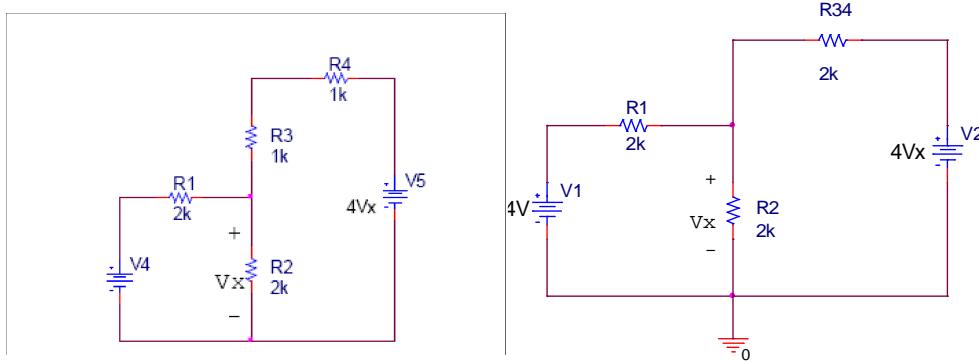


Review

Problem: d) Dependent sources



1) Determine the voltage  $V_5$  when  $V_4=4V$

Need to find voltage  $V_x$  then multiply by dependent source multiplier (4)

Node analysis

$$4 - 2 - 1 = 1$$

$$\frac{V_x - 4}{R_1} + \frac{V_x - 0}{R_2} + \frac{V_x - 4 \cdot V_x}{R_{34}} = 0$$

$$\frac{V_x}{2k} - \frac{4}{2k} + \frac{V_x}{2k} + \frac{V_x}{2k} - \frac{4 \cdot V_x}{2k} = 0$$

$$V_x \left( \frac{1}{2k} + \frac{1}{2k} + \frac{1}{2k} - \frac{4}{2k} \right) = \frac{4}{2k} \quad \left( \frac{1}{2 \cdot 10^3} \right) \cdot 3 - \frac{4}{2 \cdot 10^3} = -5 \times 10^{-4}$$

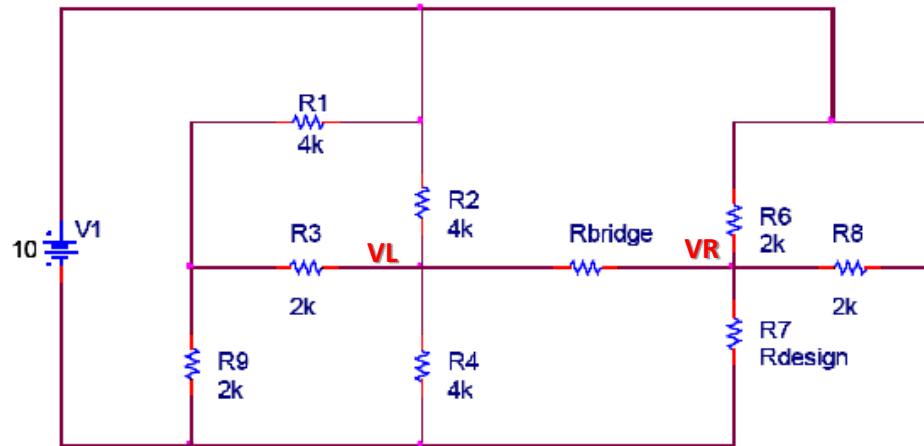
$$V_{x4} := \left[ \left( \frac{1}{2 \cdot 10^3} \right) \cdot 3 - \frac{4}{2 \cdot 10^3} \right]^{-1} \cdot \frac{4}{2 \cdot 10^3}$$

$$V_{x4} = -4$$

$$V_{54V} := -4 \cdot 4V$$

$$V_{54V} = -16V$$

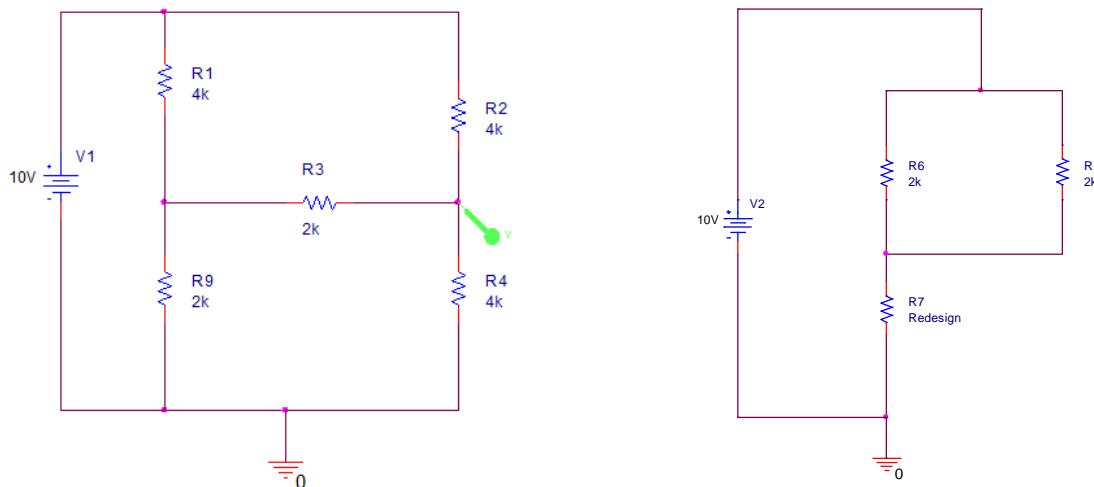
### Silly Bridge Circuit



- a) Find  $R_{design}$  such that no current goes through  $R_{bridge}$  ( $R_{design} = 0.778k$ )

How do you make it so NO CURRENT goes through  $R_{bridge}$ ?

**KEY:** The voltages must be equal at points  $VL$  and  $VR$



We have to find a way to get voltage across  $R_4$ .  
The resistors are not in series so you can't use a voltage divider (stupid  $R_3$  made it harder).

Meh....mesh analysis, why not?

3 meshes

On right side, it should be an easy equation to find  $VR$  since we have two resistors in parallel that we can combine.  
We can use a voltage divider equation if we get the voltage from  $VL$ !

Mesh loop 1

$$-10 + I_1 \cdot 4k - I_2 \cdot 4k + I_1 \cdot 2k - I_3 \cdot 2k = 0$$

$$I_1 \cdot 6k - I_2 \cdot 4k - I_3 \cdot 2k = 10$$

Mesh loop 2

$$I_2 \cdot 4k - I_1 \cdot 4k + I_2 \cdot 4k + I_2 \cdot 2k - I_3 \cdot 2k = 0$$

$$-I_1 \cdot 4k + I_2 \cdot 10k - I_3 \cdot 2k = 0$$

Mesh loop 3

$$I_3 \cdot 2k - I_1 \cdot 2k + I_3 \cdot 2k - I_2 \cdot 2k + I_3 \cdot 4k = 0$$

$$-I_1 \cdot 2k - I_2 \cdot 2k + I_3 \cdot 8k = 0$$

$$M := \begin{pmatrix} 6 \times 10^3 & -4 \times 10^3 & -2 \times 10^3 \\ -4 \times 10^3 & 10 \times 10^3 & -2 \times 10^3 \\ -2 \times 10^3 & -2 \times 10^3 & 8 \times 10^3 \end{pmatrix}$$

$$X := \begin{pmatrix} 10 \\ 0 \\ 0 \end{pmatrix}$$

$$M^{-1} \cdot X = \begin{pmatrix} 2.969 \times 10^{-3} \\ 1.406 \times 10^{-3} \\ 1.094 \times 10^{-3} \end{pmatrix}$$

$$I_{1a} := 2.97 \text{mA}$$

$$I_{2a} := 1.406 \text{mA}$$

$$I_{3a} := 1.094 \text{mA}$$

Use mesh loop current I3 to get voltage across R4 (ohms law)

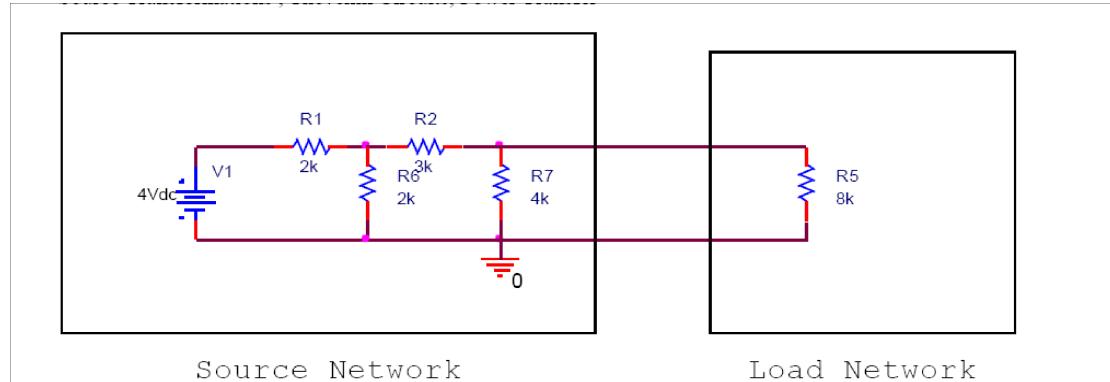
$$I_{3a} \cdot 4k\Omega = 4.376 \text{V}$$

$$4.376 = 10 \cdot \frac{R_7}{R_7 + 1k}$$

$$R_7 := 0.778 \text{k}\Omega$$

Source Transformations, Thevenin Circuits, Power Transfer

b)



1. Find  $V_{open\ circuit}$   $V_{thevenin}$

Combine resistors then use the voltage divider equation to find the voltage across the load.

$$R_{1b} := 2k\Omega$$

$$R_{5b} := 8k\Omega$$

$$R_{2b} := 3k\Omega$$

$$V_{1b} := 4V$$

$$R_{6b} := 2k\Omega$$

$$R_{7b} := 4k\Omega$$

Add R2 and R7 in series

$$R_{72b} := R_{7b} + R_{2b}$$

$$R_{72b} = 7 \cdot k\Omega$$

Add R72 and R6 in parallel

$$R_{726b} := \frac{R_{72b} \cdot R_{6b}}{R_{72b} + R_{6b}}$$

$$R_{726b} = 1.556 \cdot k\Omega$$

Voltage divider

$$V_{726b} := \frac{V_{1b} \cdot R_{726b}}{R_{1b} + R_{726b}}$$

$$V_{726b} = 1.75 \text{ V}$$

The is the voltage across R6 and R27 so use voltage divider again to find R7

$$V_{Tb} := \frac{V_{726b} \cdot R_{7b}}{R_{2b} + R_{7b}}$$

$$V_{Tb} = 1 \text{ V}$$

2. Find Ishortcircuit or INorton

Short circuit across R7 which makes it go away

$$R_{62b} := \frac{R_{2b} \cdot R_{6b}}{R_{2b} + R_{6b}}$$

$$R_{62b} = 1.2 \cdot k\Omega$$

$$V_{R62b} := \frac{V_{1b} \cdot R_{62b}}{R_{62b} + R_{1b}}$$

$$V_{R62b} = 1.5 \text{ V}$$

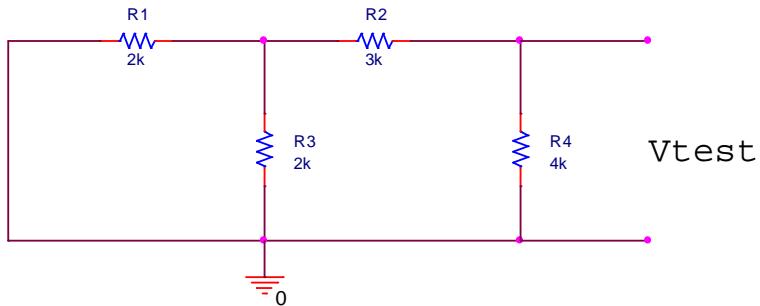
$$I_{R3} := \frac{V_{R62b}}{R_{2b}} \quad I_{Nb} := I_{R3}$$

$$I_{Nb} = 0.5 \cdot \text{mA}$$

This is the norton current

3. Find Rthevenin

Long way



Find Req

$$R_{16b} := \frac{R_{1b} \cdot R_{6b}}{R_{1b} + R_{6b}}$$

$$R_{16b} = 1 \cdot k\Omega$$

$$R_{16b} + R_{2b} = 4 \cdot k\Omega$$

Two 4k ohms in parallel

$$R_{eq} := 2k\Omega$$

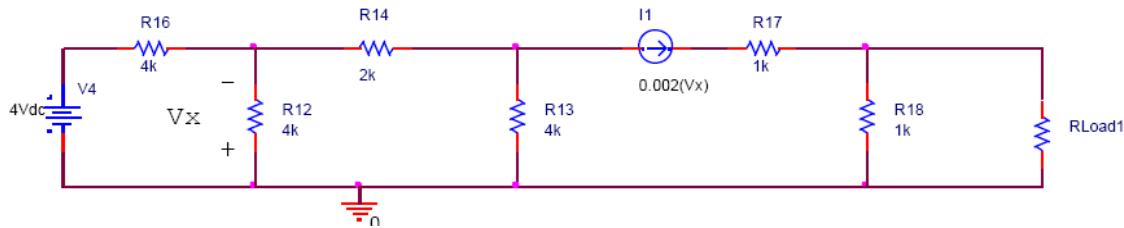
4. Is it possible to change R7 such that maximum power is delivered to R5?

$R_T = R_L$  You need  $R_T$  to be 8k but you have a 4k resistor in parallel

Any resistor you put in parallel with it will result in a Req of <4k

Thevenin Equivalence - Dependent source

c)



1. Find Vopen circuit VT

Do nodal analysis at points Va, Vb and Vc

$$(1) \quad V_a = -V_x$$

At node Va

$$\frac{V_a - 4}{4k} + \frac{V_a}{4k} + \frac{V_a - V_b}{2k} = 0$$

$$(2) \quad V_a \left( \frac{1}{4k} + \frac{1}{4k} + \frac{1}{2k} \right) - V_b \left( \frac{1}{2k} \right) = \frac{4}{4k}$$

At node Vb

$$\frac{V_b - V_a}{2k} + \frac{V_b}{4k} + 0.002 \cdot -V_a = 0$$

$$(3) \quad -V_a \left( \frac{1}{2k} + 0.002 \right) + V_b \left( \frac{1}{2k} + \frac{1}{4k} \right) = 0$$

At node Vc

$$(4) \quad -(-V_a \cdot 0.002) + \frac{V_c}{1k} = 0 \quad \frac{1}{4 \cdot 10^3} + \frac{1}{4 \cdot 10^3} + \frac{1}{2 \cdot 10^3} = 1 \times 10^{-3}$$

$$M_c := \begin{bmatrix} \frac{1}{4 \cdot 10^3} + \frac{1}{4 \cdot 10^3} + \frac{1}{2 \cdot 10^3} & \frac{-1}{2 \cdot 10^3} & 0 \\ -\left(\frac{1}{2 \cdot 10^3} + 0.002\right) & \frac{1}{2 \cdot 10^3} + \frac{1}{4 \cdot 10^3} & 0 \\ 0.002 & 0 & \frac{1}{1 \cdot 10^3} \end{bmatrix}$$

$$C_c := \begin{pmatrix} \frac{4}{4 \cdot 10^3} \\ 0 \\ 0 \end{pmatrix}$$

$$X_c := M_c^{-1} \cdot C_c$$

$$X_c = \begin{pmatrix} -1.5 \\ -5 \\ 3 \end{pmatrix} \quad V_a := -1.5V \quad V_b := -5V \quad V_c := 3V$$

$$V_{Tc} := V_c$$

2) Find Ishortcircuit IN

If you short Rload, then the current through R18 is zero so R18 goes away. The current will be I1

$$0.002 \cdot (-V_a) = 3 \times 10^{-3} V \quad \text{units here don't matter}$$

$$I_{sc} := 3mA$$

$$I_N := I_{sc}$$

$$\frac{V_c}{I_{sc}} = 1 \times 10^3 \Omega$$

This is the expected  
thevenin resistance  
value.

3) Find RThevenin using a test voltage source of 1V

Short circuit voltage sources, open circuit INDEPENDENT current sources

Find current through VTest

Need  $V_x$ , what is it when  $V_{test} = 1$

$$-V_{a2} = V_x$$

$$\frac{V_{a2}}{2k} + \frac{V_{a2} - V_{b2}}{2k} = 0$$

$$(1) \quad V_{a2} \cdot \left( \frac{1}{2k} + \frac{1}{2k} \right) - V_{b2} \cdot \left( \frac{1}{2k} \right) = 0$$

$$\frac{V_{b2} - V_{a2}}{2k} + \frac{V_{b2}}{4k} + 0.002 \cdot -V_{a2} = 0$$

$$(2) \quad -V_{a2} \cdot \left( \frac{1}{2k} + 0.002 \right) + V_{b2} \cdot \left( \frac{1}{2k} + \frac{1}{4k} \right) = 0$$

*This equation below is really all we need....*

$$-0.002 \cdot -V_{a2} + \frac{1}{1k} = 0 \quad \text{note } V_c = V_{test}=1$$

$$(3) \quad V_{a2} \cdot 0.002 = \frac{-1}{1k}$$

$$V_{a2} := \frac{-1}{1 \cdot 10^3} \cdot \frac{1}{0.002}$$

$$V_{a2} = -0.5$$

$$V_x := 0.5$$

$$0.002 \cdot 0.5 = 1 \times 10^{-3}$$

Current I1 = 1mA

Current through R18 is also 1mA

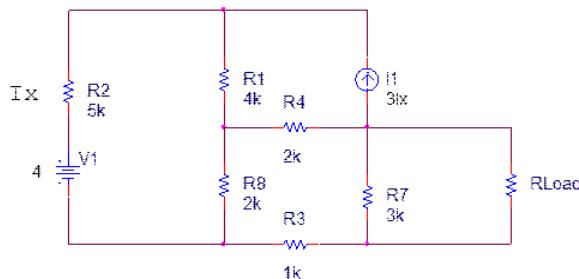
$$R_{eq2} := \frac{1V}{1mA}$$

*Anytime you have a resistor in parallel with a voltage source it is as if the voltage is dropping across that resistor....the node above the resistor must be VTest. Therefore, it would make sense that the current through the resistor is VTest/R18.... 1V/1k = 1mA*

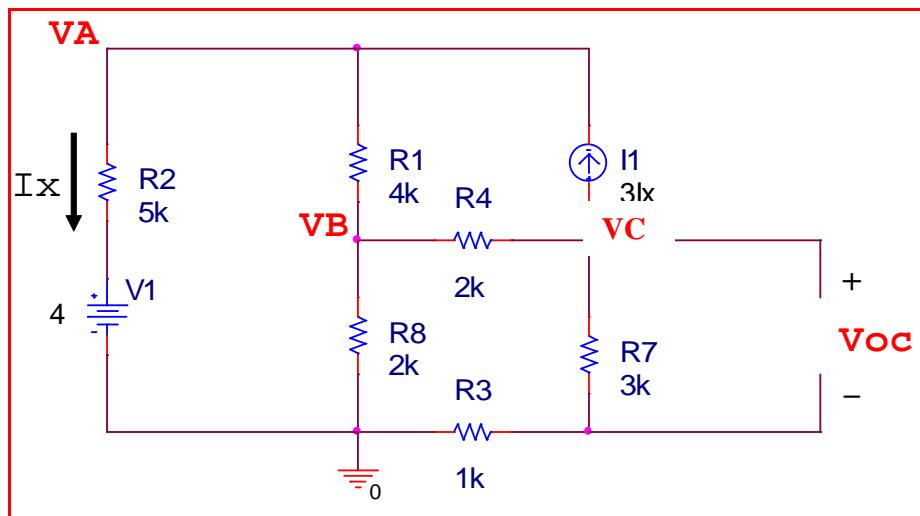
*Making Req 1kΩ*

$$R_{eq2} = 1\cdot k\Omega$$

4)



a) Find  $V_{thevenin}$  using the open circuit method



KCL at node A

$$\frac{V_A - 4}{5k} + \frac{V_A - V_B}{4k} - 3 \cdot I_x = 0$$

$$(1) \quad V_A \left( \frac{1}{5k} + \frac{1}{4k} \right) - V_B \cdot \frac{1}{4k} - 3 \cdot I_x = 0$$

KCL at node B

$$\frac{V_B - V_A}{4k} + \frac{V_B - V_C}{2k} + \frac{V_B}{2k} = 0$$

$$(2) \quad -V_A \cdot \frac{1}{4k} + V_B \left( \frac{1}{4k} + \frac{1}{2k} + \frac{1}{2k} \right) - V_C \frac{1}{2k} = 0$$

KCL at node C

$$\frac{V_C - V_B}{2k} + 3 \cdot I_x + \frac{V_C}{4k} = 0 \quad \text{notice two resistors in series!}$$

$$(3) \quad -V_B \cdot \frac{1}{2k} + V_C \left( \frac{1}{2k} + \frac{1}{4k} \right) = 0$$

Dependent source

$$\frac{V_A - 4}{5k} = I_x$$

$$(4) \quad V_A \cdot \frac{1}{5k} - I_x = \frac{4}{5k}$$

4 equations 4 unknowns VA, VB, VC, and Ix

$$M_d := \begin{bmatrix} \left( \frac{1}{5 \cdot 10^3} + \frac{1}{4 \cdot 10^3} \right) & \frac{-1}{4 \cdot 10^3} & 0 & -3 \\ \frac{-1}{4 \times 10^3} & \frac{1}{4 \cdot 10^3} + \frac{1}{2 \cdot 10^3} + \frac{1}{2 \cdot 10^3} & \frac{-1}{2 \cdot 10^3} & 0 \\ 0 & \frac{-1}{2 \cdot 10^3} & \frac{1}{2 \cdot 10^3} + \frac{1}{4 \cdot 10^3} & 3 \\ \frac{1}{5 \cdot 10^3} & 0 & 0 & -1 \end{bmatrix}$$

write variables  
as

$$v_d = \begin{pmatrix} v_A \\ v_B \\ v_C \\ i_x \end{pmatrix}$$

$$C_d := \begin{pmatrix} \frac{4}{5 \cdot 10^3} \\ 0 \\ 0 \\ \frac{4}{5 \cdot 10^3} \end{pmatrix}$$

$$X_d := M_d^{-1} \cdot C_d$$

$$X_d = \begin{pmatrix} 10.667 \\ 0 \\ -5.333 \\ 1.333 \times 10^{-3} \end{pmatrix}$$

$$v_A := 10.66V$$

$$v_B := 0V$$

$$v_C := -5.33V$$

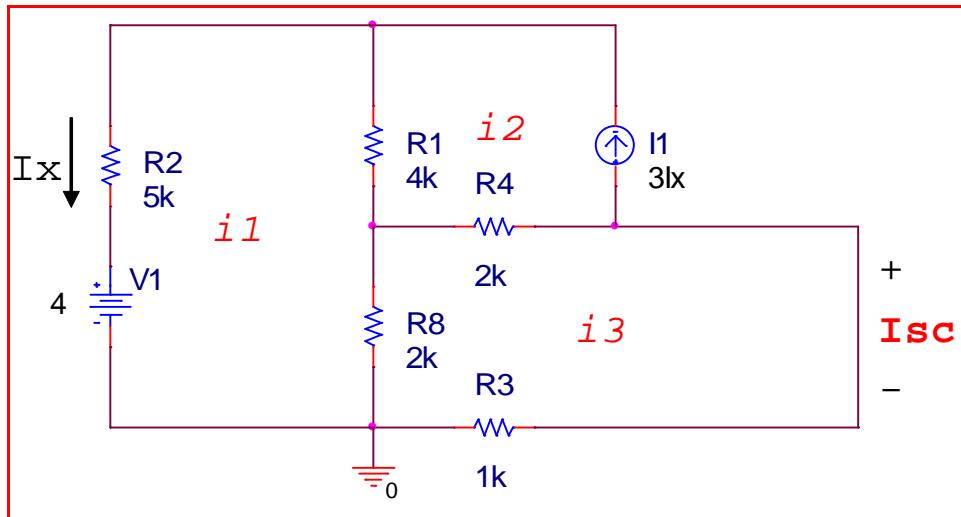
$$i_x := 1.33mA$$

Use voltage divider now to get  $V_{oc}$ , be careful with resistors

$$V_{ocd} := V_C \cdot \frac{3 \cdot k\Omega}{3k\Omega + 1k\Omega}$$

$$V_{ocd} = -3.998V$$

b) Find INorton using the Short circuit method



Source constraint  $i_2 := -3I_x$

KVL on loop 1

$$(i_1 - i_2) \cdot 4k + (i_1 - i_3) \cdot 2k - 4 + i_1 \cdot 5k = 0$$

KVL on loop 3

$$(i_3 - i_2) \cdot 2k + i_3 \cdot 1k + (i_3 - i_1) \cdot 2k = 0$$

Dependent source

$$i_1 := -I_x$$

$$M_{d2} := \begin{pmatrix} 11 \cdot 10^3 & -4 \cdot 10^3 & -2 \cdot 10^3 & 0 \\ -2 \cdot 10^3 & -2 \cdot 10^3 & 5 \cdot 10^3 & 0 \\ 0 & 1 & 0 & 3 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

write down variables,  $i_1$ ,  $i_2$ ,  $i_3$ , and  $I_x$

$$V_{d2} = \begin{pmatrix} i_1 \\ i_2 \\ i_3 \\ I_x \end{pmatrix}$$

$$C_{d2} := \begin{pmatrix} 4 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$X_{d2} := M_{d2}^{-1} \cdot C_{d2}$$

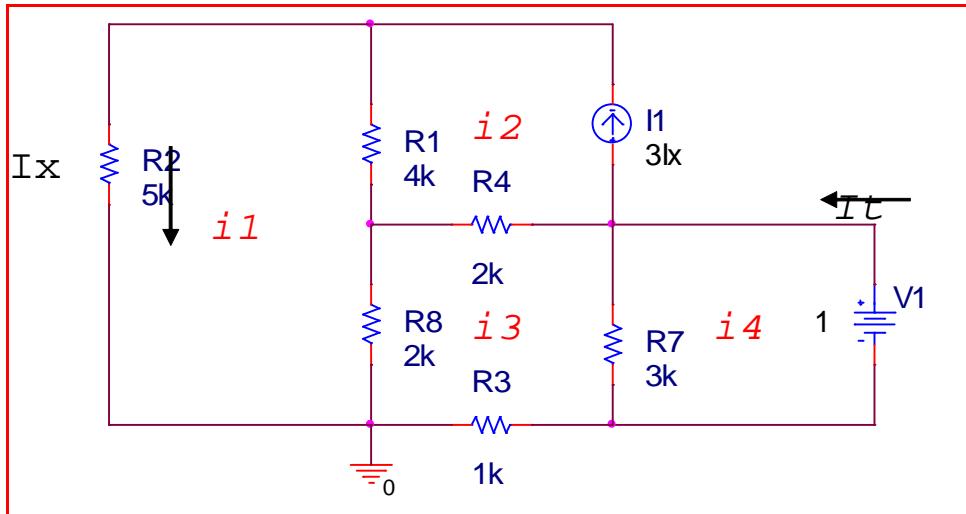
$$X_{d2} = \begin{pmatrix} -9.524 \times 10^{-4} \\ -2.857 \times 10^{-3} \\ -1.524 \times 10^{-3} \\ 9.524 \times 10^{-4} \end{pmatrix}$$

$i_1 = -0.95\text{mA}$   
 $i_2 = -2.85\text{mA}$   
 $i_3 = -1.52\text{mA} = I_{sc}$

$$i_4 = 0.95\text{mA}$$

$$I_{scd} := -1.52\text{mA}$$

c) Find R<sub>Thevenin</sub> using the test voltage/current method



Source constraint:

$$i_{2a} := -3I_x$$

KVL on loop 1

$$(i_1 - i_2) \cdot 4k + (i_1 - i_3) \cdot 2k + i_1 \cdot 5k = 0$$

KVL on loop 3:

$$(i_3 - i_2) \cdot 2k + (i_3 - i_4) \cdot 3k + i_3 \cdot 1k + (i_3 - i_1) \cdot 2k = 0$$

KVL on loop 4:

$$1 + (i_4 - i_3) \cdot 3k = 0$$

Dependent source

$$i_1 = -I_x$$

$$M_{d3} := \begin{pmatrix} 11 \cdot 10^3 & -4 \cdot 10^3 & -2 \cdot 10^3 & 0 & 0 \\ -2 \cdot 10^3 & -2 \cdot 10^3 & 8 \cdot 10^3 & -3 \cdot 10^3 & 0 \\ 0 & 0 & -3 \cdot 10^3 & 3 \cdot 10^3 & 0 \\ 0 & 1 & 0 & 0 & 3 \\ 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

variables  
i1, i2, i3, i4, Ix

$$C_{d3} := \begin{pmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 0 \end{pmatrix}$$

$$X_{d3} := M_{d3}^{-1} \cdot C_{d3}$$

$$X_{d3} = \begin{pmatrix} 9.524 \times 10^{-5} \\ 2.857 \times 10^{-4} \\ -4.762 \times 10^{-5} \\ -3.81 \times 10^{-4} \\ -9.524 \times 10^{-5} \end{pmatrix}$$

$i_{1d3} := 0.095\text{mA}$   
 $i_{2d3} := 0.285\text{mA}$   
 $i_{3d3} := -0.047\text{mA}$   
 $i_{4d3} := -0.38\text{mA}$

$$I_{\text{test}} := -i_{4d3}$$

$$I_{xd3} := -0.0956\text{mA}$$

$$R_{\text{th}} := \frac{1\text{V}}{I_{\text{test}}}$$

$$R_{\text{th}} = 2.632 \cdot k\Omega$$

Check  $V_{\text{th}}/I_{\text{sc}}$

$$\frac{V_{\text{ocd}}}{I_{\text{scd}}} = 2.63 \cdot k\Omega$$