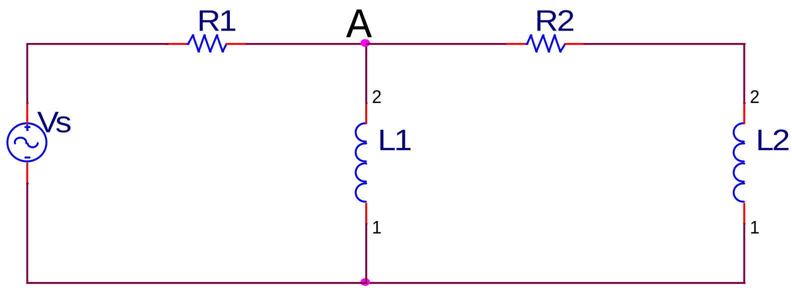
1)



a). In the above circuit, find a differential equation for the current through L2, IL2(t). The source is an arbitrary source. (Hint: Consider applying KCL at A and determine an expression for VA in terms of IL2.)

KCL at A:
$$\frac{V_A - V_s}{R_1} + I_{L1} + I_{L2} = 0$$

Using the differential relationship between current and voltage in a capacitor

$$I_{L1} = \frac{1}{L1} \cdot \int V_{L1}\, dt$$

Recognizing that VL1=VA

$$I_{L1} = \frac{1}{L1} \cdot \int V_A\, dt$$

Substituting and rearranging

$$\frac{V_A}{R_1} + \frac{1}{L1} \cdot \int V_A\, dt + I_{L2} = \frac{V_s}{R_1}$$

Recognizing that VA = VR2+VL2. Applying ohm's law and the differential relationship between current and the voltage in an inductor.

$$V_A = I_{L2} \cdot R_2 + L_2 \cdot \frac{dI_{L2}}{dt}$$

substituting

$$\frac{1}{R_1}\cdot\left(I_{L2}\cdot R_2 + L_2\cdot\frac{dI_{L2}}{dt}\right) + \frac{1}{L1}\cdot\int\left(I_{L2}\cdot R_2 + L_2\cdot\frac{dI_{L2}}{dt}\right)dt + I_{L2} = \frac{V_s}{R_1}$$

Differentiating to get rid of the integral term

$$\frac{1}{R_1}\cdot\left(R_2\cdot\frac{dI_{L2}}{dt} + L_2\cdot\frac{d^2I_{L2}}{dt^2}\right) + \frac{1}{L1}\cdot\left(I_{L2}\cdot R_2 + L_2\cdot\frac{dI_{L2}}{dt}\right) + \frac{dI_{L2}}{dt} = \frac{1}{R_1}\cdot\frac{dV_s}{dt}$$

Rearranging

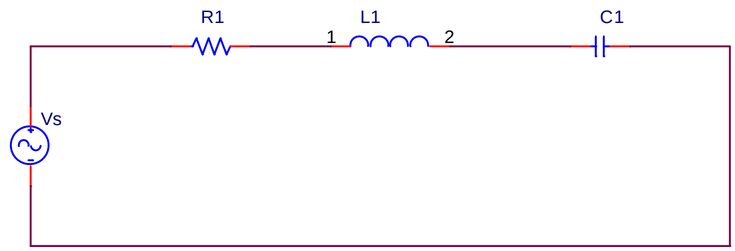
$$\frac{L_2}{R_1}\cdot\frac{d^2I_{L2}}{dt^2} + \left(\frac{R_2}{R_1} + \frac{L_2}{L_1} + 1\right)\frac{dI_{L2}}{dt} + \frac{R_1\cdot R_2}{L_1\cdot L_2}\cdot I_{L2} = \frac{1}{R_1}\frac{dV_s}{dt}$$

For the differential equation, determine symbolic expressions for the attenuation constant, α, and the resonant frequency, ω0, in terms of R1, R2, L1 and L2.

$$2\alpha = \frac{R_2}{L_2} + \frac{R_1}{L1} + \frac{R_1}{L_2} \qquad \text{so} \qquad \alpha = \frac{1}{2}\cdot\left(\frac{R_2}{L_2} + \frac{R_1}{L1} + \frac{R_1}{L_2}\right)$$

$$\omega_0^{\ 2} = \frac{R_1\cdot R_2}{L_1\cdot L_2} \qquad \text{so} \qquad \omega_0 = \sqrt{\frac{R_1\cdot R_2}{L_1\cdot L_2}}$$

2) RLC series circuits



In the above circuit, Vs is a step function source and that turns on at t = 0. Determine the form of the solution for the following conditions (you do not need to solve for the coefficients A1, A2, and A3). Indicate the damping for each case and include calculations of the attenuation constant and resonant frequency in your solution.

a). $R_1 := 10\Omega$ $\quad L_1 := 1\cdot 10^{-2}\mathrm{H}$ $\quad C_1 := 1\cdot 10^{-6}\mathrm{F}$

$$\alpha := \frac{R_1}{2\cdot L_1}$$

$$\alpha = 500\frac{1}{s}$$ don't use units

$$\omega_0 := \sqrt{\frac{1}{L_1\cdot C_1}}$$

$$\omega_0 = 1\times 10^4\frac{1}{s}$$

$\alpha < \omega_0$ therefor the circuit is (strongly) underdamped and the solution takes the form

$$\beta := \sqrt{\omega_0^{\,2} - \alpha^2} = 9.987\times 10^3\frac{1}{s}$$

$$f(t) = e^{-500t}\cdot\left(A_1\cdot\cos(9987t) + A_2\cdot\sin(9987t)\right) + A_3$$

A3 is because Vs is a step function!

b). $R_{1b} := 200\Omega \qquad L_{1b} := 1\cdot 10^{-2}\text{H} \qquad C_{1b} := 1\cdot 10^{-6}\text{F}$

$$\alpha_b := \frac{R_{1b}}{2L_{1b}}$$

$$\alpha_b = 1\times 10^4 \frac{1}{s}$$

$$\omega_{0b} := \sqrt{\frac{1}{L_{1b}\cdot C_{1b}}}$$

$$\omega_{0b} = 1\times 10^4 \frac{1}{s}$$

$\alpha = \omega_0$ therefore the circuit is critically damped

$$f(t) = A_1\cdot \exp\left(-10^4 t\right) + A_1\cdot t\cdot \exp\left(-10^4 t\right) + A_3$$

c) $R_{1c} := 1000\Omega \qquad L_{1c} := 1\cdot 10^{-2}\text{H} \qquad C_{1c} := 1\cdot 10^{-6}\text{F}$

$$\alpha_c := \frac{R_{1c}}{2L_{1c}} = 5\times 10^4 \frac{1}{s}$$

$$\omega_{0c} := \sqrt{\frac{1}{L_{1c}\cdot C_{1c}}}$$

$$\omega_{0c} = 1\times 10^4 \frac{1}{s}$$

$\alpha > \omega_0$ therefore the circuit is overdamped

$$s_1 := -\alpha_c + \sqrt{\alpha_c^{\,2} - \omega_{0c}^{\,2}} = -1.01 \times 10^3 \frac{1}{s}$$

$$s_2 := -\alpha_c - \sqrt{\alpha_c^{\,2} - \omega_{0c}^{\,2}} = -9.899 \times 10^4 \frac{1}{s}$$

$$f(t) = A_1 \cdot \exp\left(-9.899 \times 10^4 t\right) + A_2 \cdot \exp\left(-1.01 \times 10^3 t\right) + A_3$$