

Questions:

What is a series circuit?

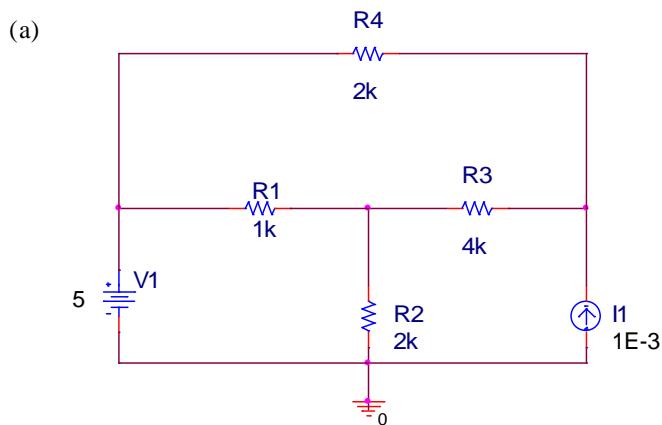
What is a parallel circuit?

How do we combine resistors in series? in parallel?

What is a voltage divider?

What is a current divider?

Review problem



1. Determine the voltage across each resistor
2. Determine the power produced/consumed by V1 and I1

$$I_{1a} := 1 \text{ mA}$$

$$R_{1a} := 1 \text{ k}\Omega$$

$$R_{2a} := 2 \text{ k}\Omega$$

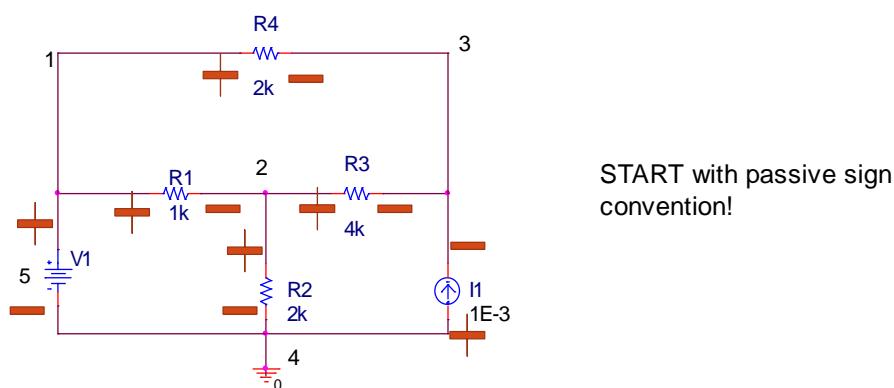
$$V_{1a} := 5 \text{ V}$$

$$R_{3a} := 4 \text{ k}\Omega$$

$$R_{4a} := 2 \text{ k}\Omega$$

Logic: Can't use current divider or voltage divider because components are not in series or parallel!

Label terminals so KVL can be done, also label nodes for KCL



We have four resistors so we need four linearly independent equations to determine the voltage across each resistors. Obtain them using combinations of KCL and KVL. Make the following observations

KCL at node 1: Not a good choice because of the voltage source connected to the node

KCL at node 3: Good choice

KCL at node 2: Good choice

KCL at node 4: Not a good choice because of voltage source connected to the node

KVL in loop 1: Good choice

KVL in loop 2: Good choice

KVL in loop 3: Not a good choice because of current source in the loop

4 equations and 4 unknowns I feel a matrix coming.....

Apply KCL at node 3 (including ohms law)

(all current going in)

$$-\frac{V_{R4a}}{R_{4a}} - \frac{V_{R3a}}{R_{3a}} - I_{1a} = 0$$

For matrix get to standard form

$$0 + 0 + \frac{1}{R_{3a}} \cdot V_{R3a} + \frac{1}{R_{4a}} \cdot V_{R4a} = 0$$

$$0 + 0 - \frac{1}{4k} V_{R3} - \frac{1}{2k} \cdot V_{R4a} = 1 \cdot 10^{-3}$$

Apply KCL at node 2 (including ohms law)

$$-\frac{V_{R1a}}{R_{1a}} + \frac{V_{R2a}}{R_{2a}} + \frac{V_{R3a}}{R_{3a}} = 0$$

$$-\frac{1}{R_{1a}} \cdot V_{R1a} + \frac{1}{R_{2a}} \cdot V_{R2a} + \frac{1}{R_{3a}} \cdot V_{R3a} + 0 = 0$$

$$-\frac{1}{1k} \cdot V_{R1a} + \frac{1}{2k} \cdot V_{R2a} + \frac{1}{4k} \cdot V_{R3a} + 0 = 0$$

Apply KVL around loop 1

$$V_{R4a} - V_{R3a} - V_{R1a} = 0$$

$$-V_{R1a} + 0 - V_{R3a} + V_{R4a} = 0$$

Apply KVL around loop 2

$$-V_{1a} + V_{R1a} + V_{R2a} = 0$$

$$V_{R1a} + V_{R2} + 0 + 0 = 5$$

Set up the matrix/vector form

$$M := \begin{pmatrix} 0 & 0 & -\frac{1}{4 \cdot 10^3} & -\frac{1}{2 \cdot 10^3} \\ -\frac{1}{1 \cdot 10^3} & \frac{1}{2 \cdot 10^3} & \frac{1}{4 \cdot 10^3} & 0 \\ -1 & 0 & -1 & 1 \\ 1 & 1 & 0 & 0 \end{pmatrix} \quad C := \begin{pmatrix} 1 \cdot 10^{-3} \\ 0 \\ 0 \\ 5 \end{pmatrix}$$

$$X := M^{-1} \cdot C$$

$$V_{R1a} := 1.3V$$

$$X = \begin{pmatrix} 1.3 \\ 3.7 \\ -2.2 \\ -0.9 \end{pmatrix}$$

$$V_{R2a} := 3.7V$$

$$V_{R3a} := -2.2V$$

$$V_{R4a} := -0.9V$$

Determine the power produced/consumed by V1 and I1

$$P_{I1} := [3.7V - (-2.2V)] \cdot (1 \cdot 10^{-3} A)$$

$$P_{I1} = 5.9 \times 10^{-3} W \quad \text{produced or supplied (negative using passive sign convention)}$$

To find V1 we know that the total power in the circuit must be zero therefore

$$P_{I1} + P_{V1} = P_{RTotal} \quad \text{where} \quad P_{RTotal} = P_{R1} + P_{R2} + P_{R3} + P_{R4}$$

Important definition of Power $P = VI$ use ohms law and substitute

$$P = \frac{V^2}{R} \quad P = I^2 R$$

$$P_{R1} := \frac{V_{R1a}^2}{R_{1a}} \quad P_{R2} := \frac{V_{R2a}^2}{R_{2a}} \quad P_{R3} := \frac{V_{R3a}^2}{R_{3a}} \quad P_{R4} := \frac{V_{R4a}^2}{R_{4a}}$$

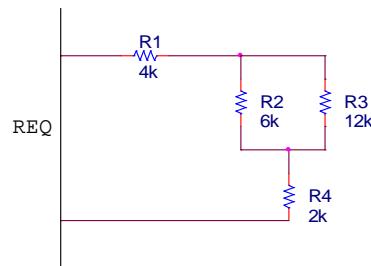
$$P_{R1} = 1.69 \times 10^{-3} W \quad P_{R2} = 6.845 \times 10^{-3} W \quad P_{R3} = 1.21 \times 10^{-3} W \quad P_{R4} = 4.05 \times 10^{-4} W$$

$$P_{RTotal} := P_{R1} + P_{R2} + P_{R3} + P_{R4} \quad P_{RTotal} = 0.01 W$$

$$P_{V1} := P_{RTotal} - P_{I1}$$

$$P_{V1} = 4.25 \times 10^{-3} W$$

Equivalent resistance

b) Determine R_{EQ} So R_2 and R_3 are in parallel, combine first, then combine R_{23} R_1 and R_4 in series

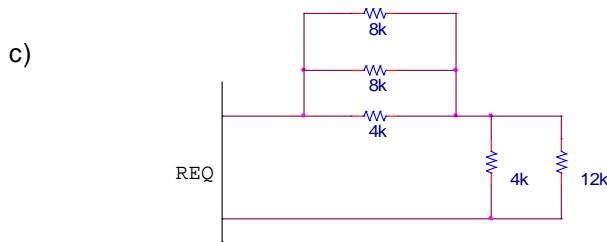
$$R_{1b} := 4\text{k}\Omega \quad R_{2b} := 6\text{k}\Omega \quad R_{3b} := 12\text{k}\Omega \quad R_{4b} := 2\text{k}\Omega$$

$$R_{23b} := \frac{R_{2b} \cdot R_{3b}}{R_{2b} + R_{3b}}$$

$$R_{23b} = 4 \times 10^3 \Omega$$

$$R_{EQ} := R_{1b} + R_{4b} + R_{23b}$$

$$R_{EQ} = 10 \cdot \text{k}\Omega$$

Determine REQ

Two sets of resistors in parallel

$$R_{right} := \frac{4\text{k}\Omega \cdot 12\text{k}\Omega}{4\text{k}\Omega + 12\text{k}\Omega}$$

$$R_{right} = 3 \cdot \text{k}\Omega$$

$$R_{top} := \frac{1}{\left(\frac{1}{4\text{k}\Omega} + \frac{1}{8\text{k}\Omega} + \frac{1}{8\text{k}\Omega} \right)}$$

or

$$R_{top88} := \frac{1}{\frac{1}{8\text{k}\Omega} + \frac{1}{8\text{k}\Omega}}$$

$$R_{top88} = 4 \cdot \text{k}\Omega$$

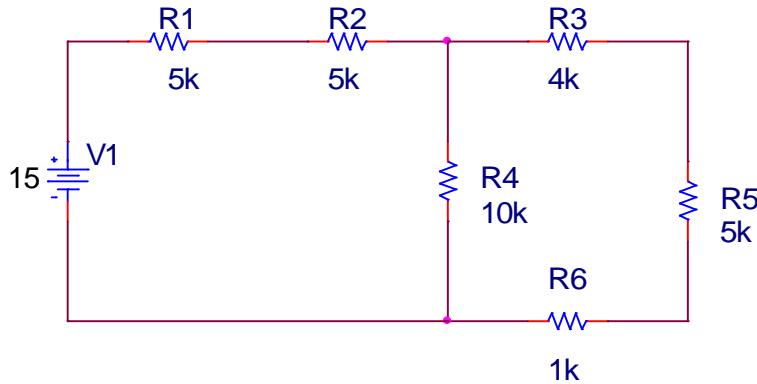
Could also combine two at a time.

Two equal resistor in parallel is half the resistance

Do that twice to get 2k

$$R_{EQc} := R_{right} + R_{top}$$

$$R_{EQc} = 5 \cdot \text{k}\Omega$$



d)

Use circuit reduction to determine the voltage across R4 and R5

$$R_{1d} := 5\text{k}\Omega \quad R_{2d} := 5\text{k}\Omega \quad R_{3d} := 4\text{k}\Omega \quad R_{4d} := 10\text{k}\Omega \quad R_{5d} := 5\text{k}\Omega \quad R_{6d} := 1\text{k}\Omega$$

$$V_{1d} := 15\text{V}$$

$$R_{356d} := R_{3d} + R_{5d} + R_{6d}$$

$$R_{356d} = 10\cdot\text{k}\Omega$$

$$R_{3564} := \frac{R_{356d} \cdot R_{4d}}{R_{356d} + R_{4d}}$$

$$R_{3564} = 5\cdot\text{k}\Omega$$

Voltage divide across R3564

$$V_{n1} := V_{1d} \cdot \frac{R_{3564}}{R_{1d} + R_{2d} + R_{3564}}$$

$$V_{n1} = 5\text{V}$$

V_{n1} is also the voltage across R4

$$V_{R4d} := V_{n1}$$

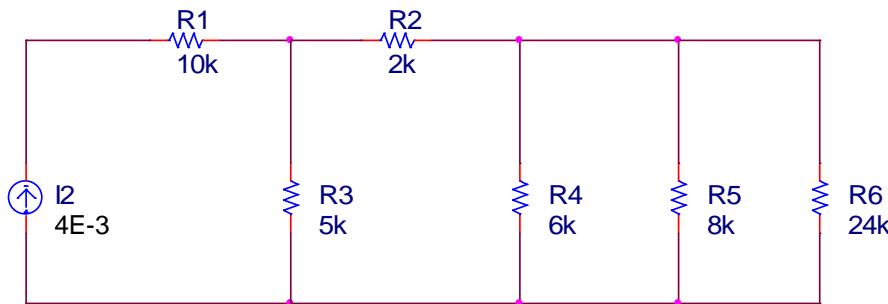
$$V_{R4d} = 5\text{V}$$

Voltage divide for R5

$$V_{R3d} := V_{R4d} \cdot \frac{R_{5d}}{R_{3d} + R_{5d} + R_{6d}}$$

$$V_{R3d} = 2.5 \text{ V}$$

e)



$$R_{1e} := 10\text{k}\Omega \quad R_{2e} := 2\text{k}\Omega \quad R_{3e} := 5\text{k}\Omega \quad R_{4e} := 6\text{k}\Omega \quad R_{5e} := 8\text{k}\Omega \quad R_{6e} := 24\text{k}\Omega$$

$$I_{2e} := 4 \cdot 10^{-3} \text{ A}$$

Combine R4 R5 and R6 in parallel

$$R_{456e} := \frac{1}{\frac{1}{R_{4e}} + \frac{1}{R_{5e}} + \frac{1}{R_{6e}}}$$

$$R_{456e} = 3 \cdot \text{k}\Omega$$

$$R_{2456} := 5\text{k}\Omega$$

$$R_{32456} := 2.5\text{k}\Omega$$

$$V_{R3e} := I_{2e} \cdot R_{32456}$$

$$V_{R3e} = 10 \text{ V}$$

Use the circuit reduction to find the current through R4

Voltage divider from VR3 with R2 and R456

$$V_{4563} := V_{R3e} \cdot \frac{R_{456e}}{R_{2e} + R_{456e}}$$

$$V_{4563} = 6 \text{ V}$$

Use ohms law to find IR\$

$$I_{R4e} := \frac{V_{4563}}{R_{4e}}$$

$$I_{R4e} = 1 \cdot \text{mA}$$