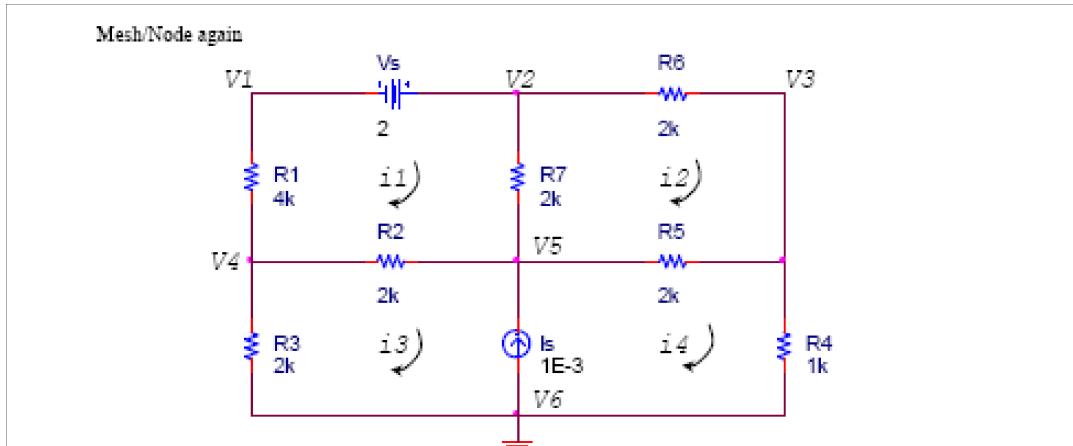


What is superposition?
What is a dependent source?

Review Problem:

a)



1. Using the labeled nodes and mesh loops, determine the linear equations for both methods.

Node analysis equations.

of nodes (6) - voltage sources (1)-reference node (1)

4 unknown equations

$$(1) \quad V_1 - V_2 = -2V \quad (\text{think like going from ground to node, so negative 2})$$

Supernode V1 and V2

$$\frac{V_1 - V_4}{4k} + \frac{V_2 - V_5}{2k} + \frac{V_2 - V_3}{2k} = 0$$

$$(2) \quad \frac{V_1}{4k} + V_2 \left(\frac{1}{2k} + \frac{1}{2k} \right) - V_3 \cdot \frac{1}{2k} - \frac{V_4}{4k} - \frac{V_5}{2k} = 0$$

At node V3 (center right point)

$$\frac{V_3 - V_2}{2k} + \frac{V_3 - V_5}{2k} + \frac{V_3 - 0}{1k} = 0$$

$$(3) \quad -V_2 \cdot \left(\frac{1}{2k} \right) + V_3 \cdot \left(\frac{1}{2k} + \frac{1}{2k} + \frac{1}{1k} \right) - V_5 \cdot \left(\frac{1}{2k} \right) = 0$$

At node V5

$$\frac{V_5 - V_4}{2k} + \frac{V_5 - V_3}{2k} + \frac{V_5 - V_2}{2k} - 1 \cdot 10^{-3} = 0$$

$$(4) \quad \frac{-V_2}{2k} - \frac{V_3}{2k} - \frac{V_4}{2k} + V_5 \cdot \left(\frac{1}{2k} + \frac{1}{2k} + \frac{1}{2k} \right) = 1 \cdot 10^{-3}$$

At node V4

$$\frac{V_4 - V_1}{4k} + \frac{V_4 - 0}{2k} + \frac{V_4 - V_5}{2k} = 0$$

$$(5) \quad \frac{-V_1}{4k} + V_4 \cdot \left(\frac{1}{4k} + \frac{1}{2k} + \frac{1}{2k} \right) - V_5 \cdot \left(\frac{1}{2k} \right) = 0$$

2) Matrix solve

sorry: full matrix doesn't fit on one page!

$$M_a := \begin{bmatrix} 1 & -1 & 0 & 0 \\ \frac{1}{4 \times 10^3} \left(\frac{1}{2 \times 10^3} + \frac{1}{2 \times 10^3} \right) & \frac{-1}{2 \times 10^3} & \frac{-1}{2 \times 10^3} & \frac{-1}{4 \times 10^3} \\ 0 & \frac{-1}{2 \times 10^3} & \left(\frac{1}{2 \times 10^3} + \frac{1}{2 \times 10^3} + \frac{1}{1 \times 10^3} \right) & 0 \\ 0 & \frac{-1}{2 \times 10^3} & \frac{-1}{2 \times 10^3} & \frac{-1}{2 \times 10^3} \\ \frac{-1}{4 \times 10^3} & 0 & 0 & \left(\frac{1}{4 \times 10^3} + \frac{1}{2 \times 10^3} + \frac{1}{2 \times 10^3} \right) \end{bmatrix}$$

$$C_a := \begin{pmatrix} -2 \\ 0 \\ 0 \\ 1 \cdot 10^{-3} \\ 0 \end{pmatrix}$$

$$X_a := M_a^{-1} \cdot C_a$$

$$X_a = \begin{pmatrix} -0.576 \\ 1.424 \\ 0.746 \\ 0.508 \\ 1.559 \end{pmatrix}$$

$V_1 = -0.57V$

$V_2 = 1.42V$

$V_3 = 0.74V$

$V_4 = 0.51V$

$V_5 = 1.56V$

$V_6 = 0V$

Mesh analysis

Need supermesh

$4 - 1 = 0 \quad 3$
equations

$$(1) \quad I_4 - I_3 = I_s$$

Loop i1

$$-2V + I_1 \cdot R_7 - I_2 \cdot R_7 + I_1 \cdot R_2 - I_3 \cdot R_2 + I_1 \cdot R_1 = 0$$

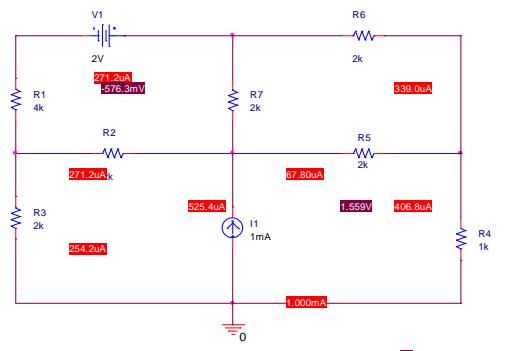
$$(2) \quad I_1 \cdot (2k + 2k + 4k) - I_2 \cdot (2k) - I_3 \cdot (2k) = 2$$

Loop i2

$$I_2 \cdot R_6 + I_2 \cdot R_5 - I_4 \cdot R_5 + I_2 \cdot R_7 - I_1 \cdot R_7 = 0$$

$$(3) \quad -I_1 \cdot 2k + I_2 \cdot (2k + 2k + 2k) - I_4 \cdot (2k) = 0$$

Loop (supermesh entire bottom)



$$I_3 \cdot R_3 + I_3 \cdot R_2 - I_1 \cdot R_2 + I_4 \cdot R_5 - I_2 \cdot R_5 + I_4 \cdot R_4 = 0$$

$$(4) \quad -I_1 \cdot (2k) - I_2 \cdot 2k + I_3 \cdot (2k + 2k) + I_4 \cdot (2k + 1k) = 0$$

$$M_{a1} := \begin{pmatrix} 8 \times 10^3 & -2 \times 10^3 & -2 \times 10^3 & 0 \\ -2 \times 10^3 & 6 \times 10^3 & 0 & -2 \times 10^3 \\ -2 \times 10^3 & -2 \cdot 10^3 & 4 \times 10^3 & 3 \cdot 10^3 \\ 0 & 0 & -1 & 1 \end{pmatrix} \quad C_{a1} := \begin{pmatrix} 2 \\ 0 \\ 0 \\ 1 \cdot 10^{-3} \end{pmatrix}$$

$$X_{a1} := M_{a1}^{-1} \cdot C_{a1}$$

$$X_{a1} = \begin{pmatrix} 2.712 \times 10^{-4} \\ 3.39 \times 10^{-4} \\ -2.542 \times 10^{-4} \\ 7.458 \times 10^{-4} \end{pmatrix} \quad I_{1a} := 0.271\text{mA}$$

$$I_{2a} := 0.339\text{mA}$$

$$I_{3a} := -0.254\text{mA}$$

$$I_{4a} := 0.746\text{mA}$$

3) Determine the voltage across R7

$$V_2 - V_5 = V_{R7}$$

$$1.42 - 1.56 = -0.14$$

just means terminals are switched if negative

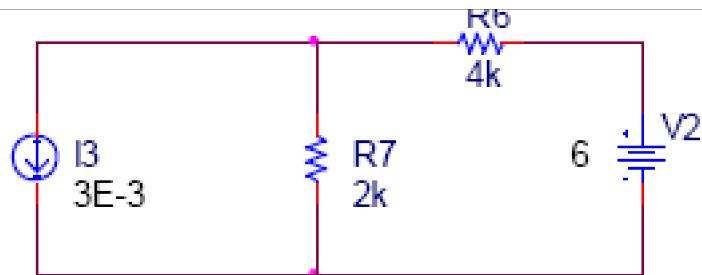
or from mesh analysis

$$2k\Omega \cdot (I_{1a} - I_{2a}) = -0.136\text{V}$$

Superposition

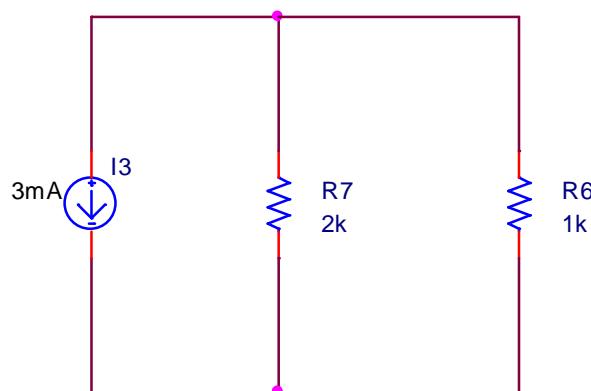
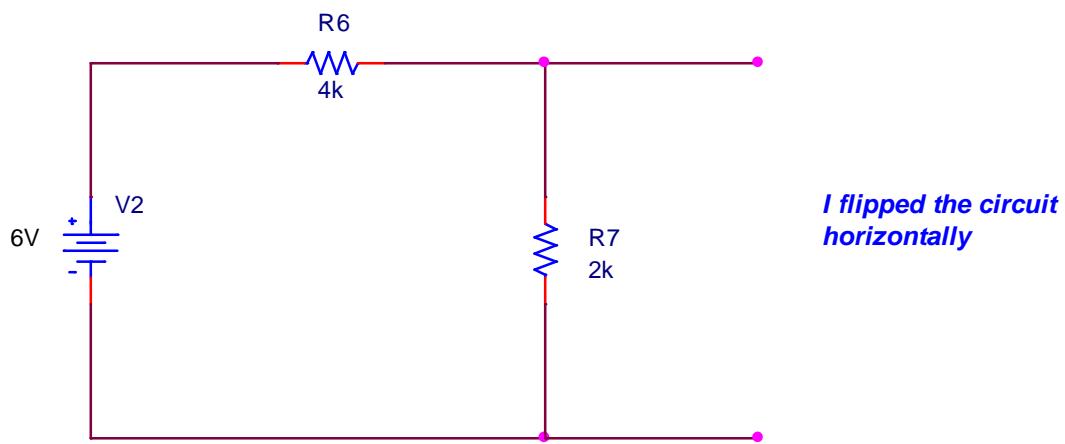
NOTE: There is a typo on their CP, R6 should be 4k and R7 should be 2k (make sure to say this before you start the problem!)

b)



1) Draw the superposition circuit for each source

**Voltage source is a short
Current source is open**



2) Use superposition to determine the voltage across R7

$$V_{2b} := 6V \quad R_{6b} := 4k\Omega \quad R_{7b} := 2k\Omega \quad I_{3b} := 3mA$$

For voltage source use voltage divider

$$V_{R7b} := V_{2b} \cdot \left(\frac{R_{7b}}{R_{6b} + R_{7b}} \right)$$

$$V_{R7b} = 2V$$

For current source use current divider then ohms law

$$I_{R7b} := -I_{3b} \cdot \left(\frac{R_{6b}}{R_{7b} + R_{6b}} \right)$$

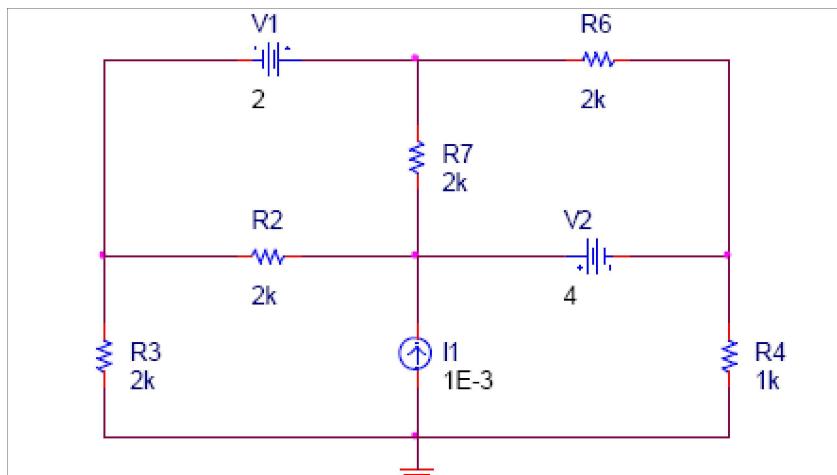
$$I_{R7b} = -2mA$$

$$V_{R7b2} := I_{R7b} \cdot R_{7b}$$

$$V_{R7b2} = -4V$$

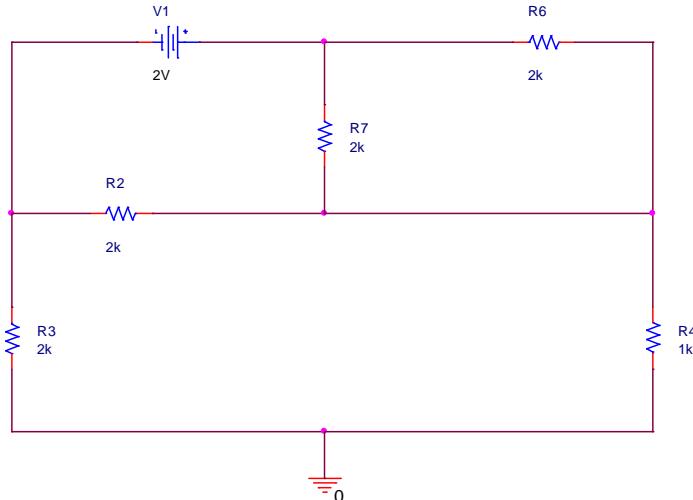
$$V_{Tot} := V_{R7b} + V_{R7b2} = -2V$$

c)



Apply superposition to the circuit shown above to find VR_7

Start with Voltage source V_1



$$R_{3c} := 2\text{k}\Omega \quad R_{2c} := 2\text{k}\Omega \quad R_{7c} := 2\text{k}\Omega \quad R_{6c} := 2\text{k}\Omega \quad R_{4c} := 1\text{k}\Omega$$

$$V_{1c} := 2\text{V} \quad V_{2c} := 4\text{V}$$

$$I_{1c} := 1\text{mA}$$

MESH ANALYSIS

Loop 1

$$-2V + I_1 \cdot R_{7c} - I_2 \cdot R_{7c} + I_1 \cdot R_2 = 0$$

$$-2 + I_1 \cdot 2k - I_2 \cdot 2k + I_1 \cdot 2k - I_3 \cdot 2k = 0$$

$$(1) \quad I_1 \cdot (2k + 2k) - I_2 \cdot (2k) - I_3 \cdot 2k = 2$$

Loop 2

$$I_2 \cdot R_7 - I_1 \cdot R_7 + I_2 \cdot R_6 = 0$$

$$M_C := \begin{bmatrix} (2 \cdot 10^3 + 2 \cdot 10^3) & -2 \cdot 10^3 & -2 \cdot 10^3 \\ -2 \cdot 10^3 & (2 \cdot 10^3 + 2 \cdot 10^3) & 0 \\ -2 \cdot 10^3 & 0 & (2 \cdot 10^3 + 2 \cdot 10^3 + 1 \cdot 10^3) \end{bmatrix}$$

$$I_2 \cdot 2k - I_1 \cdot 2k + I_2 \cdot 2k = 0$$

$$-I_1 \cdot 2k + I_2 \cdot (2k + 2k) = 0$$

$$C_C := \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$$

Loop 3

$$I_3 \cdot R_3 + I_3 \cdot R_2 - I_1 \cdot R_2 + I_3 \cdot R_4 = 0$$

$$X_C := M_C^{-1} \cdot C_C$$

$$-I_1 \cdot (2k) + I_3 \cdot (2k + 2k + 1k) = 0$$

$$X_C = \begin{pmatrix} 9.091 \times 10^{-4} \\ 4.545 \times 10^{-4} \\ 3.636 \times 10^{-4} \end{pmatrix}$$

$$I_{1c} := 0.91 \text{ mA}$$

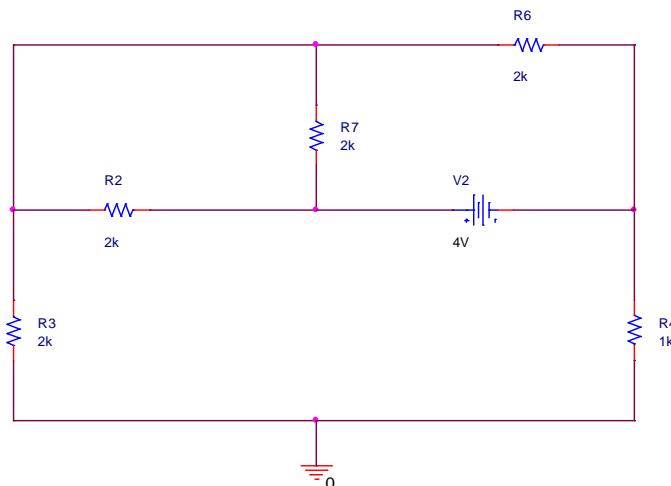
$$I_{2c} := 0.45 \text{ mA}$$

$$I_{3c} := 0.36 \text{ mA}$$

$$V_{R7} := (I_{1c} - I_{2c}) \cdot R_{7c} \quad I_{1c} - I_{2c} = 0.46 \text{ mA}$$

$$V_{R7} = 0.92 \text{ V}$$

For voltage source V_2



MESH

$$I_1 \cdot R_2 - I_3 \cdot R_2 + I_1 \cdot R_7 - I_2 \cdot R_7 = 0$$

$$I_2 \cdot R_7 - I_1 \cdot R_7 + I_2 \cdot R_6 - V_2 = 0$$

$$(1) \quad I_1 \cdot (2k + 2k) - I_2 \cdot 2k - I_3 \cdot 2k = 0$$

$$(2) \quad -I_1 \cdot 2k + I_2 \cdot (2k + 2k) = 4$$

$$I_3 \cdot R_3 + I_3 \cdot R_2 - I_1 \cdot R_2 + V_2 + I_3 \cdot R_4 = 0$$

$$(3) \quad -I_1 \cdot 2k + I_3 \cdot (2k + 2k + 1k) = -4V$$

$$M_{c2} := \begin{bmatrix} (2 \cdot 10^3 + 2 \cdot 10^3) & -2 \times 10^3 & -2 \cdot 10^3 \\ -2 \cdot 10^3 & (2 \cdot 10^3 + 2 \cdot 10^3) & 0 \\ -2 \cdot 10^3 & 0 & (2 \cdot 10^3 + 2 \cdot 10^3 + 1 \cdot 10^3) \end{bmatrix} \quad C_{c2} := \begin{pmatrix} 0 \\ 4 \\ -4 \end{pmatrix}$$

$$X_{c2} := M_{c2}^{-1} \cdot C_{c2}$$

$$X_{c2} = \begin{pmatrix} 1.818 \times 10^{-4} \\ 1.091 \times 10^{-3} \\ -7.273 \times 10^{-4} \end{pmatrix} \quad \begin{aligned} I_{1c2} &:= 0.182 \text{mA} \\ I_{2c2} &:= 1.09 \text{mA} \\ I_{3c2} &:= -0.727 \text{mA} \end{aligned}$$

$$V_{R7c2} := (-I_{2c2} + I_{1c2}) \cdot R_{7c} \quad (\text{Check direction of loops relative to what it should be!!!})$$

$$V_{R7c2} = -1.816 \text{V}$$

OR find voltage at Va and Vb (still need to watch signs of current based on loop direction)

$$-I_{3c2} \cdot R_{3c} = 1.454 \text{V}$$

$$I_{3c2} \cdot R_{4c} = -0.727 \text{V}$$

$$4 + -0.727 = 3.273$$

$$1.454 - 3.273 = -1.819$$

Last current source I1

KCL at Va

$$\frac{V_a - V_b}{R_6} + \frac{V_a - V_b}{R_7} + \frac{V_a - 0}{R_3} + \frac{V_a - V_b}{R_2} = 0$$

$$(1) \quad V_a \left(\frac{1}{2k} + \frac{1}{2k} + \frac{1}{2k} + \frac{1}{2k} \right) - V_b \left(\frac{1}{2k} + \frac{1}{2k} + \frac{1}{2k} \right) = 0$$

KCL at Vb

$$\frac{V_b - V_a}{R_7} + \frac{V_b - 0}{R_4} + \frac{V_b - V_a}{R_6} + \frac{V_b - V_a}{R_2} + (-I_1) = 0$$

$$(2) \quad -V_a \left(\frac{1}{2k} + \frac{1}{2k} + \frac{1}{2k} \right) + V_b \left(\frac{1}{2k} + \frac{1}{1k} + \frac{1}{2k} + \frac{1}{2k} \right) = 1mA$$

$$M_{c3} := \begin{bmatrix} \left(\frac{1}{2 \cdot 10^3} + \frac{1}{2 \cdot 10^3} + \frac{1}{2 \cdot 10^3} + \frac{1}{2 \cdot 10^3} \right) & -\left(\frac{1}{2 \cdot 10^3} + \frac{1}{2 \cdot 10^3} + \frac{1}{2 \cdot 10^3} \right) \\ -\left(\frac{1}{2 \cdot 10^3} + \frac{1}{2 \cdot 10^3} + \frac{1}{2 \cdot 10^3} \right) & \left(\frac{1}{2 \times 10^3} + \frac{1}{1 \times 10^3} + \frac{1}{2 \times 10^3} + \frac{1}{2 \times 10^3} \right) \end{bmatrix}$$

$$C_{c3} := \begin{pmatrix} 0 \\ 1 \cdot 10^{-3} \end{pmatrix}$$

$$X_{c3} := M_{c3}^{-1} \cdot C_{c3}$$

$$X_{c3} = \begin{pmatrix} 0.545 \\ 0.727 \end{pmatrix} \quad \begin{aligned} V_a &:= 0.545V \\ V_b &:= 0.727V \end{aligned}$$

$$V_{R7c3} := V_a - V_b$$

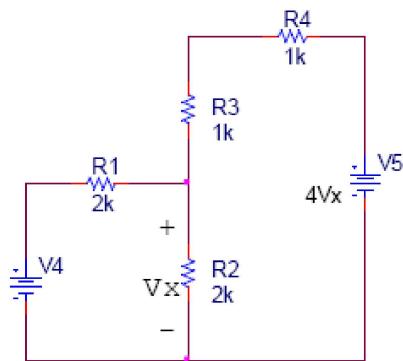
$$V_{R7c3} = -0.182 \text{ V}$$

$$V_{TOTc} := V_{R7} + V_{R7c2} + V_{R7c3}$$

$$V_{TOTc} = -1.078 \text{ V}$$

Dependent sources

d)



1) Determine the voltage V_5 when $V_4=4\text{V}$

Need to find voltage V_x then multiply by dependent source multiplier (4)

$$4 - 2 - 1 = 1$$

Node analysis

$$\frac{V_x - 4}{R_1} + \frac{V_x - 0}{R_2} + \frac{V_x - 4 \cdot V_x}{2k} = 0$$

$$\frac{V_x}{2k} - \frac{4}{2k} + \frac{V_x}{2k} + \frac{V_x}{2k} - \frac{4 \cdot V_x}{2k} = 0$$

$$V_x \cdot \left(\frac{1}{2k} + \frac{1}{2k} + \frac{1}{2k} - \frac{4}{2k} \right) = \frac{4}{2k} \quad \left(\frac{1}{2 \cdot 10^3} \right) \cdot 3 - \frac{4}{2 \cdot 10^3} = -5 \times 10^{-4}$$

$$V_{x4} := \frac{\frac{4}{2 \cdot 10^3}}{\left[\left(\frac{1}{2 \cdot 10^3} \right) \cdot 3 - \frac{4}{2 \cdot 10^3} \right]}$$

$$V_{x4} = -4$$

$$V_{54V} := -4 \cdot 4V$$

$$V_{54V} = -16V$$

2) Determine V_5 when $V_4 = 1V$

$$\frac{V_x - 1}{R_1} + \frac{V_x - 0}{R_2} + \frac{V_x - 4 \cdot V_x}{2k} = 0$$

$$\frac{V_x}{2k} - \frac{1}{2k} + \frac{V_x}{2k} + \frac{V_x}{2k} - \frac{4 \cdot V_x}{2k} = 0$$

$$V_x \left(\frac{1}{2k} + \frac{1}{2k} + \frac{1}{2k} - \frac{4}{2k} \right) = \frac{1}{2k}$$

$$V_{x1} := \left[\left(\frac{1}{2 \cdot 10^3} \right) \cdot 3 - \frac{4}{2 \cdot 10^3} \right]^{-1} \cdot \frac{1}{2 \cdot 10^3}$$

$$V_{x1} = -1$$

$$V_{51V} := V_{x1} \cdot 4$$

$$V_{51V} = -4$$

3) Determine the voltage V5 when V4 = 2V

$$\frac{V_x - 2}{R_1} + \frac{V_x - 0}{R_2} + \frac{V_x - 4 \cdot V_x}{2k} = 0$$

$$\frac{V_x}{2k} - \frac{2}{2k} + \frac{V_x}{2k} + \frac{V_x}{2k} - \frac{4 \cdot V_x}{2k} = 0$$

$$V_x \cdot \left(\frac{1}{2k} + \frac{1}{2k} + \frac{1}{2k} - \frac{4}{2k} \right) = \frac{2}{2k}$$

$$V_{x2} := \left[\left(\frac{1}{2 \cdot 10^3} \right) \cdot 3 - \frac{4}{2 \cdot 10^3} \right]^{-1} \cdot \frac{2}{2 \cdot 10^3}$$

$$V_{x2} = -2$$

$$V_{52V} := V_{x2} \cdot 4$$

$$V_{52V} = -8$$