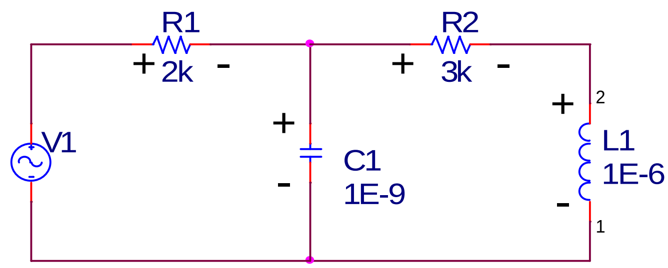
Voltage/Current continuity

1) In the circuit below, the voltage source is defined as follows:

$$Vs = \begin{cases} 0 & t < 0 \\ 10V & 0 < t \end{cases} \text{ (the voltage source turns on at t = 0)}$$



a. What are the initial conditions for the circuit?

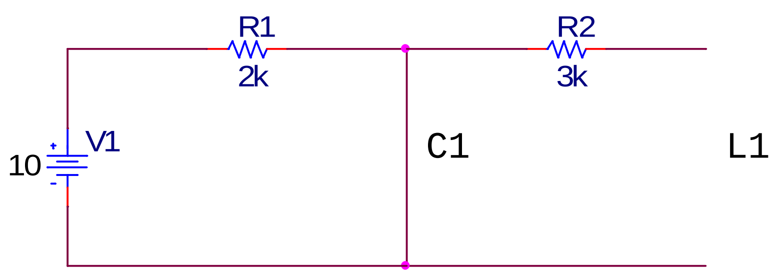
The source is zero for t<0, indicting that the intial conditions are zero.

b. Determine the mathematical expression for the source.

Vs=10u(t)

c. At t = 0^+ , (just after the voltage source turns on), for the polarities indiated in the circuit, determine the voltage acorss each component and the current through each component.

At t = 0+, the source is 10V. We enforce the continuity condition for inductors to get IL(0+)=IL(0-) and for capacitors VC(0+)=VC(0-). since the initial conditions are zero, the current through the inductor at t=0+, must be zero, effectively an open circuit and the voltage across the capaictor at t= 0+ must be zero, effectively a short circuit. The t=0+ circuit can be drawn as



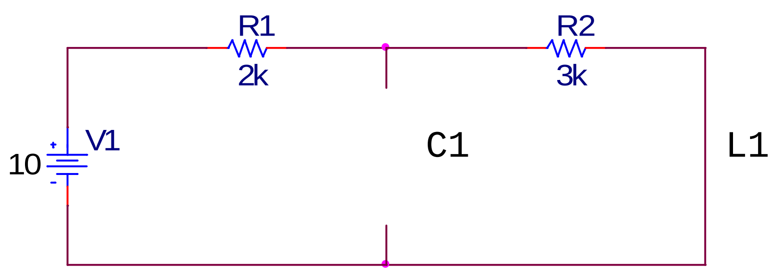
Circuit analysis gives the following results

Component	Voltage	Current
R1	10 V	5mA
R2	0	0
C1	0 (continuity)	5mA
L1	0	0 (continuity)

d. At t goes to ∞, the polarities indicated in the circut, determine the voltage across each component and the current through each component.

As t goes to ∞ for a step function source, we approach the DC steady state conditions. At DC steady state, a capacitor acts as an open circuit and an inductor acts as a short circuit.

When t goes to ∞ the circuit can be drawn as



Circuit analysis gives the following results

Component	Voltage	Current
R1	4 V	2mA
R2	6 V	2mA
C1	6 V	0 (open circuit)
L1	0 (short circuit)	2mA

Voltage across R1 is a voltage divider

$$10\text{V}\cdot\frac{2\text{k}\Omega}{2\text{k}\Omega + 3\text{k}\Omega} = 4\text{ V} \qquad I_{R1} := \frac{4\text{V}}{2\text{k}\Omega} = 2\text{ mA}$$

Voltage across R2 is a voltage divider

$$I_{R2} := \frac{6\text{V}}{3\text{k}\Omega} = 2\text{ mA}$$

$$10\text{V}\cdot\frac{3\text{k}\Omega}{2\text{k}\Omega + 3\text{k}\Omega} = 6\text{ V}$$

Voltage across capacitor

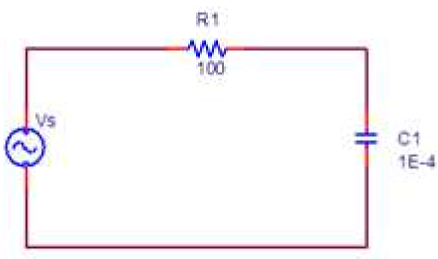
Same a voltage across R2 = 6V $\qquad I_{C1} := 0\text{mA} \quad \text{(open circuit)}$

Voltage across the inductor

I.IL is the same as the current through R1 and R2

$V_{L1} := 0$ short circuit $\qquad I_{lL} := I_{R1} = 2\,mA$

2) RC series circuit

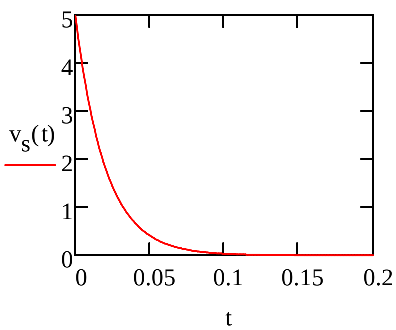


a) Exponential source input

$t := 0, 0.001 .. 0.2$

$v_s(t) = 5e^{-50t}u(t)$

$v_s(t) := 5 \cdot e^{-50t}$



exponential source that turns on at t =0

Initial conditions

$v_c\left(0^-\right) = 0$ still this way because of the step function

$v_c(t) = A_1 \cdot e^{\frac{-t}{\tau}} + A_2 \cdot e^{-50t}$ VCF must look like input

VCN + VCF

$\tau := 100 \cdot 1 \times 10^{-4}$

$\tau = 0.01 \qquad \frac{1}{\tau} = 100$

$v_c(t) = A_1 \cdot e^{-100t} + A_2 \cdot e^{-50t}$

Find A2 coefficient from differential equation for a particular solution subsitute VCF into diff eq. formed by KVL

$$RC\frac{dv_{CF}}{dt} + v_{CF} = V_s$$

$$0.01\cdot\frac{d}{dt}\cdot\left(A_2\cdot e^{-50t}\right) + A_2\cdot e^{-50t} = 5\cdot e^{-50t}$$

$$-0.5\cdot A_2\cdot e^{-50t} + A_2\cdot e^{-50t} = 5\cdot A_2\cdot e^{-50t}$$

Divide out e^-50t

$$-0.5A_2 + A_2 = 5$$

$$A_2\cdot(-0.5 + 1) = 5$$

$$A_2 = 10$$

Find A1 using inital conditions

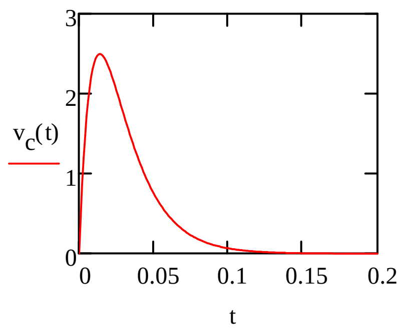
$$v_c(t) = A_1\cdot e^{-100t} + 10\cdot e^{-50}$$

$$v_c(0^-) = v_c\left(0^+\right) = 0$$

$$v_c(t) := -10e^{-100t} + 10e^{-50t}$$

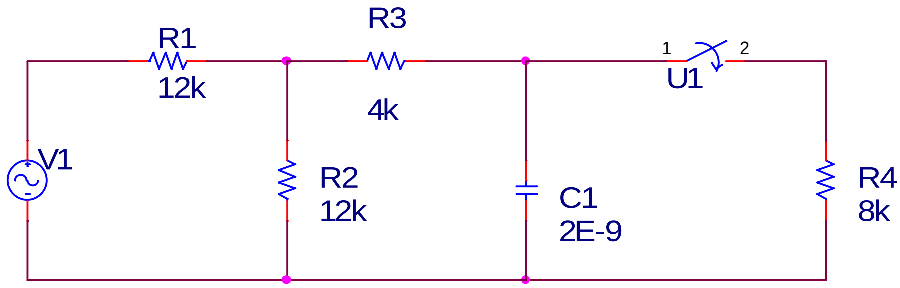
$$A_1 + 10 = 0$$

$$A_1 = -10$$



$t := 0, 0.001..0.2$

3) Circuit Analysis and Thevenin/Norton Circuits



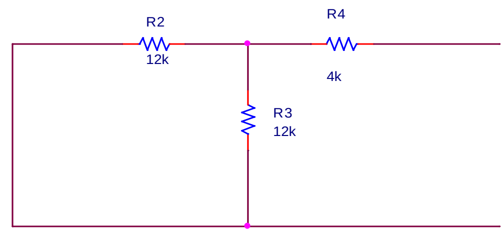
In the above circuit,
1.The voltage source turns on at t=0 with a voltage of 20V
2. Switch U1 closes at 15E-6s
3. The voltage source turns off at t = 25E-6

a. Determine the voltage across C1 as a function of time for t>0.

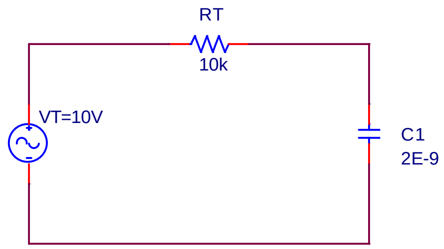
For 0<t<15E-6

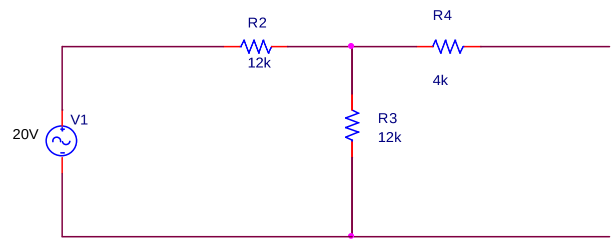
Find the thevenin equivalent then solve the simple RC circuit with a step function source

$$R_{Thevenin} = \frac{12k \cdot 12k}{24k} + 4k = 10k$$



$$V_{Thevenin} = 20 \cdot \frac{12k}{24k} = 10V$$





The simple RC circuit with a step function source has a solution of the form

$$V_{C1} = A_1 \cdot e^{\frac{-t}{2 \cdot 10^{-5}}} + A_2$$

$$\tau := 10 \cdot 10^{3} \cdot 2 \cdot 10^{-9}$$

$$\tau = 2 \times 10^{-5}$$

Since the source is a step function, the DC steady state as t goes to ∞ is Vthevenin, giving A2

(note: what happens to the capacitor at DC steady state? open circuit)

$$A_2 = 10V$$

The intial conditions are zero giving

$$v_c\left(0^{+}\right) = 0 = A_1 + A_2$$

$$A_1 = -10V$$

$$V_{C1}(t) = -10\exp\left(\frac{-t}{2 \times 10^{-5}}\right) + 10 \quad V$$

For 15E-6s<t<25E-6

Switch closes and now a 8k resistor is added

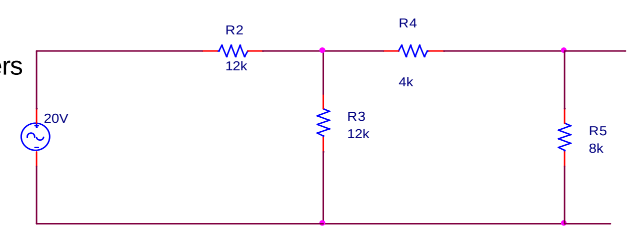
$$R_{Thevenin} = \frac{10k \cdot 8k}{18k} = 4.4k$$

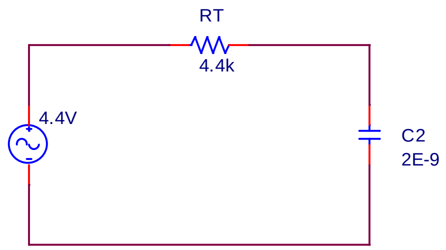
two voltage dividers

$$20 \cdot \frac{6}{18} = 6.667$$

$$6.6 \cdot \frac{8}{12} = 4.4$$

$$V_{Thevenin} = 4.4V$$





$$\tau_1 := 4.4\cdot 10^{3}\cdot 2\cdot 10^{-9}$$

$$\tau_1 = 8.8\times 10^{-6}$$

$$V_{c2}(t) = A_3\cdot e^{\frac{-\left(t-15\cdot 10^{-6}\right)}{8.88\cdot 10^{-6}}} + A_4$$

Since the source is a step function source, the DC steady state as t approaches ∞ is VThevenin giving

$$A_4 = 4.4v$$

Enforcing continuity conditions, Vc1(15E-6-)=Vc2(15E6+)=5.28

$$-10\cdot e^{\frac{-15\cdot 10^{-6}}{2\cdot 10^{-5}}} + 10 = 5.276$$

$$5.28 = A_3\cdot e^{0} + A_4$$

$$5.28 = A_3 + 4.4$$

$$V_{c2}(t) = 0.84e^{-\left(\frac{t-15\cdot 10^{-6}}{8.88\cdot 10^{-6}}\right)} + 4.44$$

For t>25E-6 switch is sitll down so Rthev is the same

$$R_{Thevenin} = 4.4k$$

$$V_{Thevenin} = 0$$

the source is off

$$\tau_1 := 4.4\cdot 10^{3}\cdot 2\cdot 10^{-9}$$

$$\tau_1 = 8.8\times 10^{-6}$$

$$V_{c3}(t) = A_5\cdot e^{\frac{-\left(t-25\cdot 10^{-6}\right)}{8.88\cdot 10^{-6}}} + A_6$$

$$A_6 = \text{Vthevenin} = 0$$

Enforcing the continity conditions $V.c2\left(25\cdot 10^{-6-}\right) = V.C3\left(25\cdot 10^{-6+}\right)$

$$0.84e^{-\left(\frac{25\cdot 10^{-6}-15\cdot 10^{-6}}{8.88\cdot 10^{-6}}\right)} + 4.44 = 4.712$$

$$V_{c3} = 4.71 \cdot e^{\frac{-\left(t - 25 \cdot 10^{-6}\right)}{8.88 \cdot 10^{-6}}}$$

$$V_C(t) = \begin{cases} -10\exp\left(\dfrac{-t}{2E-5}\right)+10 & 0 < t < 15E-6 \\ 5.28\exp\left(\dfrac{-\left(t-15\times 10^{-6}\right)}{8.888E-6}\right) & 15E-6 < t < 25E-7 \\ 4.71\exp\left(\dfrac{-\left(t-25\times 10^{-6}\right)}{8.888E-6}\right) & 25E-7 < t \end{cases}$$