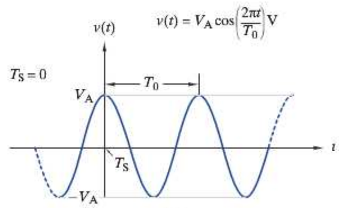
What does the time constant represent in an exponential function?
How do you define a sinusoid?
What is impedance?
How is a capacitor affected by an input signal that changes over time?
How is an inductor affected by an input signal that changes over time?
How do you derive the equation for an integrator and differentiator?
Why does a capacitor or inductor in the op amp circuit cause integration or differentiation?

Sinusoidal waveform

Draw sinusoid with



(a)

VA is the amplitude in volts and definies minimum and maximum values
To is the period in seconds) which is the time required to complete once cylce

Can be expressed with either sine or cosine function, the choice between the two depends where we choose to define t=0. If we chose t=0, v(t)=0

$$v(t) = V_A \cdot \sin\left(\frac{2\pi t}{T_0}\right) V$$

If we choose t=0, v(t)=+VA

$$v(t) = V_A \cdot \cos\left(\frac{2\pi t}{T_0}\right) V$$

typically use cosine version (still call it a sinusoid)

General expression for the sinusoid replaces t with (t-Ts) to signify a shift in time (Ts is the time shift parameter)

Peak nearest the origin occurs at t=Ts

Shift can also be definied as a phase angle or phase shift. Changing the phase angle moves the waveform to the left or right, revealing different phases of the oscillating waveform.

Based on the circular interpretation of the cosine function. A period is divided into 2πradians or 360 degrees.

$$\phi = -2\pi \cdot \frac{T_s}{T_0} = -360 \cdot \frac{T_s}{T_0}$$

http://www.youtube.com/watch?v=QFi16s4RXXY

http://www.analyzemath.com/unitcircle/unit_circle_applet.html

The phase angle is often given in degrees but must be converted to radians so units of the 2πt/T0 terms match

Fourier coefficients

An alternative form of the general sinusoid equation is obtained using the identity

$$\cos(x+y) = \cos(x)\cdot\cos(y) - \sin(x)\sin(y)$$ also $$\sin(x+y) = \sin x\cdot\cos y + \cos x\cdot\sin y$$

$$v(t) = \left(V_A\cdot\cos\phi\right)\cos\left(\frac{2\cdot\pi\,t}{T_0}\right) + \left(-V_A\cdot\sin\Phi\right)\cdot\sin\left(\frac{2\cdot\pi\,t}{T_0}\right)V$$

$$a = V_A\cdot\cos\phi$$ Fourier coefficients-constants

$$b = -V_A\cdot\sin\phi$$

$$v(t) = a\cdot\cos\left(\frac{2\pi\,t}{T_0}\right) + b\cdot\sin\left(\frac{2\pi\,t}{T_0}\right)V$$

$$\phi = \tan^{-1}\left(\frac{-b}{a}\right)$$

Cyclic frequency f0 is defined as the number of periods per unit time.

$$f_0 = \frac{1}{T_0}$$ units Hz

Angular frequency ω0 is in radians per second and related by

$$\omega_0 = 2\pi\,f_0 = \frac{2\pi}{T_0}$$

Primary parameters

1. Amplitude: either VA or the Fourier coefficients a and b
2. Time shift: either Ts or the phase angle ϕ
3. Time/frequency: either T0, f0, or ω0

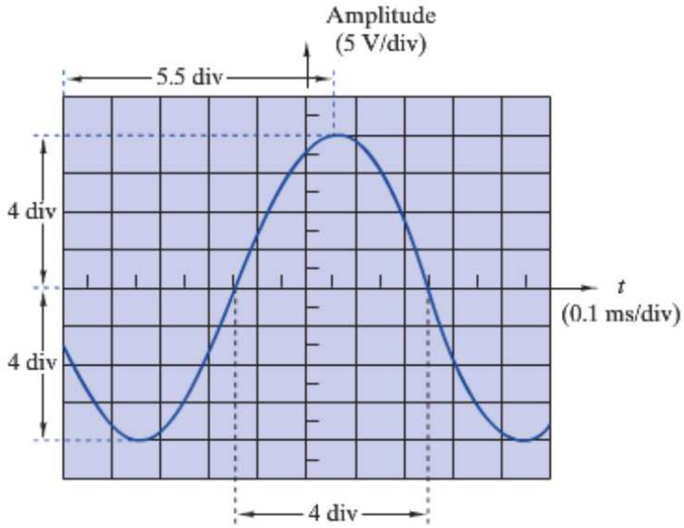
Three equivalent ways to represent a sinusoid

$$v(t) = V_A\cdot\cos\left[2\pi\frac{\left(t-T_s\right)}{T_0}\right] = V_A\cdot\cos\left(\frac{2\pi\,t}{T_0}+\phi\right) = a\cdot\cos\left(\frac{2\pi\,t}{T_0}\right) + b\sin\left(\frac{2\pi\,t}{T_0}\right)V$$

$$v(t) = V_A\cdot\cos\left[2\pi\,f_0\cdot\left(t-T_s\right)\right] = V_A\cdot\cos\left(2\cdot\pi\cdot f_0\cdot t+\phi\right) = a\cdot\cos\left(2\pi\,f_0\cdot t\right) + b\cdot\sin\left(2\pi\,f_0\cdot t\right)V$$

$$v(t) = V_A\cdot\cos\left[\omega_0\cdot\left(t-T_s\right)\right] = V_A\cdot\cos\left(\omega_0\cdot t+\phi\right) = a\cdot\cos\left(\omega_0\cdot t\right) + b\cdot\sin\left(\omega_0\cdot t\right)V$$

1) (Example 5-9 in the text book) The diagram below is an oscilloscope display of a sinusoid. The vertical axis (amplitude) is calibrated at 5V per division, and the horizontal axis (time) is calibrated at 0.1 ms per division. Derive an expression for the sinusoid (IF t=0 is in the center of the oscilloscope).



a. Find the amplitude VA

Note: the waveform is centered on the x-axis

$$V_A = (4\text{div})\cdot\left(5\frac{V}{\text{div}}\right) = 20V$$

b. Find T0

Need 1 cycle: Can find by taking 1/2 cycle and multiplying times 2

$$(4\text{div}\cdot 2)\cdot\left(0.1\frac{\text{ms}}{\text{div}}\right) = 0.8\text{ms}$$

c. Find f0 and ω0

$$f_0 := \frac{1}{0.8\text{ms}} \qquad f_0 = 1.25\cdot\text{kHz}$$

$$\omega_0 := 2\cdot\pi\cdot f_0 = 7.854\times10^3\cdot\frac{\text{rad}}{\text{s}}$$

d. Determine Ts

$$T_s := 0.5 \cdot 0.1$$

$$T_s = 0.05$$

T.s is positive and positive peak is delayed

e. Calculate ϕ0, Fourier coefficients

$$t - 0.05$$

$$\phi = -2\pi \cdot \frac{T_s}{T_0} = -360 \cdot \frac{T_s}{T_0}$$

$$\phi := -2 \cdot \pi \cdot \left(\frac{0.05}{0.8}\right)$$

$$\phi = -22.5 \cdot \mathrm{deg} \qquad \phi = -0.393 \cdot \mathrm{rad}$$

$$a := 20 \cdot \cos(\phi) \qquad a = 18.478$$

$$b := -20 \cdot \sin(\phi) \qquad b = 7.654$$

f. Write equations

$$v(t) = V_A \cdot \cos\left[(\omega t) - T_s\right] \qquad v(t) = 20 \cdot \cos\left(7854t - 0.05 \cdot 10^{-3}\right) V$$

$$v(t) = V_A \cdot \cos(\omega t + \phi)\, V \qquad v(t) = 20 \cos(7854t - 22.5\mathrm{deg})\, V$$

$$v(t) = a \cdot \cos(\omega t) + b \cdot \sin(\omega t) V \qquad v(t) = 18.48 \cdot \cos(7854\omega t) + 7.65 \sin(7854\omega t)$$

2.) Sketch the waveform described by

$$v(t) = 10\cdot\cos(2000\pi\, t - 60\text{deg})\,\text{V}$$

$$v(t) = V_A\cdot\cos\left(2\cdot\pi\cdot f_0\cdot t + \phi\right)$$

$$V_A = 10$$

$$f_{02} := \frac{2000\text{Hz}}{2} = 1\times 10^3\cdot\text{Hz}$$

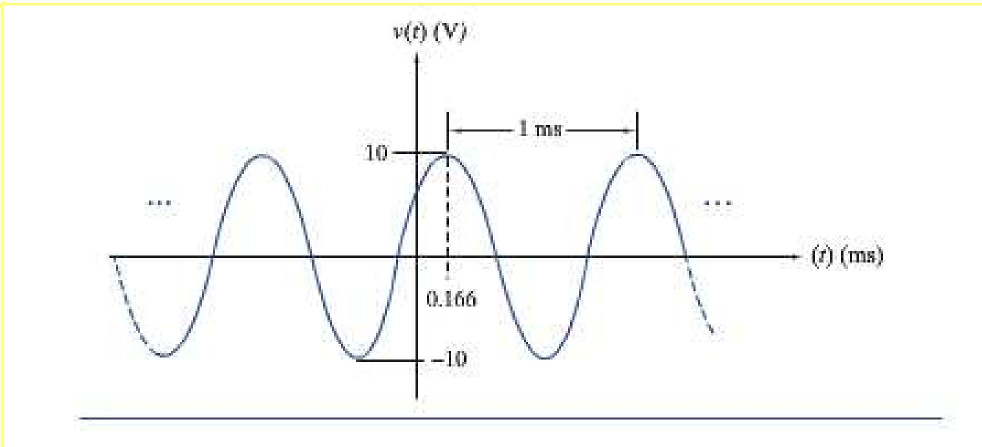
$$T_{02} := \frac{1}{f_{02}}$$

$$T_{02} = 1\cdot\text{ms}$$

$$\phi_2 = -360\cdot\frac{T_s}{T_0}$$

$$\frac{-60}{-360}\cdot 1\text{ms} = 0.167\cdot\text{ms}$$

$$T_s = 1.67\text{mS}$$



3.) Characterize the composite waveform generated by subtracting an exponential from a step function with the same amplitude

$$v(t) = V_A \cdot u(t) - \left(V_A \cdot e^{\frac{-t}{TC}}\right) \cdot u(t)\, V$$

$$= V_A \left(1 - e^{\frac{-t}{TC}}\right) \cdot u(t)\, V$$

For t<0, the waveform is 0 because the step function is 0 before that time. At t = 0 the waveform is still zero because the step and exponential cancel:

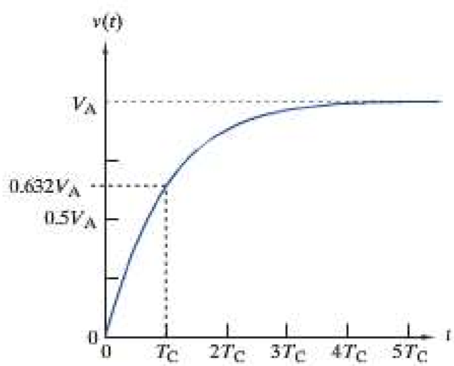
$$v(0) = V_A \cdot \left(1 - e^{0}\right) \cdot 1 = 0$$

For t>>>Tc, the waveform approaches a constant value VA because the exponential term decays to zero.

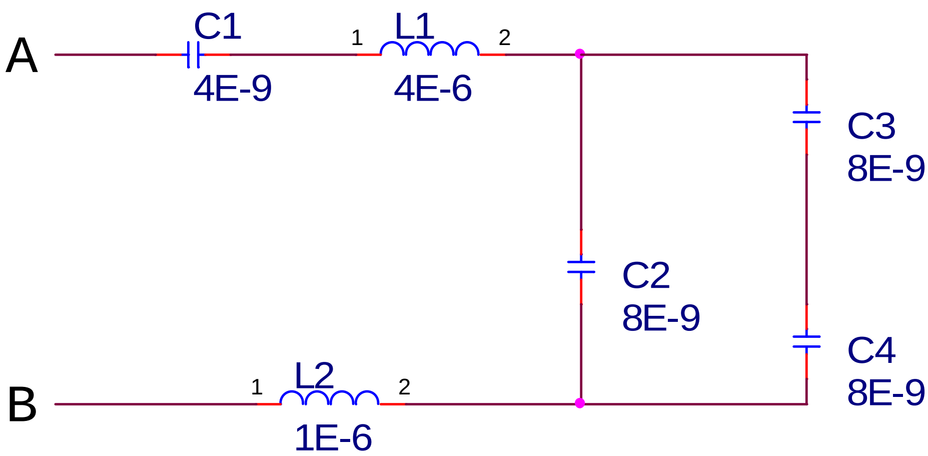
For practical purposes v(t) is within less than 1% of its final value VA when t = 5 Tc

At t=Tc $\quad v(t) = V_A \cdot \left(1 - e^{-1}\right) = 0.632 V_A \qquad 1 - e^{-1} = 0.632$

The waveform rises to about 63% of its value in one time constant $\qquad 1 - e^{-2} = 0.865$



4.) Find an expression for the equivalent impedance

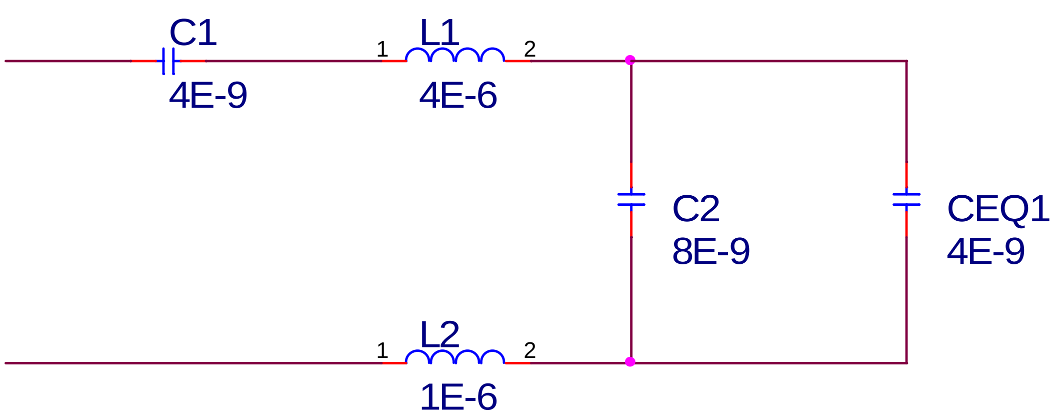


$C_1 := 4\text{nF}$ $\quad C_2 := 8\text{nF}$ $\quad C_3 := 8\text{nF}$ $\quad C_4 := 8\text{nF}$ $\quad L_1 := 4\mu\text{H}$ $\quad L_2 := 1\mu\text{H}$

C3 and C4 are in series

$$C_{34} := \frac{1}{\left(\frac{1}{C_3} + \frac{1}{C_4}\right)}$$

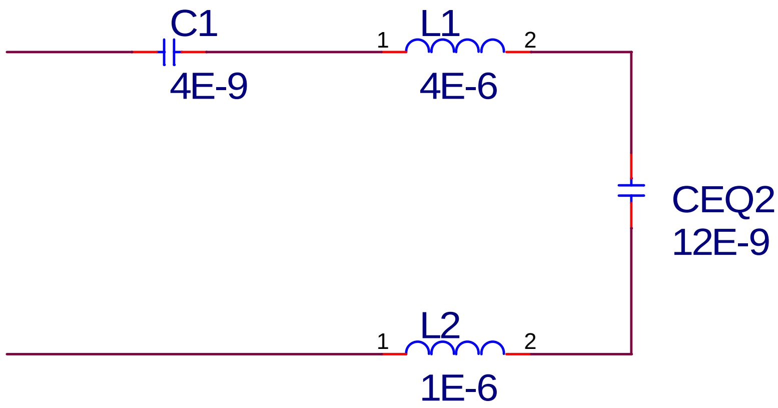
$$C_{34} = 4\cdot\text{nF}$$



C2 and C34 in parallel

$$C_{234} := C_2 + C_{34}$$

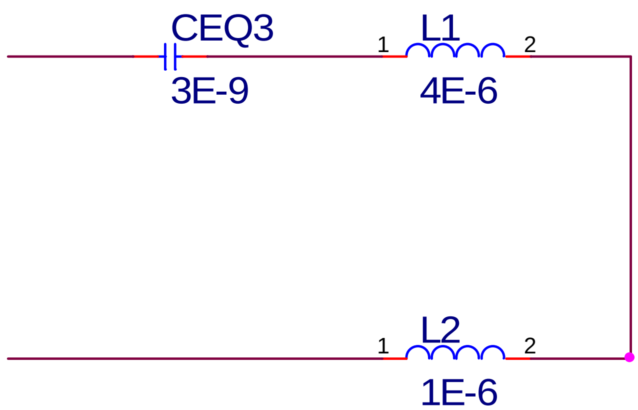
$$C_{234} = 12 \cdot nF$$



C1 and C234 in series

$$C_{1234} := \frac{1}{\left(\frac{1}{C_{234}} + \frac{1}{C_1}\right)}$$

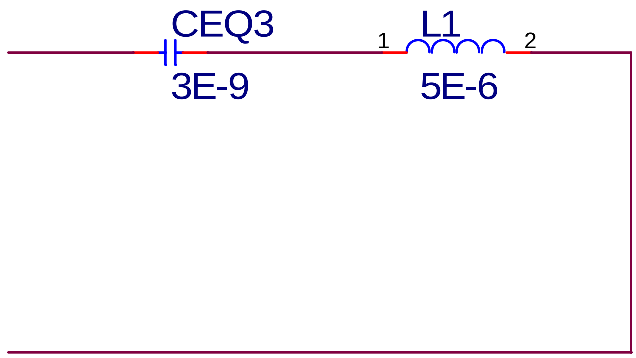
$$C_{1234} = 3 \cdot nF$$



L1 and L2 in series

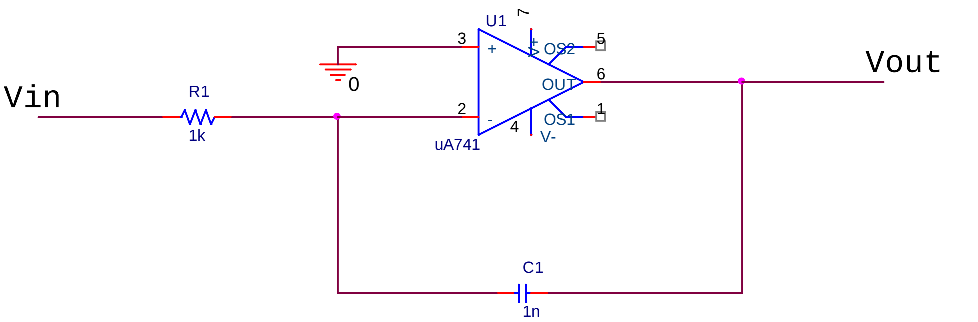
$$L_{12} := L_1 + L_2$$

$$L_{12} = 5 \cdot \mu H$$

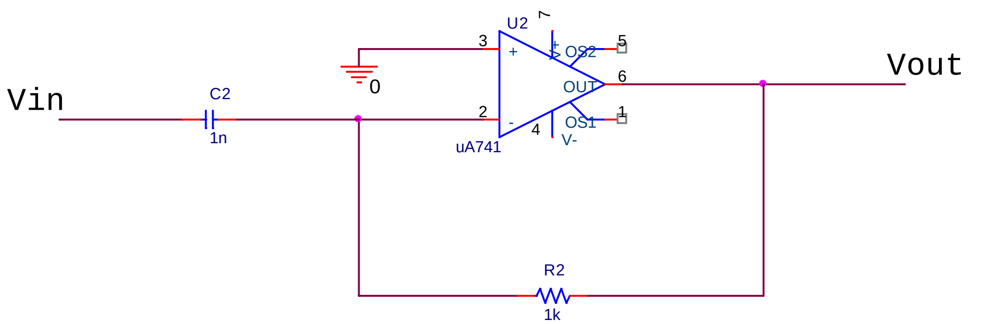


No more reduction is possible

Determine the relationship between the input voltage and the output voltage for the circuits below



(a)



(b)

(a) KCL at node V1=0

$$iR_1 + iC_1 = 0$$

$$\frac{0 - V_{in}}{R_1} + C_1 \cdot \frac{d(0 - V_{out})}{dt} = 0$$

$$\frac{-V_{in}}{R_1} + C_1 \cdot d\left(\frac{-Vout}{dt}\right) = 0$$

$$C_1 \cdot \frac{-dV_{out}}{dt} = \frac{V_{in}}{R_1}$$

$$V_{out} = \frac{-1}{R_1 \cdot C_1} \cdot \int V_{in}\, dt$$

(b) KCL at node V1=0

$$iC_2 + iR_2 = 0$$

$$C_2 \frac{d\left(0 - V_{in}\right)}{dt} + \frac{0 - V_{out}}{R_2} = 0$$

$$C_2 \cdot \frac{-dV_{in}}{dt} = \frac{V_{out}}{R_2}$$

$$\boxed{Vout = -R_2 \cdot C_2 \cdot \frac{dV_{in}}{dt}}$$