

How do we analyze voltage dividers in the s-domain?

How do we analyze current dividers in the s-domain?

How do we implement source transformations in the s-domain?

How do we apply s-domain analysis to circuits?

Review: Impedances with initial condition terms, Circuit transformation to the s-domain.

How do we use the transfer function to find the initial conditions for a circuit?

How do we use the transfer function to find the DC steady state as  $t \rightarrow \infty$ ?

### Review

$$c. \quad F(s) = \frac{4s^2 + 12s + 8}{(s+8)(s+4)^2(s+1)}$$

$$\blacksquare = \frac{4 \cdot (s+2) \cdot (s+1)}{(s+8) \cdot (s+4)^2 \cdot (s+1)} = 4 \cdot \frac{(s+2)}{(s+8) \cdot (s+4)^2}$$

Zeros  $-2$

Poles  $-8, -4$  double

$$F(s) = \frac{A_1}{s+8} + \frac{A_2}{s+4} + \frac{A_3}{(s+4)^2}$$

$$f(t) = A_1 \cdot e^{-8t} + A_2 \cdot e^{-4t} + A_3 \cdot t \cdot e^{-4t}$$

Find  $A_1$  using cover up rule

$$4 \cdot \frac{(s+2) \cdot (s+8)}{(s+8) \cdot (s+4)^2} = \frac{4 \cdot (s+2)}{(s+4)^2} \quad \text{at } s=-8$$

$$A_{1c} := \frac{4 \cdot (-8+2)}{(-8+4)^2}$$

$$A_{1c} = -1.5$$

Find A3 using the coverup rule for s+4 at s=-4

$$4 \cdot \frac{(s+2)(s+4)^2}{(s+8) \cdot (s+4)^2} = \frac{4 \cdot (s+2)}{(s+8)} \quad \text{at } s = -4$$

$$A_{3c} := \frac{4 \cdot (-4+2)}{(-4+8)}$$

$$A_{3c} = -2$$

Find A2 using F(0) or F(1)

$$4 \cdot \frac{(s+2)}{(s+8) \cdot (s+4)^2} = \frac{-1.5}{s+8} + \frac{A_2}{s+4} + \frac{-2}{(s+4)^2} \quad F(0)$$

$$0.063 = \frac{-1.5}{8} + \frac{A_2}{4} + \frac{-2}{16}$$

$$\frac{-1.5}{8} = -0.188$$

$$\frac{-2}{16} = -0.125$$

$$-0.188 + -0.125 = -0.313$$

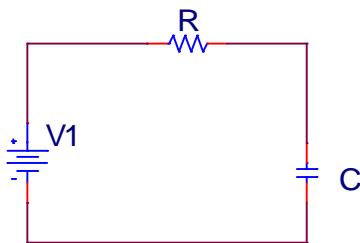
$$0.063 + 0.313 = 0.376$$

$$0.376 \cdot 4 = 1.504$$

$$A_2 = 1.5$$

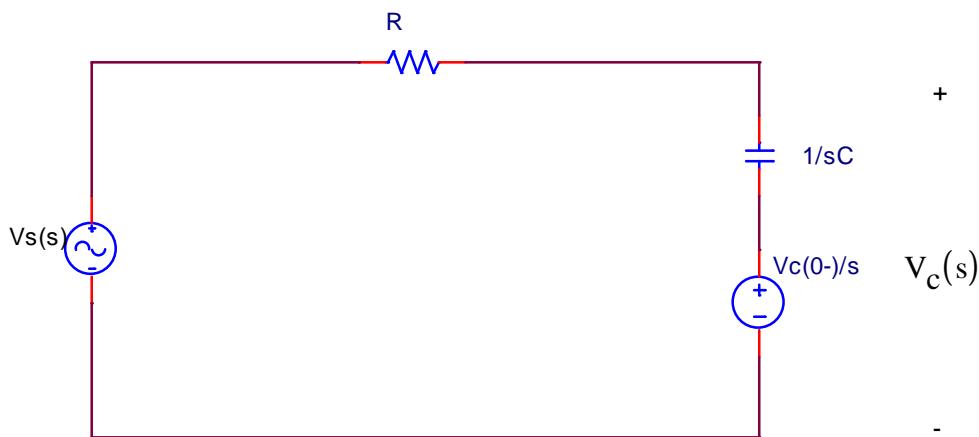
$$f(t) = -1.5 \cdot e^{-8t} + 1.5 \cdot e^{-4t} - 2 \cdot t \cdot e^{-4t}$$

S-domain analysis



- Draw the s-domain equivalent circuit.  $V_1$  is an arbitrary source.
- Symbolically, determine the transfer function for the voltage across the capacitor.
- If the initial conditions are zero and  $V_1$  is a step function  $5u(t)$ ,  $R = 1k\Omega$  and  $C = 2E-6F$ , find the voltage across the capacitor.
- If the source voltage is 10V for  $t < 0$  and 5V for  $t > 0$ ,  $R = 1k\Omega$  and  $C = 2E-6F$ , find the voltage across the capacitor.

a



Voltage is across both components!

$$V_C(s) = V_{Zc} + V_c(0^-)$$

What is  $V_{Zc}$ ? How do you find it? Voltage divider but account for the other source

b

$$V_C(s) = \frac{\frac{1}{RC}}{s + \frac{1}{RC}} \cdot \left( V_s(s) - \frac{V_c(0^-)}{s} \right) + \frac{V_c(0^-)}{s}$$

First order circuits in the s-domain  
for capacitors

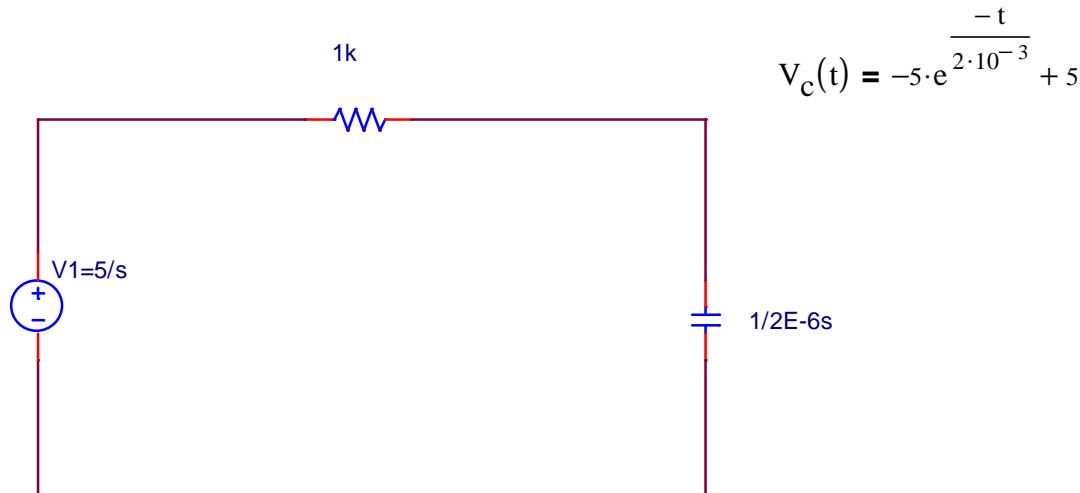
c. If the initial conditions are zero and  $V_1$  is a step function

$$V_1 = 5u(t) \quad V_1(s) = \frac{5}{s} \quad V_c(0^+) = V_c(0^-) = 0$$

$$R_1 := 1k\Omega \quad R_1(s) = 1k\Omega$$

$$C_1 := 2 \cdot 10^{-6} F \quad C_1(s) = \frac{1}{2 \cdot 10^{-6} \cdot s}$$

Just by inspection



$$V_c(s) = \frac{500}{s+500} \cdot \left( \frac{5}{s} - 0 \right)$$

$$\bullet = \frac{2500}{s \cdot (s+500)}$$

$$V_C(s) = \frac{1}{RC} \cdot \left( V_s(s) - \frac{V_c(0^-)}{s} \right) + \frac{V_c(0^-)}{s}$$

$$\frac{1}{R_1 \cdot C_1} = 500 \frac{1}{s}$$

Find poles and zeros

$$\text{poles} \quad 0, -500$$

$$\text{zeros} \quad 0$$

$$F(s) = \frac{2500}{s \cdot (s+500)} = \frac{A_1}{s} + \frac{A_2}{s+500}$$

Coverup rule for A1

$$\frac{2500 \cdot s}{s \cdot (s + 500)} = \frac{2500}{(s + 500)} \quad \text{at } s=0 \quad A_1 := \frac{2500}{500}$$

A<sub>1</sub> = 5

Coverup rule for A2

$$\frac{2500 \cdot (s + 500)}{s \cdot (s + 500)} = \frac{2500}{s} \quad \text{at } s=-500 \quad A_2 := \frac{2500}{-500}$$

A<sub>2</sub> = -5

$$V_C(s) = \frac{5}{s} + \frac{-5}{s + 500}$$

$$V_C(t) = 5 - 5e^{-500t}$$

If the source voltage is 10V for  $t < 0$  and 5V for  $t > 0$ ,  $R = 1k\Omega$  and  $C = 2E-6F$ , find the voltage across the capacitor.

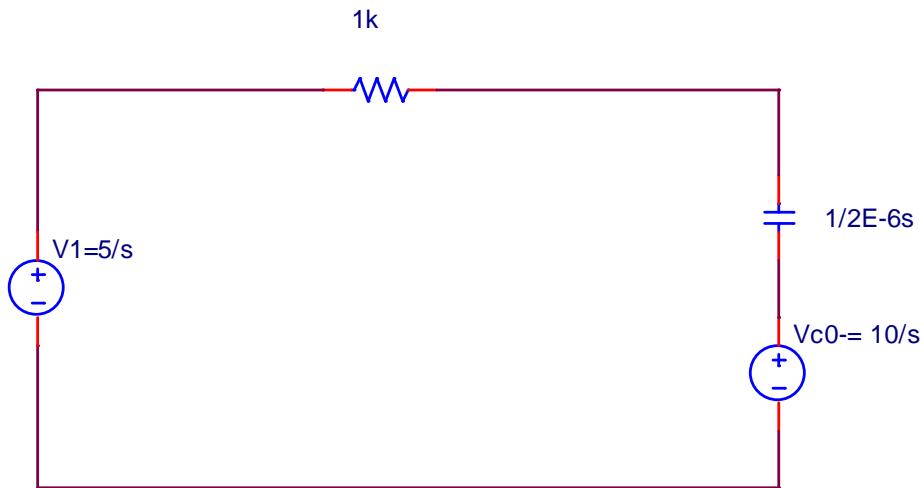
d. because at  $t < 0$  10V      Set up part d only!

$$V_C(0^+) = V_C(0^-) = 10V$$

$$V_s(s) = \frac{5}{s}$$

but because  $V_s$  goes to 5V

$$V_C(s) = \frac{\frac{1}{RC}}{s + \frac{1}{RC}} \cdot \left( V_s(s) - \frac{V_c(0^-)}{s} \right) + \frac{V_c(0^-)}{s}$$



$$V_C(s) = \frac{500}{s + 500} \cdot \left( \frac{5}{s} - \frac{10}{s} \right) + \frac{10}{s}$$

$$\left( \frac{500}{s + 500} \cdot \frac{-5}{s} \right) + \frac{10}{s}$$

$10/s$  is easy so do it separately

$$F(s) = \frac{-2500}{s \cdot (s + 500)} = \frac{A_1}{s} + \frac{A_2}{s + 500}$$

Coverup rule for  $A_1$

$$\frac{-2500 \cdot s}{s \cdot (s + 500)} = \frac{-2500}{(s + 500)} \quad \text{at } s=0$$

$$A_1 := \frac{-2500}{500}$$

$$A_1 = -5$$

Coverup rule for  $A_2$

$$\frac{-2500 \cdot (s + 500)}{s \cdot (s + 500)} = \frac{-2500}{s} \quad \text{at } s=-500$$

$$A_2 := \frac{-2500}{-500}$$

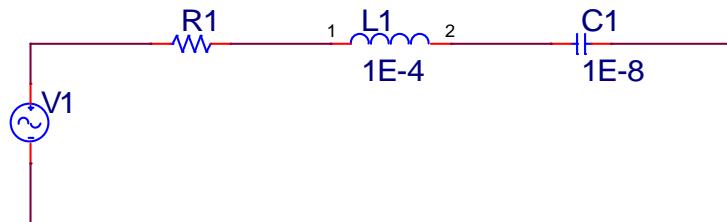
$$A_2 = 5$$

$$V_C(s) = \frac{-5}{s} + \frac{5}{s + 500} + \frac{10}{s}$$

$$V_C(t) = -5 + 10 + 5 \cdot e^{-500t}$$

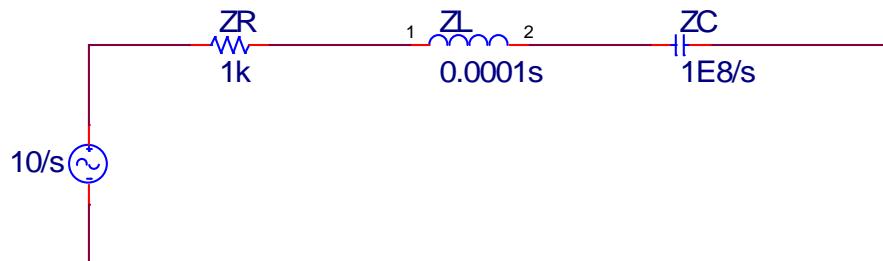
$$V_C(t) = 5 + 5 \cdot e^{-500t}$$

RLC series circuits



In the above circuit, the voltage source turns on at  $t = 0$ , with a voltage of 10V,  $V_1 = 10u(t)$  and the initial conditions are zero.

- a. Draw the s-domain equivalent circuit. Include your Laplace transform of the source term. Label your component values using symbolic notation (eg.  $sL$ ).



- b. Using impedances, determine the transfer function for the voltage across C. Use symbolic terms in your expression ( $R$ ,  $L$ ,  $C$ ).

$$V_C(s) = \frac{Z_C}{Z_R + Z_L + Z_C} \cdot V_s(s)$$

$$V_C(s) = \frac{\frac{1}{sC}}{R + sL + \frac{1}{sC}} \cdot V_s(s)$$

$$V_C(s) = \frac{\frac{1}{sC}}{\left( \frac{sCR + s^2 \cdot L \cdot C + 1}{sC} \right)}$$

$$V_C(s) = \frac{1}{s^2 \cdot L \cdot C + s \cdot R \cdot C + 1} \quad \text{divide by } LC$$

$$V_C(s) = \frac{\frac{1}{LC}}{s^2 + s \frac{R}{L} + \frac{1}{LC}} \cdot V_s(s)$$

c. Using your result from part b., determine the transfer function for the current through the capacitor

$$I_C(s) = \frac{V_C(s)}{Z_C} \quad \begin{aligned} &\text{How do you get the current through an element if the element is impedance} \\ &\text{Ohms law} \end{aligned}$$

$$I_C(s) = \frac{\frac{1}{LC} \cdot \frac{sC}{1}}{s^2 + s \frac{R}{L} + \frac{1}{LC}}$$

$$I_C(s) = \frac{\frac{s}{L}}{\left( s^2 + s \frac{R}{L} + \frac{1}{LC} \right)} \cdot V_s(s)$$

d. For  $R=1000\Omega$ , determine the voltage across the capacitor as a function of time for  $t > 0$ .

$$V_C(s) = \frac{\frac{1}{LC}}{s^2 + s \frac{R}{L} + \frac{1}{LC}} \cdot V_s(s) \quad \begin{aligned} L_d &:= 1 \cdot 10^{-4} \text{H} \\ C_d &:= 1 \cdot 10^{-8} \text{F} \\ R_d &:= 1 \text{k}\Omega \end{aligned}$$

$$\frac{1}{L_d \cdot C_d} = 1 \times 10^{12} \frac{1}{s^2} \quad \frac{R_d}{L_d} = 1 \times 10^7 \frac{1}{s}$$

$$V_C(s) = \frac{1 \times 10^{12}}{(s^2 + s \cdot 1 \cdot 10^7 + 1 \cdot 10^{12})} \cdot \frac{10}{s} \quad \frac{10}{s} \quad \text{comes from the fact that it is a step function input}$$

$$s^2 + s \cdot 1 \cdot 10^7 + 1 \cdot 10^{12} = 0$$

$$\begin{pmatrix} 2000000\cdot\sqrt{6} - 5000000 \\ -2000000\cdot\sqrt{6} - 5000000 \end{pmatrix}$$

$$2000000\cdot\sqrt{6} - 5000000 = -1.01 \times 10^5$$

$$-2000000\cdot\sqrt{6} - 5000000 = -9.899 \times 10^6$$

$$V_c(s) = \frac{1 \cdot 10^{12}}{(s + 1.01 \cdot 10^5) \cdot (s + 9.899 \cdot 10^6)} \cdot \frac{10}{s}$$

Zeros: none

Poles:  $0, -1.01 \cdot 10^5, -9.899 \cdot 10^6$

$$V_c(s) = \frac{1 \cdot 10^{13}}{s \cdot (s + 1.01 \cdot 10^5) \cdot (s + 9.899 \cdot 10^6)} = \frac{A_1}{s} + \frac{A_2}{s + 1.01 \cdot 10^5} + \frac{A_3}{s + 9.899 \cdot 10^6}$$

Coverup rule For A1

$$\frac{1 \cdot 10^{13} \cdot s}{s \cdot (s + 1.01 \cdot 10^5) \cdot (s + 9.899 \cdot 10^6)} \quad \text{at } s = 0$$

$$\frac{1 \cdot 10^{13}}{(0 + 1.01 \cdot 10^5) \cdot (0 + 9.899 \cdot 10^6)} = 10.002$$

Cover up rule for A2

$$\frac{1 \cdot 10^{13} \cdot (s + 1.01 \cdot 10^5)}{s \cdot (s + 1.01 \cdot 10^5) \cdot (s + 9.899 \cdot 10^6)} \quad \text{at } s = -1.01 \cdot 10^5$$

$$\frac{1 \cdot 10^{13}}{(-1.01 \cdot 10^5 + 9.899 \cdot 10^6) \cdot (-1.01 \cdot 10^5)} = -10.105$$

Cover up rule for A3

$$\frac{1 \cdot 10^{13} \cdot (s + 9.899 \cdot 10^6)}{s \cdot (s + 1.01 \cdot 10^5) \cdot (s + 9.899 \cdot 10^6)} \quad \text{at} \quad s = -9.899 \cdot 10^6$$

$$\frac{1 \cdot 10^{13}}{-9.899 \cdot 10^6 \cdot (-9.899 \cdot 10^6 + 1.01 \cdot 10^5)} = 0.103$$

$$V_C(s) = \frac{1 \cdot 10^{13}}{s \cdot (s + 1.01 \cdot 10^5) \cdot (s + 9.899 \cdot 10^6)} = \frac{10}{s} + \frac{-10.1}{s + 1.01 \cdot 10^5} + \frac{0.103}{s + 9.899 \cdot 10^6}$$

$$V_C(t) = -10.1 \cdot e^{-1.01 \cdot 10^5 t} + 0.103 \cdot e^{-9.899 \cdot 10^6 t} + 10$$

e. For  $R_1 = 200\Omega$ , determine the voltage across the capacitor as a function of time for  $t > 0$ .

Done in team assignment 07

$$V_C(t) = 10 \cdot e^{-1 \cdot 10^6 t} - 1 \cdot 10^7 \cdot t \cdot e^{-1 \cdot 10^6 t} + 10$$

f. For  $R_1 = 25\Omega$ , determine the voltage across the capacitor as a function of time for  $t>0$

$$L_f := 1 \cdot 10^{-4} \text{H}$$

$$C_f := 1 \cdot 10^{-8} \text{F}$$

$$R_f := 25\Omega$$

$$\frac{1}{L_f \cdot C_f} = 1 \times 10^{12} \frac{1}{\text{s}^2} \quad \frac{R_f}{L_f} = 2.5 \times 10^5 \frac{1}{\text{s}}$$

$$V_c(s) = \frac{1 \cdot 10^{12}}{s^2 + s \cdot 2.5 \cdot 10^5 + 1 \cdot 10^{12}} \cdot \frac{10}{s}$$

$$s^2 + s \cdot 2.5 \cdot 10^5 + 1 \cdot 10^{12} = 0$$

$$\begin{pmatrix} -125000.0 - 992156.74164922147144i \\ -125000.0 + 992156.74164922147144i \end{pmatrix}$$

$$\text{pole } 0, -1.25 \cdot 10^5 + 9.9 \cdot 10^5 j, -1.25 \cdot 10^5 - 9.9 \cdot 10^5 j$$

$$V_c(s) = \frac{1 \cdot 10^{13}}{s \cdot [s - (-1.25 \cdot 10^5 - 9.9 \cdot 10^5 j)] \cdot [s - (-1.25 \cdot 10^5 + 9.9 \cdot 10^5 j)]} = *$$

$$V_c(s) = \frac{A_1}{s} + \frac{A_2}{s + 1.25 \cdot 10^5 + 9.9 \cdot 10^5 j} + \frac{A_2^*}{s + 1.25 \cdot 10^5 - 9.9 \cdot 10^5 j}$$

For A1 at s=0

$$\frac{1 \cdot 10^{13} \cdot s}{s \cdot (s^2 + s \cdot 2.5 \cdot 10^5 + 1 \cdot 10^{12})}$$

$$\frac{1 \cdot 10^{13}}{0^2 + 0 \cdot 2.5 \cdot 10^5 + 1 \cdot 10^{12}} = 10$$

For A2 at  $s = -1.25 \cdot 10^5 - 9.9 \cdot 10^5 j$

$$\frac{1 \cdot 10^{13} \cdot (s + 1.25 \cdot 10^5 + 9.9 \cdot 10^5 j)}{s \cdot (s + 1.25 \cdot 10^5 + 9.9 \cdot 10^5 j) \cdot (s + 1.25 \cdot 10^5 - 9.9 \cdot 10^5 j)}$$

$$\frac{1 \cdot 10^{13}}{(-1.25 \cdot 10^5 - 9.9 \cdot 10^5 j) \cdot (-1.25 \cdot 10^5 - 9.9 \cdot 10^5 j + 1.25 \cdot 10^5 - 9.9 \cdot 10^5 j)}$$

$$\frac{1 \cdot 10^{13}}{(-1.25 \cdot 10^5 - 9.9j \cdot 10^5) \cdot (-1.98j \cdot 10^6)} = -5.021 - 0.634i$$

For A2\* is its complex conjugate

$$-5.021 + 0.634j$$

$$V_c(s) = \frac{A_1}{s} + \frac{A_2}{s + 1.25 \cdot 10^5 + 9.9 \cdot 10^5 j} + \frac{A_2^*}{s + 1.25 \cdot 10^5 - 9.9 \cdot 10^5 j}$$

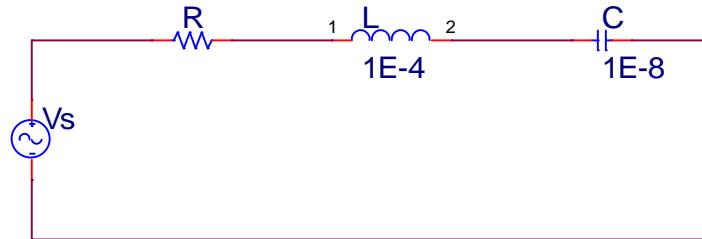
$$V_c(s) = \frac{10}{s} + \frac{-5.021 - 0.634j}{s + 1.25 \cdot 10^5 + 9.9 \cdot 10^5 j} + \frac{-5.021 + 0.634j}{s + 1.25 \cdot 10^5 - 9.9 \cdot 10^5 j}$$

Now take the inverse transform

remember  $\frac{1 \cdot 10^{13}}{s \cdot [s - (-1.25 \cdot 10^5 - 9.9 \cdot 10^5 j)] \cdot [s - (-1.25 \cdot 10^5 + 9.9 \cdot 10^5 j)]}$

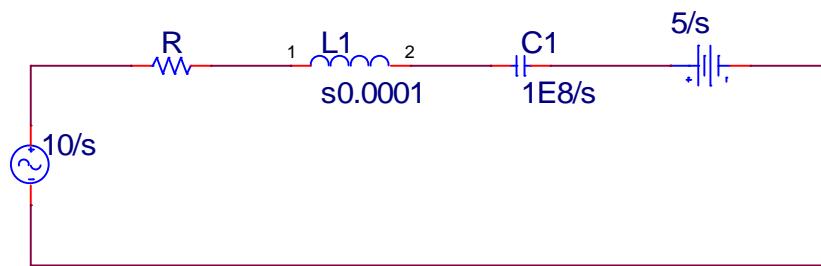
$$V_c(t) = (-5.021 - 0.634j) \cdot e^{(-1.25 \cdot 10^5 - 9.9 \cdot 10^5 j)t} + (-5.021 + 0.634j) \cdot e^{(-1.25 \cdot 10^5 + 9.9 \cdot 10^5 j)t} + 10$$

## RLC series circuits



In the above circuit, the voltage source is  $V_s = \begin{cases} 5V & t < 0 \\ 10V & 0 < t \end{cases}$

- a. Draw the s-domain equivalent circuit. Include your Laplace transform of the source term. Label your component values using symbolic notation. Remember to include any initial value source terms.



- b. Using impedances, determine the transfer function for the voltage across C. Use symbolic terms in your expression (RLC).

$$V_C(s) = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L} \cdot s + \frac{1}{LC}} \cdot \left( V_s(s) - \frac{V_c(0^-)}{s} \right) + \frac{V_c(0^-)}{s}$$

- b. Using your result from b., determine the transfer function for the current through the capacitor. Use symbolic terms in your expression (R, L, C).

$$I_C(s) = sC \cdot V_{Zc}(s) = \frac{\frac{s}{L}}{\left( s^2 + \frac{R}{L} \cdot s + \frac{1}{LC} \right)} \cdot \left( V_s(s) - \frac{V_c(0^-)}{s} \right)$$

d. Using partial fraction expansion and inverse Laplace transforms, for  $R_1 = 2000\Omega$ , determine the voltage across the capacitor as a function of time for  $t > 0$ . Verify that the initial conditions and the DC steady state conditions as  $t$  goes to infinity are satisfied.

$$L_d = 1 \times 10^{-4} \text{ H} \quad R_{d2} := 2000 \Omega \quad \frac{1}{L_d \cdot C_d} = 1 \times 10^{12} \frac{1}{\text{s}^2}$$

$$C_d = 1 \times 10^{-8} \text{ F} \quad \frac{R_{d2}}{L_d} = 2 \times 10^7 \frac{1}{\text{s}}$$

$$V_C(s) = \frac{1E12}{s^2 + 2E7s + 1E12} \left( \frac{10}{s} - \frac{5}{s} \right) + \frac{5}{s}$$

$$V_C(s) = \frac{1E12}{s^2 + 2E7s + 1E12} \left( \frac{5}{s} \right) + \frac{5}{s}$$

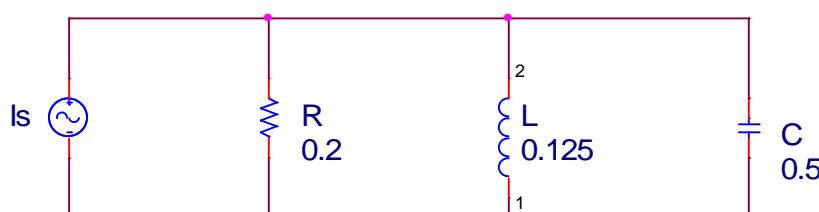
Apply partial fraction expansion

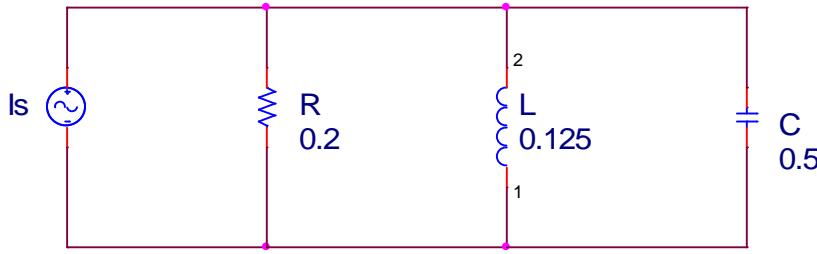
$$V_C(s) = \frac{0.0126}{s + 1.995E7} + \frac{-5.0126}{s + 5E4} + \frac{5}{s} + \frac{5}{s}$$

$$V_C(t) = 0.0126 \exp(-1.995E7t) - 5.0126 \exp(-5E4t) + 10$$

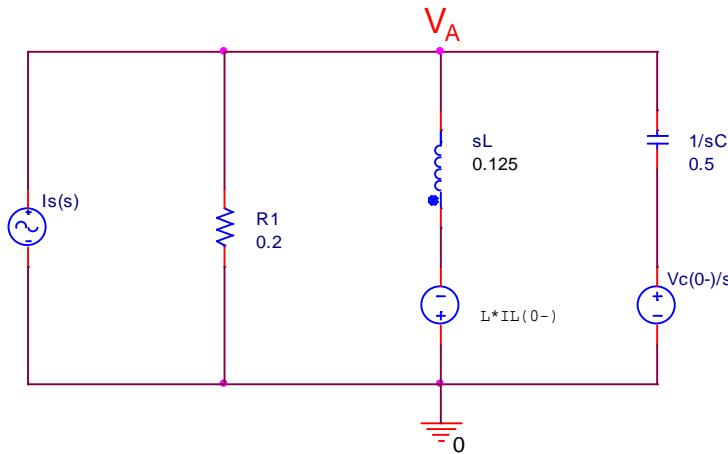
$$V_C(0) = 0.0126 - 5.0126 + 10 = 5 \text{ V (check)}$$

$$V_C(t \rightarrow \infty) = 0 + 0 + 10 = 10 \text{ V (check)}$$





- Symbolically, determine the general expression for  $I_L(s)$ . Include initial condition terms.
- Using s-domain analysis, determine the current through the inductor for source  $Is = 5u(t)$  mA



Use node analysis

$$I_L(s) + I_R(s) + I_C(s) = I_s(s)$$

parameter of interest

$$I_L(s) + \frac{V_A - 0}{R} + \frac{V_A - \frac{V_c(0^-)}{s}}{\frac{1}{sC}} = I_s(s)$$

Symbolically

$$I_L(s) + \frac{V_A(s)}{Z_R} + \frac{V_A(s) - \frac{V_c(0^-)}{s}}{Z_C} = I_s(s)$$

$$V_A = V_L = sL \cdot I_L(s) - L \cdot I_L(0^-)$$

remember  $V_L$  is across both components

ohms law

$$I_L(s) + \frac{sL \cdot I_L(s) - L \cdot I_L(0^-)}{Z_R} + \frac{sL \cdot I_L(s) - L \cdot I_L(0^-) - \frac{V_C(0^-)}{s}}{Z_C} = I_s(s)$$

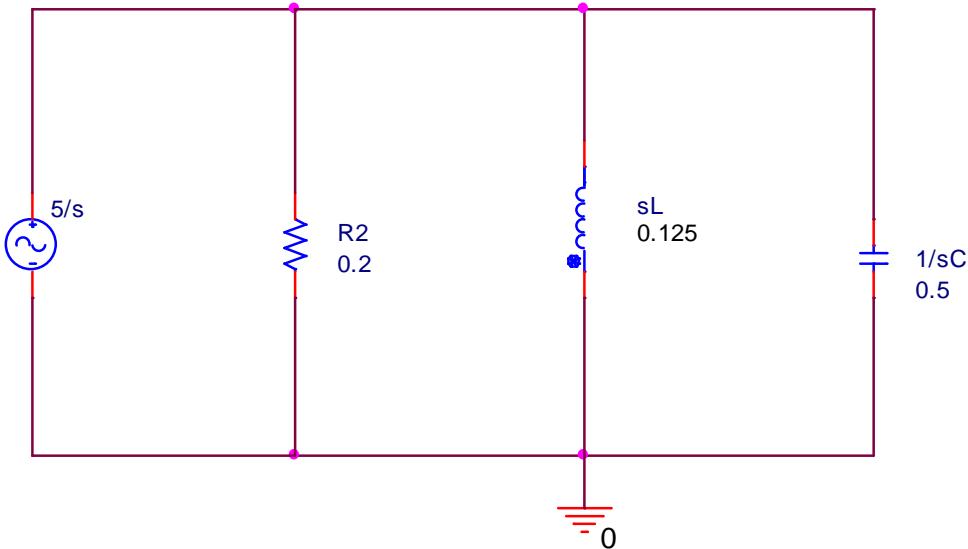
$$I_L(s) + \frac{sL \cdot I_L(s) - L \cdot I_L(0^-)}{R} + \frac{sL \cdot I_L(s) - L \cdot I_L(0^-) - \frac{V_C(0^-)}{\frac{1}{sC}}}{\frac{1}{sC}} = I_s(s)$$

$$I_L(s) \cdot \left( \frac{sL}{R} + 1 + s^2 LC \right) = I_s(s) + \frac{L}{R} \cdot I_L(0^-) + s \cdot L \cdot C I_L(0^-) + C \cdot V_C(0^-)$$

these are all initial conditions

$$I_L(s) = \frac{\frac{1}{LC}}{s^2 + \frac{1}{RC} \cdot s + \frac{1}{LC}} \cdot \text{above}$$

b.  $I_s = 5 \cdot u(t)$ , mA      initial conditions are ZERO!



$$I_L(s) = \frac{\frac{1}{LC}}{s^2 + \frac{1}{RC} \cdot s + \frac{1}{LC}} \cdot I_s(s)$$

$L_b := 0.125$   
 $C_b := 0.5$   
 $R_b := 0.2$

$$\frac{1}{L_b \cdot C_b} = 16$$

$$I_L(s) = \frac{16}{s^2 + 10s + 16} \cdot \frac{5}{s}$$

$\frac{1}{R_b \cdot C_b} = 10$

$$s = -2$$

$$s = -8$$

Partial fraction expansion

$$\frac{80}{s \cdot (s+2) \cdot (s+8)} = \frac{A_1}{s} + \frac{A_2}{s+2} + \frac{A_3}{s+8}$$

Cover up rule for A1 at s=0

$$\frac{80}{2 \cdot 8} = 5$$

Cover up rule for A2 at s=-2

$$\frac{80}{-2 \cdot (-2+8)} = -6.667$$

Cover up rule for A3 at s = -8

$$\frac{80}{-8(-8+2)} = 1.667$$

$$I_L(s) = (5 - 6.67 \cdot e^{-2t} + 1.667 \cdot e^{-8t}) \cdot u(t)$$

you can check this by looking at what happens to this circuit in DC steady state, Cap open, inductor shorts current through inductor must be 5 mA

then look at t=0, the initial conditions are zero so  $I_L(s)$  should be zero at t=0 substitute in

$$5 - 6.667 + 1.667 = 0$$