Questions

What is partial fraction expansion?
What is the cover-up rule?
What is the impedance of a capacitor, including initial conditions?
What is the impedance of an inductor, including initial conditions?
How do we analyze voltage dividers in the s-domain?
How do we analyze current dividers in the s-domain?
How do we implement source transformations in the s-domain?
How do we apply s-domain analysis to circuits?

1) $$F(s) = \frac{s+1}{(s+4)\cdot(s+3)\cdot(s+2)}$$

a. Find the poles and zeros
b. Draw the pole zero diagram

Zeros -1

Poles $-2, -3, -4$

$$\frac{s+1}{(s+4)\cdot(s+3)\cdot(s+2)} = \frac{A_1}{s+4} + \frac{A_2}{s+3} + \frac{A_3}{s+2}$$

Find A1 using the cover up rule

$$\frac{s+1}{(s+3)\cdot(s+2)} = A_1 \quad \text{at } s = -4 \qquad A_1 = \frac{-4+1}{(-4+3)\cdot(-4+2)} = -1.5$$

Find A2 using the cover up rule

$$\frac{s+1}{(s+4)\cdot(s+2)} = A_2 \quad \text{at } s = -3 \qquad A_2 = \frac{-3+1}{(-3+4)\cdot(-3+2)} = 2$$

Find A3 using the cover up rule

$$\frac{s+1}{(s+4)(s+3)} = A_3 \quad \text{at } s=-2 \qquad A_3 = \frac{-2+1}{(-2+4)(2+3)} = -0.5$$

$$f(t) = (-1.5\cdot\exp(-4t) + 2\cdot\exp(-3t) - 0.5\exp(-2t))\cdot u(t)$$

Pole zero diagram

2) Transfer functions
Find the poles and zeros for the following functions
Apply partial fraction expansion to the following functions to find f(t)

a. $$F(s) = \frac{2 \cdot s}{s^2 + 8s + 25}$$

See course notes

b. $$F(s) = \frac{2s}{(s+3)^2}$$

$$p_1, p_2 = -3$$

To find $L^{-1}(F(s))$

Expand $$F(s) = \frac{A_1}{s+3} + \frac{A_2}{(s+3)^2}$$

$f(t) = A_1 \cdot e^{-3t} + A_2 \cdot t \cdot e^{-3t}$ $\quad t > \blacksquare = 0$

Need only to find A1 and A2

To find A2 use the cover up rule

$$A_2 = \left[(s+3)^2 \cdot F(s)\right] \cdot \left|\blacksquare\right._{s=-3} \qquad A_2 = 2 \cdot s \cdot \left|\blacksquare\right._{s=-3} \qquad A_2 = -6$$

Cannot use cover up rule for A1

Use F(0) with A2 subsituted in

$$F(s) = \frac{A_1}{s+3} + \frac{A_2}{(s+3)^2} \qquad F(s) = \frac{2s}{(s+3)^2}$$

$$F(0) = \frac{2\cdot 0}{(0+3)^2} = 0 = \frac{A_1}{0+3} + \frac{-6}{(0+3)^2} \qquad \frac{A_1}{3} + \frac{-2}{3} = 0 \qquad A_1 = 2$$

$$f(t) = 2\cdot 3^{-3t} - 6\cdot t\cdot e^{-3t}$$

can also do

$$F(1) \quad \frac{2}{16} = \frac{A_1}{4} + \frac{-6}{16} \qquad \frac{8}{16} = \frac{A_1}{4} \qquad A_1 = 2$$

c. $$F(s) = \frac{4s^2 + 12s + 8}{(s+8)(s+4)^2(s+1)}$$

$$= \frac{4\cdot(s+2)\cdot(s+1)}{(s+8)\cdot(s+4)^2\cdot(s+1)} = 4\cdot\frac{(s+2)}{(s+8)\cdot(s+4)^2}$$

Zeros $\quad -2$

Poles $\quad -8, -4\text{double}$

$$F(s) = \frac{A_1}{s+8} + \frac{A_2}{s+4} + \frac{A_3}{(s+4)^2}$$

$$f(t) = A_1\cdot e^{-8t} + A_2\cdot e^{-4t} + A_3\cdot t\cdot e^{-4t}$$

Find A1 using cover up rule

$$4\cdot\frac{(s+2)\cdot(s+8)}{(s+8)\cdot(s+4)^2} = \frac{4\cdot(s+2)}{(s+4)^2}$$ at s=-8

$$A_{1c} := \frac{4\cdot(-8+2)}{(-8+4)^2}$$

$$A_{1c} = -1.5$$

Find A3 using the coverup rule for s+4 at s=-4

$$4\cdot\frac{(s+2)(s+4)^2}{(s+8)\cdot(s+4)^2} = \frac{4\cdot(s+2)}{(s+8)} \qquad \text{at s= -4}$$

$$A_{3c} := \frac{4\cdot(-4+2)}{(-4+8)}$$

$$A_{3c} = -2$$

Find A2 using F(0) or F(1)

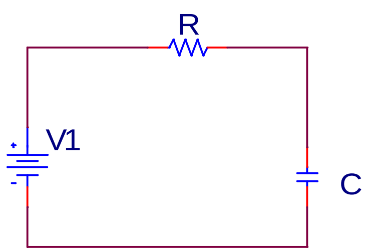
$$4\cdot\frac{(s+2)}{(s+8)\cdot(s+4)^2} = \frac{-1.5}{s+8} + \frac{A_2}{s+4} + \frac{-2}{(s+4)^2} \qquad F(0)$$

$$0.063 = \frac{-1.5}{8} + \frac{A_2}{4} + \frac{-2}{16}$$

$$A_2 = 1.5$$

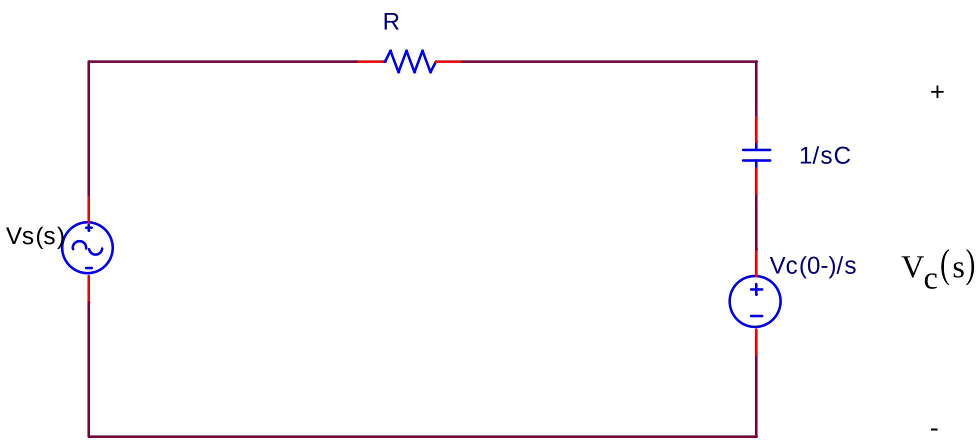
$$f(t) = -1.5\cdot e^{-8t} + 1.5\cdot e^{-4t} - 2\cdot t\cdot e^{-4t}$$

S-domain analysis



a. Draw the s-domain equivalent circuit. V1 is an arbitrary source.
b. Symbolically, determine the transfer function for the voltage across the capacitor.
c. If the initial conditions are zero and V1 is a step function 5u(t), R = 1kΩ and C = 2E-6F, find the voltage across the capacitor.
d. If the source voltage is 10V for t < 0 and 5V for t >0, R = 1kΩ and C = 2E-6F, find the voltage across the capacitor.

a



Voltage is across both components!

$$V_C(s) = V_{Zc} + V_c(0^-)$$

What is VZc? How do you find it? Voltage divider but account for the other source

b
$$V_C(s) = \frac{\frac{1}{RC}}{s+\frac{1}{RC}}\cdot\left(V_s(s) - \frac{V_c(0^-)}{s}\right) + \frac{V_c(0^-)}{s}$$

First order circuits in the s-domain for capacitors

c. If the intial conditions are zero and V1 is a step function

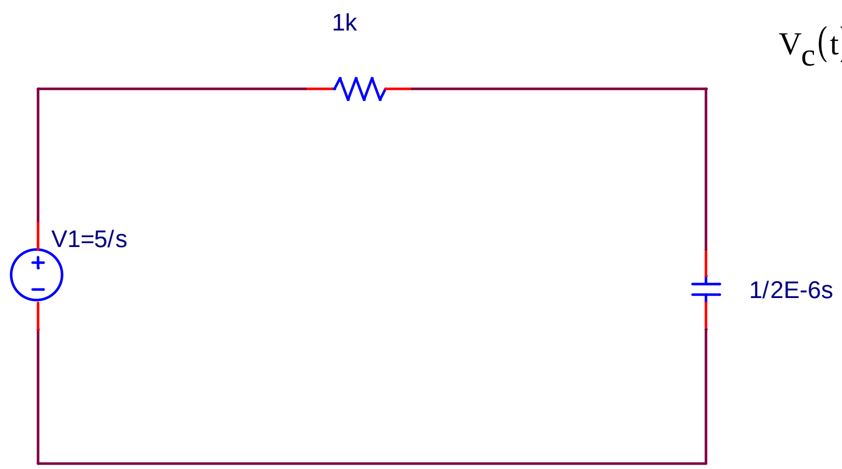
$$V_1 = 5\,u(t) \qquad V_1(s) = \frac{5}{s} \qquad V_c(0^+) = V_c(0^-) = 0$$

$$R_1 := 1k\Omega \qquad R_1(s) = 1k\Omega$$

$$C_1 := 2\cdot 10^{-6}F \qquad C_1(s) = \frac{1}{2\cdot 10^{-6}\cdot s}$$

Just by inspection

$$V_c(t) = -5\cdot e^{\frac{-t}{2\cdot 10^{-3}}} + 5$$



$$V_c(s) = \frac{500}{s+500}\cdot\left(\frac{5}{s} - 0\right)$$

$$= \frac{2500}{s\cdot(s+500)}$$

$$V_C(s) = \frac{\frac{1}{RC}}{s+\frac{1}{RC}}\cdot\left(V_s(s) - \frac{V_c(0^-)}{s}\right) + \frac{V_c(0^-)}{s}$$

$$\frac{1}{R_1\cdot C_1} = 500\,\frac{1}{s}$$

Find poles and zeros

poles 0, -500

zeros 0

$$F(s) = \frac{2500}{s\cdot(s+500)} = \frac{A_1}{s} + \frac{A_2}{s+500}$$

Coverup rule for A1

$$\frac{2500\cdot s}{s\cdot(s+500)} = \frac{2500}{(s+500)}$$ at s=0

$$A_1 := \frac{2500}{500}$$

$$A_1 = 5$$

Coverup rule for A2

$$\frac{2500\cdot(s+500)}{s\cdot(s+500)} = \frac{2500}{s}$$ at s=-500

$$A_2 := \frac{2500}{-500}$$

$$A_2 = -5$$

$$V_c(s) = \frac{5}{s} + \frac{-5}{s+500}$$

$$V_c(t) = 5 - 5e^{-500t}$$

d.

because at t<0 10V

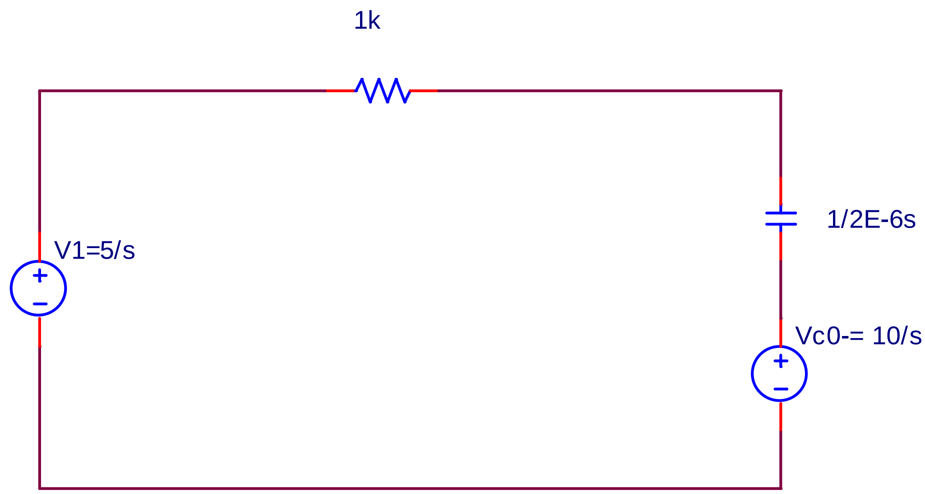
Set up part d only!

$$V_c\left(0^+\right) = V_c\left(0^-\right) = 10V$$

$$V_s(s) = \frac{5}{s}$$

but because Vs goes to 5V

$$V_C(s) = \frac{\frac{1}{RC}}{s + \frac{1}{RC}} \cdot \left(V_s(s) - \frac{V_c\left(0^-\right)}{s}\right) + \frac{V_c\left(0^-\right)}{s}$$



$$V_C(s) = \frac{500}{s+500} \cdot \left(\frac{5}{s} - \frac{10}{s}\right) + \frac{10}{s}$$

$$\left(\frac{500}{s+500} \cdot \frac{-5}{s}\right) + \frac{10}{s}$$

10/s is easy so do it separately

$$F(s) = \frac{-2500}{s \cdot (s+500)} = \frac{A_1}{s} + \frac{A_2}{s+500}$$

Coverup rule for A1

$$\frac{-2500 \cdot s}{s \cdot (s + 500)} = \frac{-2500}{(s + 500)} \quad \text{at s=0}$$

$$A_1 := \frac{-2500}{500}$$

$$\boxed{A_1 = -5}$$

Coverup rule for A2

$$\frac{-2500 \cdot (s + 500)}{s \cdot (s + 500)} = \frac{-2500}{s} \quad \text{at s=-500}$$

$$A_2 := \frac{-2500}{-500}$$

$$\boxed{A_2 = 5}$$

$$V_C(s) = \frac{-5}{s} + \frac{5}{s + 500} + \frac{10}{s}$$

$$V_C(t) = -5 + 10 + 5 \cdot e^{-500t}$$

$$\boxed{V_C(t) = 5 + 5 \cdot e^{-500t}}$$