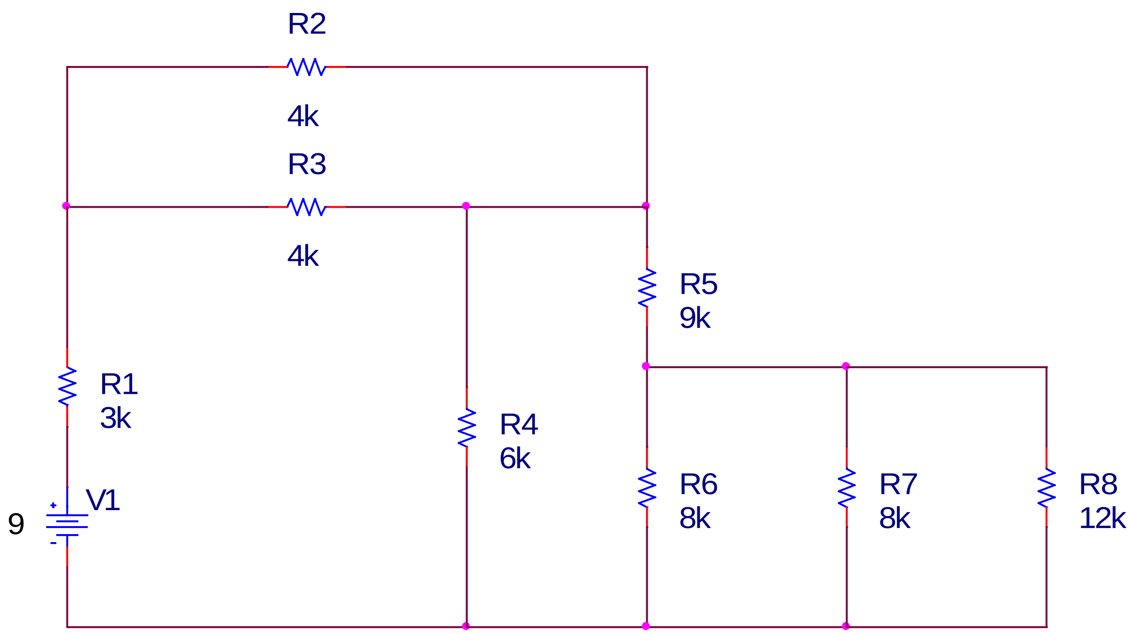
Questions:
What is the effective voltage when two voltage sources are in series?
What happens when two voltage sources are placed in parallel?
What is the effective current when two current sources are in parallel?
What happens when two current sources are placed in series?
What is the effect of placing a resistor in parallel with a voltage source?
What is the effect of placing a resistor in series with a current source?
What is source conversion?
What is node analysis?
What nodes are constrained in node analysis?
How many independent equations are needed in node analysis?
What is mesh analysis?
What loop currents are constrained in mesh analysis?
How many independent equations are needed in mesh analysis? (a little bit more tricky to answer)

Review problem
a)



1. If we solved this circuit using the method in homework 1, how many independent equations are needed to determine the voltage across each resistor?

8 resistors, eight unknowns, means 8 equations needed

2. How many nodes are in the circuit?

4: Start from one element and go to the next one. Keep doing that until you're done.

3. Use circuit reduction techniques to find the voltage across the resistor.

$R_1 := 3k\Omega$ $R_2 := 4k\Omega$ $R_3 := 4k\Omega$ $R_4 := 6k\Omega$ $R_5 := 9k\Omega$ $R_6 := 8k\Omega$ $R_7 := 8k\Omega$

$R_8 := 12k\Omega$ $V_1 := 9V$

$$R_{678} := \frac{1}{\frac{1}{R_8} + \frac{1}{R_7} + \frac{1}{R_6}} = 3 \cdot k\Omega$$

$R_{5678} := R_{678} + R_5$ 3k + 9k

$R_{5678} = 12 \cdot k\Omega$

$$R_{45678} := \frac{R_4 \cdot R_{5678}}{R_4 + R_{5678}}$$

$R_{45678} = 4 \cdot k\Omega$

Combine the top two resistors R2 and R3

They are equal and in parallel so

$R_{23} := 2k\Omega$

All in series, now use voltage divider

$$V_{R1} := V_1 \cdot \frac{R_1}{R_{23} + R_1 + R_{45678}}$$

$V_{R1} = 3\,V$

$$V_{R23} := V_1 \cdot \frac{R_{23}}{\left(R_{23} + R_1 + R_{45678}\right)}$$

$V_{R23} = 2\,V$ $V_{R2} := 2V$ $V_{R3} := 2V$

Node find voltage at node above R5 (will have to divide further!)

$$V_{R45678} := V_1 \cdot \frac{R_{45678}}{\left(R_{23} + R_1 + R_{45678}\right)}$$

$$V_{R45678} = 4\ \mathrm{V}$$

R.4 goes to ground so

$$V_{R4} := V_{R45678}$$

$$V_{R4} = 4\ \mathrm{V}$$

Voltage divide for voltage across R5 using node voltage

$$V_{R5} := V_{R45678} \cdot \frac{R_5}{R_5 + R_{678}}$$

$$V_{R5} = 3\ \mathrm{V}$$

Do the same for voltage across R6 R7 and R8 (all equal)

$$V_{R678} := V_{R45678} \cdot \frac{R_{678}}{R_5 + R_{678}}$$

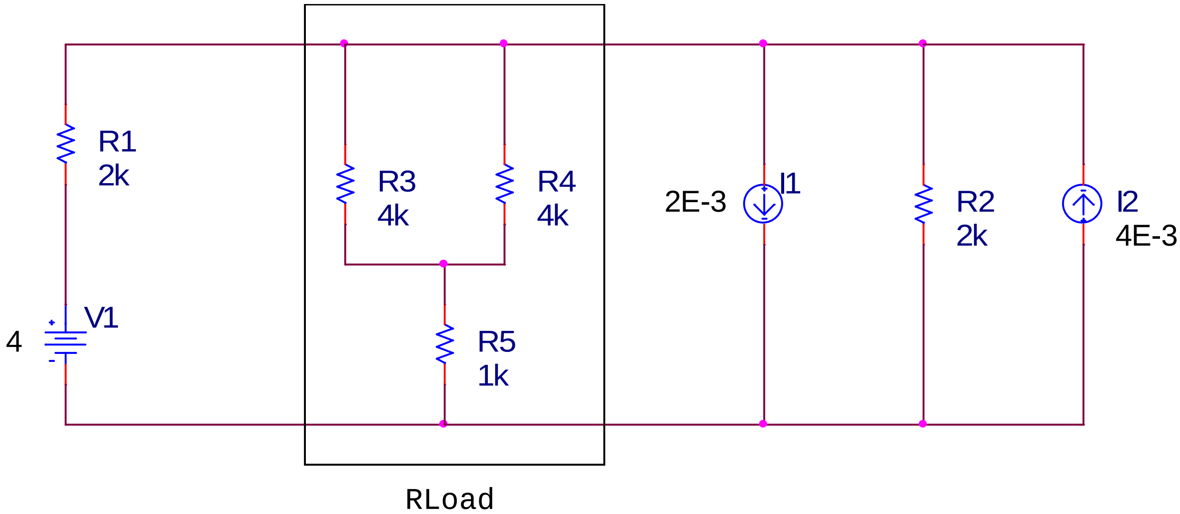
$$V_{R678} = 1\ \mathrm{V}$$

$V_{R6} := V_{R678}$ $\qquad$ $V_{R7} := V_{R678}$ $\qquad$ $V_{R8} := V_{R678}$

$V_{R6} = 1\ \mathrm{V}$ $\qquad$ $V_{R7} = 1\ \mathrm{V}$ $\qquad$ $V_{R8} = 1\ \mathrm{V}$

Source Transformation

b)



1. Use source transformation techniques to obtain a simlified circuit with Vs, Rs, and RLoad

$V_{1b} := 4V$ $\quad R_{3b} := 4k\Omega$ $\quad R_{5b} := 1k\Omega$ $\quad R_{2b} := 2k\Omega$

$R_{1b} := 2k\Omega$ $\quad R_{4b} := 4k\Omega$ $\quad I_{1b} := 2mA$ $\quad I_{2b} := 4mA$

Transform the voltage source into current source

$$I_{sv1} := \frac{V_{1b}}{R_{1b}}$$

$I_{sv1} = 2 \cdot mA$ note direction of arrow from derivation

arrow goes up from plus to minus with voltage source with minus closest to ground

Combine all current source and resistors in parallel outside of Rload

$I_s := I_{sv1} - I_{1b} + I_{2b}$

$I_{sv1} - I_{1b} + I_{2b} = 4 \cdot mA$

$R_s := 1k\Omega$

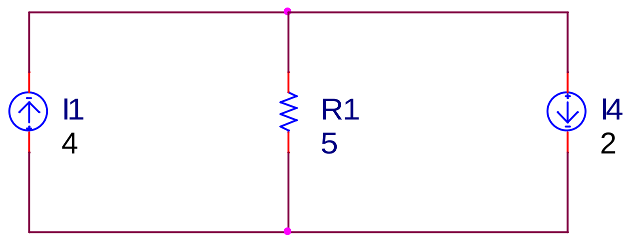
Transform combined current source and resistances back to voltage source and RS

$V_s := I_s \cdot R_s$ $\quad V_s = 4\,V$ $\quad R_s = 1 \cdot k\Omega$

$$R_{Load} := \frac{R_{3b} \cdot R_{4b}}{R_{3b} + R_{4b}} + R_{5b} \qquad R_{Load} = 3 \cdot k\Omega$$

Simple mesh/node analysis

c)



$I_{1c} := 4A$ $\quad I_{4c} := 2A$ $\quad R_{1c} := 5\Omega$

1. How many nodes are in the circuit?

Two

How many mesh loops?

Two

2. Use node analysis to find VR1 and IR1

Step 1: Label all node voltages, known and unknown identifying variables v1, v2 etc.

a. # of unknown node voltages = # of nodes (2) - # of voltage sources (0) -1 reference

$2 - 0 - 1 = 1$ v1 at top of R1 (also VR1)

Step 2: Write a KCL at each unknown node voltage

$I_{1c} - I_{v1} + I_{4c} = 0$

$-I_{1c} + \frac{v_1}{R_1} + I_{4c} = 0$

$-4A + \frac{v_1}{5\Omega} + 2A = 0$

$\frac{v_1}{5\Omega} = 2A$

$v_1 := 10V$

$V_{R1c} := v_1$ $\quad V_{R1c} = 10\,V$

$I_{R1c} := \frac{V_{R1c}}{5\Omega}$

$I_{R1c} = 2\,A$

3. Use mesh analysis to find VR1 and IR1.

Step 1: Label all mesh currents

a. Unknown mesh currents and currents from current sources, i1, i2, etc.

c. # of unknown mesh currents = # of meshes (2) - # of current sources (1)

$$2 - 1 = 1$$

Step 2: Write a KVL around each unknown mesh current

a. Sum of voltages due to all mesh currents = 0
b. Best to go backwards around current arrow
c. Express v's in terms of mesh currents using ohms law

See book pg. 97 on what to do when there are current sources! It is assumed that we have voltage sources in mesh analysis.

Three ways to handle it

1. Source transformation (if current source is connected in parallel with a resistor)

2. Mesh current determined by source curent and is no longer an unknown
Write mesh equatoins for all others and move known mesh current to source side of the equation in final step. (If current source is only in one mesh)

3. Neither works wen a current source is contained in two meshes or is not connected in parallel with a resistance. Create a supermesh.

Take number 1.

$$I_{12c} := I_{1c} - I_{4c}$$

$$I_{12c} = 2 \cdot A$$

$$V_{sc} := I_{12c} \cdot R_{1c}$$

$$V_{sc} = 10\,V$$

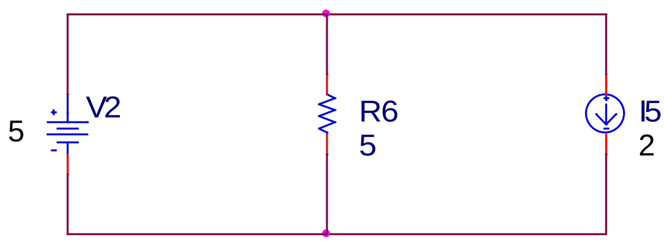
Mesh

$$-10V + V_{R1c} = 0$$

$$V_{R1c} = 10\,V$$

$$I_{R1c} = 2\,A$$

d)



2. Use node analysis to find VR1 and IR1 Note should be VR6

$I_{5d} := 2A$ $V_{2d} := 5V$ $R_{6d} := 5\Omega$ $Va := V_{2d}$

Adding voltage sources to circuits modifies node analysis (contrained nodes)

Three ways to handle it:

Method 1: Use source transformation to replace the votlage source and series resistance with an equivalent current source and parallel resistance

Method 2: Strategically select reference node and write node equations in usual way

Method 3: Combine nodes to make a supernode

Use method 2

If node VB become ground then the voltage source makes VA = 5

The voltage acorss the resistor is therefore simply 5V $V_{R6d} := 5V$

$$I_{R6d} := \frac{V_{R6d}}{R_{6d}} \qquad I_{R6d} = 1\,A$$

3. Use mesh analysis to find VR1 and IR1 (should be VR6 and IR6)

Using method 2

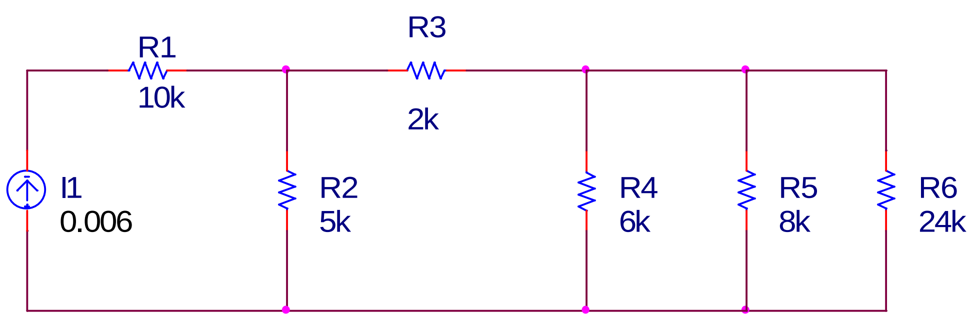
Mesh current is no longer unknown

Only need 1 loop. Choose the one on the right and use I5 to get that voltage.

$$-V_{2d} + R_{6d} \cdot I_{5d} = 5\ V$$

$$I_{R6d} := \frac{V_{R6d}}{R_{6d}} \qquad I_{R6d} = 1\ A$$

e)



1. How many nodes are in the circuit? Loops?

4 nodes
4 loops

2. How many nodes are contrained? (depends no whether a voltage source is there)

None

3. How many loops are contrained? (One current source there)

One

4. Is node analysis or mesh analysis easier in this circuit?

Node analysis yields three (not two) independent equations, mesh analysis yields four independent equations (without circuit reduction)

\# of unknown node voltages = # of nodes (4) - # of voltage sources (0) -1 reference

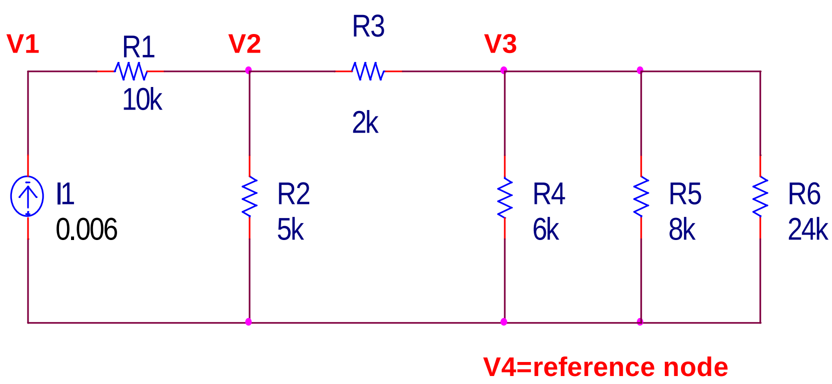
$$4 - 0 - 1 = 3$$

\# of unknown mesh currents = # of meshes (4) - # of current sources (1)

$$4 - 1 = 3$$

5. Determine VR3 using either mesh or node analysis:

Node analysis



At node V1:

$$\frac{V_1 - V_2}{10k} - 0.006 = 0$$

(1) $$V_1 \cdot \frac{1}{10k} - V_2 \cdot \frac{1}{10k} = 0.006$$

At node V2:

$$\frac{V_2 - V_1}{10k} + \frac{V_2 - 0}{5k} + \frac{V_2 - V_3}{2k} = 0$$

(2) $$-V_1 \cdot \left(\frac{1}{10k}\right) + V_2 \cdot \left(\frac{1}{10k} + \frac{1}{5k} + \frac{1}{2k}\right) - V_3 \cdot \left(\frac{1}{2k}\right) = 0$$

At node V3:

$$\frac{V_3 - V_2}{2k} + \frac{V_3 - 0}{6k} + \frac{V_3 - 0}{6k}$$

note combined parallel resistors at that node for convenience

(3) $$-V_2 \cdot \frac{1}{2k} + V_3 \cdot \left(\frac{1}{2k} + \frac{1}{6k} + \frac{1}{6k}\right) = 0$$

$$M_e := \begin{bmatrix} \frac{1}{10 \times 10^3} & \frac{-1}{10 \cdot 10^3} & 0 \\ \frac{-1}{10 \cdot 10^3} & \left(\frac{1}{10 \cdot 10^3} + \frac{1}{5 \cdot 10^3} + \frac{1}{2 \cdot 10^3}\right) & \frac{-1}{2 \cdot 10^3} \\ 0 & -\frac{1}{2 \cdot 10^3} & \left(\frac{1}{2 \cdot 10^3} + \frac{1}{6 \cdot 10^3} + \frac{1}{6 \cdot 10^3}\right) \end{bmatrix}$$

$$C_e := \begin{pmatrix} 0.006 \\ 0 \\ 0 \end{pmatrix}$$

$$X_e := M_e^{-1} \cdot C_e$$

$$X_e = \begin{pmatrix} 75 \\ 15 \\ 9 \end{pmatrix}$$

$V_{1e} := 75V$

$V_{2e} := 15V$

$V_{3e} := 9V$

Voltage across VR3

$V_{2e} - V_{3e} = 6\,V$

Now do mesh analysis

We know that $I_{1e} := 6mA$

So do mesh analysis for I2, I3 and I4 (with I1 in mind)

Around Mesh 2

$$I_2 \cdot R_2 - 0.006 \cdot R_2 + I_2 \cdot R_3 + I_2 \cdot R_4 - I_3 \cdot R_4 = 0$$

$$I_2 \cdot 5k - 30 + I_2 \cdot 2k + I_2 \cdot 6k - I_3 \cdot 6k = 0$$

(1) $I_2 \cdot (5k + 2k + 6k) - I_3 \cdot 6k = 30$

Around Mesh 3

$$I_3 \cdot R4 - I_2 \cdot R_4 + I_3 \cdot R5 - I_4 \cdot R_5 = 0$$

$$I_3 \cdot 6k - I_2 \cdot 6k + I_3 \cdot 8k - I_4 \cdot 8k = 0$$

(2) $$-I_2 \cdot 6k + I_3 \cdot (6k + 8k) - I_4 \cdot (8k) = 0$$

Around Mesh 4

$$I_4 \cdot R5 - I_3 \cdot R_5 + I_4 \cdot R_6 = 0$$

(3) $$-I_3 \cdot 8k + I_4 \cdot (8k + 24k) = 0$$

$$M_{e2} := \begin{bmatrix} 2 \cdot 10^3 + 6 \cdot 10^3 + 5 \cdot 10^3 & -6 \cdot 10^3 & 0 \\ -6 \cdot 10^3 & \left(6 \cdot 10^3 + 8 \cdot 10^3\right) & -8 \cdot 10^3 \\ 0 & -8 \cdot 10^3 & 8 \cdot 10^3 + 24 \cdot 10^3 \end{bmatrix}$$

$$C_{e2} := \begin{pmatrix} 30 \\ 0 \\ 0 \end{pmatrix}$$

$$X_{e2} := M_{e2}^{-1} \cdot C_{e2}$$

$$X_{e2} = \begin{pmatrix} 3 \times 10^{-3} \\ 1.5 \times 10^{-3} \\ 3.75 \times 10^{-4} \end{pmatrix}$$

$I_2 = 3mA$

$I_3 = 1.5mA$

$I_4 = 0.375mA$

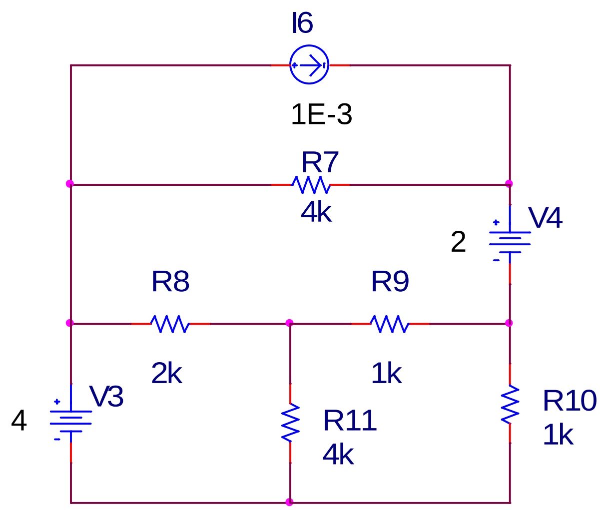
$I_1 = 6mA$

To find VR3

$$I_2 \cdot R_3 = 6V$$

$$3mA \cdot 2k\Omega = 6\ V$$

f)



1. How many nodes are in the above circuit? Loops?

5 nodes
4 loops

2. How many nodes are contrained? 2

3. How many loops are constrained 1

4. Is KCL or KVL easier in this circuit actually node or mesh is a better question.

# of unknown node voltages = # of nodes (5) - # of voltage sources (2) -1 reference

$$5 - 2 - 1 = 2$$

Don't do mesh

# of unknown mesh currents = # of meshes (4) - # of current sources (1) -

$$4 - 1 = 3$$

5. Apply the method you (we choose) to find VR9

We'll choose node.

We have two voltage sources. One becomes a node voltage. The other will become a supernode.

$V_1 = 4V$ (we'll incorporate this into the equation directly)

$$(1) \quad V_2 - V_3 = 2V$$

Start at supernode

$$-I_6 + \frac{V_2 - 4V}{R_7} + \frac{V_3 - V_4}{R_9} + \frac{V_3 - 0}{R_{10}} = 0$$

$$-1mA + V_2 \cdot \frac{1}{4k} - 1mA + \frac{V_3}{1k} - \frac{V_4}{1k} + \frac{V_3}{1k} = 0$$

$$(2) \quad V_2 \cdot \left(\frac{1}{4k}\right) + V_3 \cdot \left(\frac{1}{1k} + \frac{1}{1k}\right) - V_4 \cdot \left(\frac{1}{1k}\right) = 2 \times 10^{-3}$$

At node V4

$$\frac{V_4 - 4}{R_8} + \frac{V_4 - 0}{R_{11}} + \frac{V_4 - V_3}{R_9} = 0$$

$$\frac{V_4}{2k} - 2mA + \frac{V_4}{4k} + \frac{V_4}{1k} - \frac{V_3}{1k} = 0$$

$$(3) \quad -V_3 \cdot \left(\frac{1}{1k}\right) + V_4 \cdot \left(\frac{1}{2k} + \frac{1}{4k} + \frac{1}{1k}\right) = 2 \times 10^{-3}$$

$$M_{f1} := \begin{bmatrix} \frac{1}{4 \cdot 10^3} & \left(\frac{1}{1 \cdot 10^3} + \frac{1}{1 \cdot 10^3}\right) & \frac{-1}{1 \cdot 10^3} \\ 0 & \frac{-1}{1 \cdot 10^3} & \left(\frac{1}{2 \cdot 10^3} + \frac{1}{4 \cdot 10^3} + \frac{1}{1 \cdot 10^3}\right) \\ 1 & -1 & 0 \end{bmatrix}$$

$$C_{f1} := \begin{pmatrix} 2\cdot 10^{-3} \\ 2\cdot 10^{-3} \\ 2 \end{pmatrix}$$

$$X_{f1} := M_{f1}^{-1}\cdot C_{f1}$$

$$X_{f1} = \begin{pmatrix} 3.574 \\ 1.574 \\ 2.043 \end{pmatrix}$$

$V_2 = 3.57V$

$V_3 = 1.57V$

$V_4 = 2.04V$

$V_1 = 0$

Nodes 3 and 4 switched in answer on the sheet

$$V_{R9} = \frac{V_4 - V_3}{1k\Omega}$$

$2.04V - 1.57V = 0.47\,V$