V}_{\mathrm{R} 1 \mathrm{~d}}:=\frac{\mathrm{R}_{1 \mathrm{~d}} \cdot \mathrm{~V}_{\mathrm{R} 3 \mathrm{~d}}+\mathrm{I}_{1 \mathrm{~d}} \cdot \mathrm{R}_{1 \mathrm{~d}} \cdot \mathrm{R}_{2 \mathrm{~d}}}{\mathrm{R}_{1 \mathrm{~d}}+\mathrm{R}_{2 \mathrm{~d}}} \\
& \mathrm{~V}_{\mathrm{R} 1 \mathrm{~d}}=2.857 \mathrm{~V}
\end{aligned}
$$

To find the voltage across R2d you subtract the node 1 from node 2

$$
\begin{aligned}
\mathrm{M}_{2} & :=\left[\begin{array}{ccc}
-1 & 1 & 1 \\
-\left(\frac{1}{4 \cdot 10^{3}}+\frac{1}{2 \cdot 10^{3}}\right) & 0 & \frac{1}{2 \cdot 10^{3}} \\
\frac{1}{2 \cdot 10^{3}} & 0 & -\left(\frac{1}{2 \cdot 10^{3}}+\frac{1}{1 \cdot 10^{3}}\right)
\end{array}\right] \\
\mathrm{C}_{2} & :=\left(\begin{array}{c}
0 \\
-2 \cdot 10^{-3} \\
1 \cdot 10^{-3}
\end{array}\right)
\end{aligned}
$$

Also need loop to solve matrix equation!

$$
\begin{equation*}
-\mathrm{V}_{\mathrm{R} 1 \mathrm{~d}}+\mathrm{V}_{\mathrm{R} 2 \mathrm{~d}}+\mathrm{V}_{\mathrm{R} 3 \mathrm{~d}}=0 \tag{3}
\end{equation*}
$$

So put all equations together

$$
-V_{R 1 d}+V_{R 2 d}+V_{R 3 d}=0
$$

$$
-\left(\frac{1}{\mathrm{R}_{1 \mathrm{~d}}}+\frac{1}{\mathrm{R}_{2 \mathrm{~d}}}\right) \cdot \mathrm{V}_{\mathrm{R} 1 \mathrm{~d}}+0+\frac{1}{\mathrm{R}_{2 \mathrm{~d}}} \cdot \mathrm{~V}_{\mathrm{R} 3 \mathrm{~d}}=-2
$$

$$
\frac{1}{\mathrm{R}_{2 \mathrm{~d}}} \cdot \mathrm{~V}_{\mathrm{R} 1 \mathrm{~d}}+0-\left(\frac{1}{\mathrm{R}_{2 \mathrm{~d}}}+\frac{1}{\mathrm{R}_{3 \mathrm{~d}}}\right) \cdot \mathrm{V}_{\mathrm{R} 3 \mathrm{~d}}=1
$$

voltages.

$$
\mathrm{V}_{\mathrm{R} 1 \mathrm{~d}}-\mathrm{V}_{\mathrm{R} 3 \mathrm{~d}}=2.571 \mathrm{~V}
$$

$$
\mathrm{X}_{2}:=\mathrm{M}_{2}^{-1} \cdot \mathrm{C}_{2} \quad \mathrm{X}_{2}=\left(\begin{array}{l}
2.857 \\
2.571 \\
0.286
\end{array}\right)
$$

