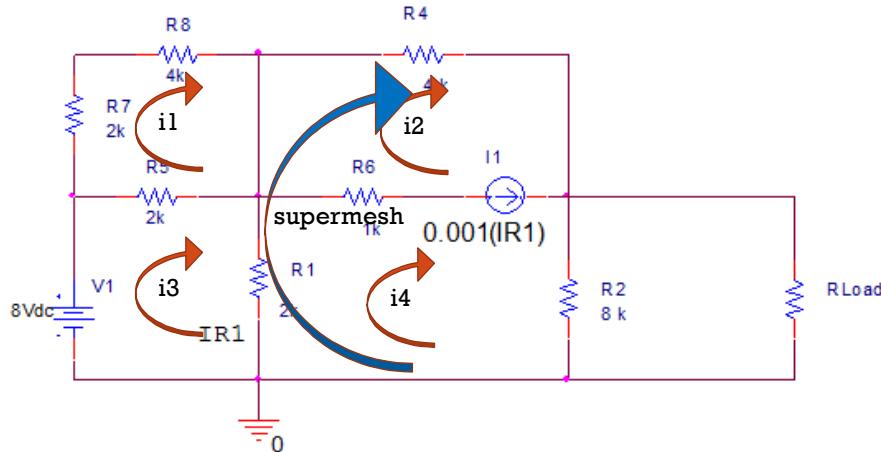


Thevenin Equivalence - Dependent sources

a)



1) Find V<sub>Thevenin</sub> using the open circuit method

Mesh around loop 1

$$i_1 \cdot 4k + i_1 \cdot 2k - i_3 \cdot 2k + i_1 \cdot 2k = 0$$

$$(1) \quad i_1 \cdot (8k) - i_3 \cdot 2k = 0$$

Supermesh loop

$$i_2 \cdot 4k + i_4 \cdot 8k + i_4 \cdot 2k - i_3 \cdot 2k = 0$$

$$(2) \quad i_2 \cdot 4k - i_3 \cdot 2k + i_4 \cdot 10k = 0$$

Current source

$$-i_2 + i_4 = 0.001(I_{R1})$$

Dependent source

$$I_{R1} = i_3 - i_4$$

Substitute in and rearrange

$$-i_2 + i_4 = 0.001(i_3 - i_4)$$

$$(3) \quad -i_2 - 0.001i_3 + 1.001i_4 = 0$$

Mesh around loop 3

$$i_3 \cdot 2k - i_1 \cdot 2k + i_3 \cdot 2k - i_4 \cdot 2k - 8 = 0$$

$$(4) \quad -i_1 \cdot 2k + i_3 \cdot 4k - i_4 \cdot 2k = 8$$

$$M_a := \begin{pmatrix} 8 \cdot 10^3 & 0 & -2 \cdot 10^3 & 0 \\ 0 & 4 \cdot 10^3 & -2 \cdot 10^3 & 10 \cdot 10^3 \\ 0 & -1 & -0.001 & 1.001 \\ -2 \cdot 10^3 & 0 & 4 \cdot 10^3 & -2 \cdot 10^3 \end{pmatrix}$$

$$C_a := \begin{pmatrix} 0 \\ 0 \\ 0 \\ 8 \end{pmatrix}$$

$$X_a := M_a^{-1} \cdot C_a$$

$$X_a = \begin{pmatrix} 6.223 \times 10^{-4} \\ 3.541 \times 10^{-4} \\ 2.489 \times 10^{-3} \\ 3.562 \times 10^{-4} \end{pmatrix} \quad \begin{aligned} i_1 &:= 0.622 \text{mA} \\ i_2 &:= 0.354 \text{mA} \\ i_3 &:= 2.49 \text{mA} \\ i_4 &:= 0.356 \text{mA} \end{aligned}$$

$$V_T := i_4 \cdot 8k\Omega$$

$$V_T = 2.848 \text{ V}$$

2) Find INorton using the short circuit method

Repeat the above analysis including a short circuit connection across the load leads. With the exception of loop 4, all equations are the same.

Mesh loop 1

$$i_1 \cdot 4k + i_1 \cdot 2k + i_1 \cdot 2k - i_3 \cdot 2k = 0$$

$$(1) \quad i_1 \cdot (8k) - i_3 \cdot 2k = 0$$

Supermesh loop

$$i_2 \cdot 4k + i_4 \cdot 2k - i_3 \cdot 2k = 0$$

$$(2) \quad i_2 \cdot 4k - i_3 \cdot 2k + i_4 \cdot 2k = 0$$

Current source

$$-i_2 + i_4 = 0.001(I_{R1})$$

Dependent  
source

$$I_{R1} = i_3 - i_4$$

Substitute in and  
rearrange

$$-i_2 + i_4 = 0.001(i_3 - i_4)$$

$$(3) \quad -i_2 - 0.001i_3 + 1.001i_4 = 0$$

Mesh around loop 3

$$i_3 \cdot 2k - i_1 \cdot 2k + i_3 \cdot 2k - i_4 \cdot 2k - 8 = 0$$

$$(4) \quad -i_1 \cdot 2k + i_3 \cdot 4k - i_4 \cdot 2k = 8$$

$$M_{a1} := \begin{pmatrix} 8 \cdot 10^3 & 0 & -2 \cdot 10^3 & 0 \\ 0 & 4 \cdot 10^3 & -2 \cdot 10^3 & 2 \cdot 10^3 \\ 0 & -1 & -0.001 & 1.001 \\ -2 \cdot 10^3 & 0 & 4 \cdot 10^3 & -2 \cdot 10^3 \end{pmatrix}$$

$$C_{a1} := \begin{pmatrix} 0 \\ 0 \\ 0 \\ 8 \end{pmatrix}$$

$$X_{a1} := M_{a1}^{-1} \cdot C_{a1}$$

$$X_{a1} = \begin{pmatrix} 7.061 \times 10^{-4} \\ 9.408 \times 10^{-4} \\ 2.824 \times 10^{-3} \\ 9.427 \times 10^{-4} \end{pmatrix} \quad i_{1a} := 0.706\text{mA}$$

$$i_{2a} := 0.941\text{mA}$$

$$i_{3a} := 2.82\text{mA}$$

$$i_{4a} := 0.942\text{mA}$$

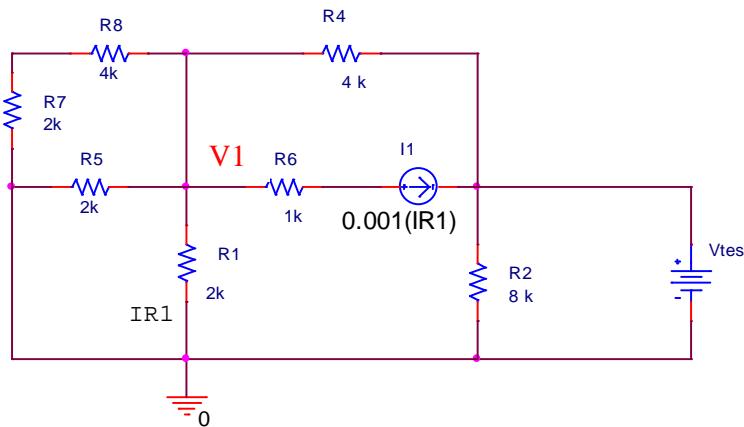
$$I_{sc} := i_{4a}$$

$$I_{sc} = 0.942 \cdot \text{mA}$$

**For future reference**

$$\frac{V_T}{I_{sc}} = 3.023 \times 10^3 \Omega$$

3) Find RThevenin



Dependent source equation

$$I_{R1} = \frac{V_1}{2k}$$

Performing KCL at V1

$$\frac{V_1 - 0}{0.857k} + \frac{V_1 - V_{\text{test}}}{4k} + 0.001I_{R1}$$

$$\frac{V_1 - 0}{0.857k} + \frac{V_1 - V_{\text{test}}}{4k} + 0.001 \frac{V_1}{2k}$$

$$(1) \quad V_1 \left( \frac{1}{0.857k} + \frac{1}{4k} + \frac{0.001}{2 \cdot 10^3} \right) = \frac{V_{\text{test}}}{4k}$$

$$V_1 = 0.175 \cdot V_{\text{test}}$$

Note Req for left

$$\frac{(4k\Omega + 2k\Omega) \cdot 2k\Omega}{4k\Omega + 2k\Omega + 2k\Omega} = 1.5 \cdot k\Omega$$

$$\frac{1.5k\Omega \cdot 2k\Omega}{3.5k\Omega} = 0.857 \cdot k\Omega$$

Perform KCL at Vtest

$$\frac{V_{\text{test}} - V_1}{4k} + \frac{V_{\text{test}}}{8k} - 0.001I_{R1} - I_{\text{Test}} = 0$$

substitute in V1 and IR1 equations

$$\frac{V_{\text{test}} - 0.175V_{\text{test}}}{4k} + \frac{V_{\text{test}}}{8k} - 0.001 \cdot \frac{0.175V_{\text{test}}}{2k} = I_{\text{Test}}$$

$$V_{\text{test}} \cdot \frac{(1 - 0.175)}{4k} + \frac{1}{8k} - \frac{0.001 \cdot 0.175}{2k} = I_{\text{Test}}$$

remember  $V_{\text{test}}=1$

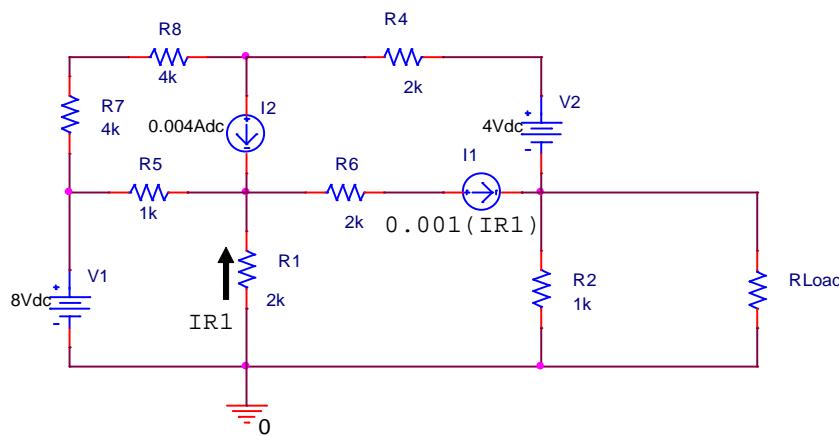
$$\frac{0.825}{4 \cdot 10^3} + \frac{1}{8 \cdot 10^3} - \frac{0.001 \cdot 0.175}{2 \cdot 10^3} = 3.312 \times 10^{-4}$$

$$I_{\text{test}} = 3.312 \cdot 10^{-4}$$

$$\frac{V_{\text{Test}}}{I_{\text{Test}}} = 3k\Omega$$

$$\frac{1}{3.31 \cdot 10^{-4}} = 3.021 \times 10^3$$

b)



- 1) Find VT using superposition
- 2) Find IN and RT

Keeping V1, short V2, open I2, upper two loops merge

Voc solution starting with the lower left loop:

Loop 1:

$$-8 + i_1 \cdot 1k - i_2 \cdot 1k + i_1 \cdot 2k - i_3 \cdot 2k = 0$$

$$(1) \quad i_1 \cdot 3k - i_2 \cdot 1k - i_3 \cdot 2k = 8$$

Supermesh!

Loop2 + Loop3 (merged loops)

$$i_2 \cdot 1k - i_1 \cdot 1k + i_2 \cdot 4k + i_2 \cdot 4k + i_2 \cdot 2k + i_3 \cdot 1k + i_3 \cdot 2k - i_1 \cdot 2k = 0$$

$$(2) -i_1 \cdot 3k + i_2 \cdot 11k + i_3 \cdot 3k = 0$$

Dependent current source substitution

$$-i_2 + i_3 = 0.001(i_3 - i_1) \quad 1 - 0.001 = 0.999$$

$$(3) 0.001 \cdot i_1 - i_2 + 0.999 \cdot i_3 = 0$$

$$M_b := \begin{pmatrix} 3 \times 10^3 & -1 \cdot 10^3 & -2 \cdot 10^3 \\ -3 \cdot 10^3 & 11 \cdot 10^3 & 3 \cdot 10^3 \\ 0.001 & -1 & 0.999 \end{pmatrix}$$

$$C_b := \begin{pmatrix} 8 \\ 0 \\ 0 \end{pmatrix}$$

$$X_b := M_b^{-1} \cdot C_b$$

$$X_b = \begin{pmatrix} 3.392 \times 10^{-3} \\ 7.275 \times 10^{-4} \\ 7.248 \times 10^{-4} \end{pmatrix}$$

$$i_{1b1} := 3.39mA$$

$$i_{2b1} := 0.7275mA$$

$$i_{3b1} := 0.7248mA$$

$$V_{ocv1} := i_{3b1} \cdot 1k\Omega$$

$$V_{ocv1} = 0.725V$$

Keeping V2, Short V1, Open I2 (upper two loops merge)

Voc solution starting with the lower left loop

Loop 1, lower left loop

(1)  $i_1 \cdot 3k - i_2 \cdot 1k - i_3 \cdot 2k = 0$

Supermesh!

Loop2 and Loop2

$-i_1 \cdot 3k + i_2 \cdot 11k + i_3 \cdot 3k + 4 = 0$

(2)  $-i_1 \cdot 3k + i_2 \cdot 11k + i_3 \cdot 3k = -4$

Dependent current source and substitution

$(-i_2 + i_3) = 0.001(i_3 - i_1)$

(3)  $0.001 \cdot i_1 - i_2 + 0.999 \cdot i_3 = 0$

$$M_{b2} := \begin{pmatrix} 3 \times 10^3 & -1 \cdot 10^3 & -2 \cdot 10^3 \\ -3 \cdot 10^3 & 11 \cdot 10^3 & 3 \cdot 10^3 \\ 0.001 & -1 & 0.999 \end{pmatrix}$$

$$C_{b2} := \begin{pmatrix} 0 \\ -4 \\ 0 \end{pmatrix}$$

$X_{b2} := M_{b2}^{-1} \cdot C_{b2}$

$$X_{b2} = \begin{pmatrix} -3.636 \times 10^{-4} \\ -3.636 \times 10^{-4} \\ -3.636 \times 10^{-4} \end{pmatrix} \quad \begin{aligned} i_{1b2} &:= -0.3646 \text{mA} \\ i_{2b2} &:= -0.3636 \text{mA} \\ i_{3b2} &:= -0.3636 \text{mA} \end{aligned}$$

$$V_{ocv2} := i_3 b_2 \cdot 1k\Omega$$

$$V_{ocv2} = -0.364 \text{ V}$$

Keep I2, Short V1 and V2 (four loops now!)

Better to reduce than use four loops

Loop 1:

$$(1) \quad i_1 \cdot 3k - i_3 \cdot 2k = 0$$

Supermesh loop2 and loop3

$$(2) \quad -i_1 \cdot 3k + i_2 \cdot 9k + i_3 \cdot 2k + i_4 \cdot 3k = 0$$

Dependent current source and substitution

$$(-i_2 + i_4) = 0.001(i_4 - i_1)$$

$$(3) \quad i_1 - i_2 + 0.999i_4 = 0$$

Independent current source

$$(4) \quad i_2 - i_3 = 0.004$$

$$M_{b3} := \begin{pmatrix} 3 \times 10^3 & 0 & 2 \cdot 10^3 & 0 \\ -3 \cdot 10^3 & 9 \cdot 10^3 & 2 \cdot 10^3 & 3 \cdot 10^3 \\ 1 & -1 & 0 & 0.999 \\ 0 & 1 & -1 & 0 \end{pmatrix}$$

$$C_{b3} := \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0.004 \end{pmatrix}$$

$$X_{b3} := M_{b3}^{-1} \cdot C_{b3}$$

$$X_{b3} = \begin{pmatrix} 1.778 \times 10^{-3} \\ 1.333 \times 10^{-3} \\ -2.667 \times 10^{-3} \\ -4.448 \times 10^{-4} \end{pmatrix} \quad i_{1b3} := 1.78 \text{mA}$$

$$i_{2b3} := 1.33 \text{mA}$$

$$i_{3b3} := -2.67 \text{mA}$$

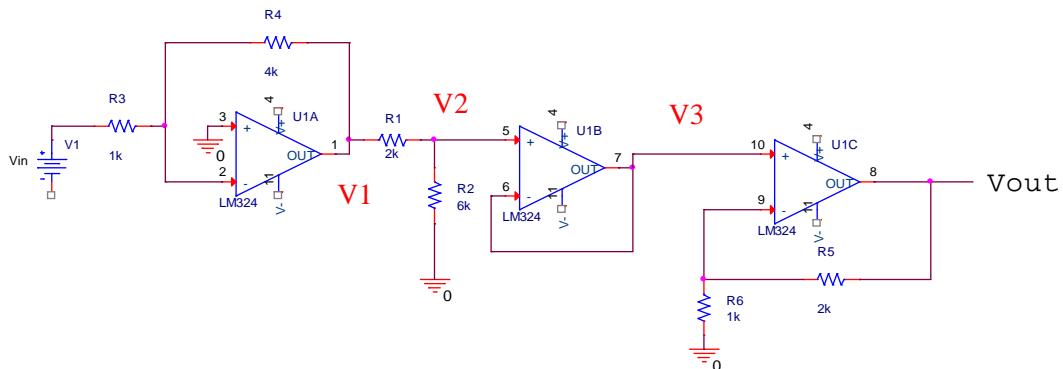
$$i_{4b3} := -0.45 \text{mA}$$

$$V_{oci2} := i_{4b3} \cdot 1k\Omega$$

$$V_{oci2} = -0.45 \text{ V}$$

$$V_{Tb} := V_{ocv1} + V_{ocv2} + V_{oci2} = -0.089 \text{ V}$$

c)



1) Determine Vout in terms of Vin

Stage 1. Inverting op amp

$$V_1 = -\left(\frac{4k}{1k}\right)V_1$$

$$V_1 = -4V_1$$

Stage 2: Voltage divider

$$V_2 = \frac{6k}{2k + 6k} \cdot V_1$$

$$V_2 = \left(\frac{3}{4}\right) \cdot -4 \cdot V_1 = -3V_1$$

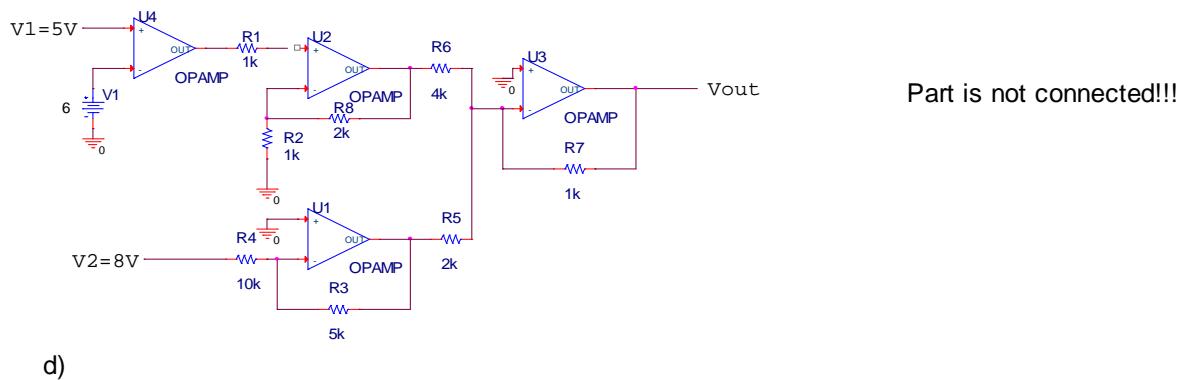
Stage 3: Unity gain amplifier/buffer/ voltage follower

$$V_3 = V_2 = -3V_1$$

Stage 4: Non-inverting amplifier

$$V_{\text{out}} = \left(1 + \frac{2k}{1k}\right) \cdot V_3$$

$$V_{\text{out}} = 3 \cdot (-3)V_1 = -9V_1$$



d)

- 1) For the above circuit, determine the output voltage,  $V_{\text{out}}$ . the voltage supplied to the op-amps is 9V and -9V, as appropriate

U4 is a comparator

U2: non inverting amplifier with a gain of 3

U1: inverting amplifier with a gain of -0.5

U3: Summing amplifier with gain of -0.25 and -0.5

Stage 1:

Output of U4

$(V_+ < V_-)$

$5 < 6$

so -9  
V

Stage 2.1:

$$\text{Output of U2} \quad \text{Output of U4*GainU2} \quad -9V \cdot 3 = -27V \quad \text{Saturated!} \quad -9V$$

$$1 + \frac{R_8}{R_2} = 1 + \frac{2}{1} = 3$$

Stage 2.2:

$$\text{Output of U1} \quad V_2 \cdot \text{Gain of U1} \quad 8V \cdot -0.5 = -4V$$

$$\frac{-R_3}{R_4} = \frac{5k}{10k} = -0.5$$

Stage 3:

$$\text{Output of U3} \quad \text{Output of U2* Gain1 of U3+} \\ \text{Output of U1*Gain2 of U3}$$

$$-9 \cdot \left( \frac{-1k}{4k} \right) + -4 \left( \frac{-1k}{2k} \right)$$

$$-9V \cdot -0.25 + -4V \cdot -0.5 = 4.25V$$