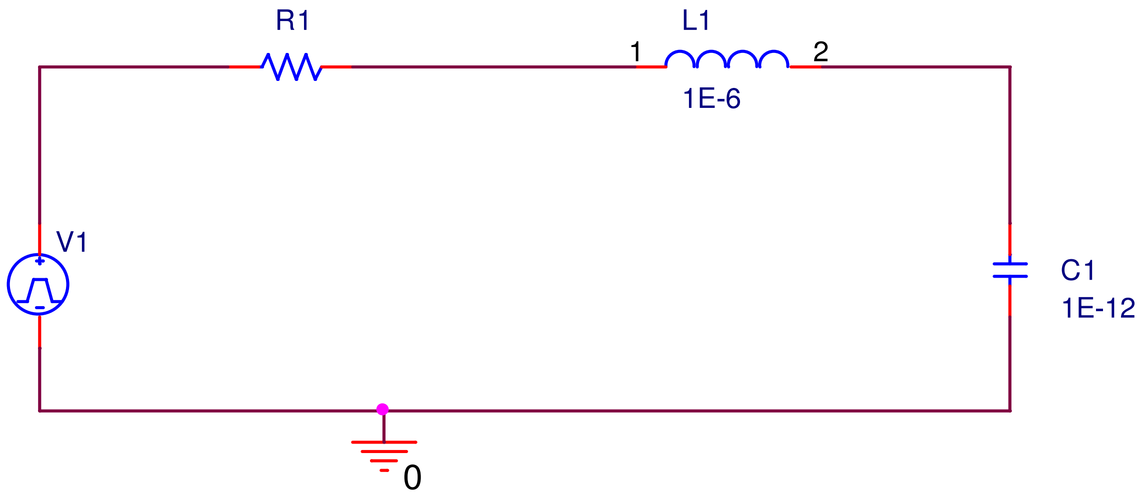
S-domain analysis with initial conditions

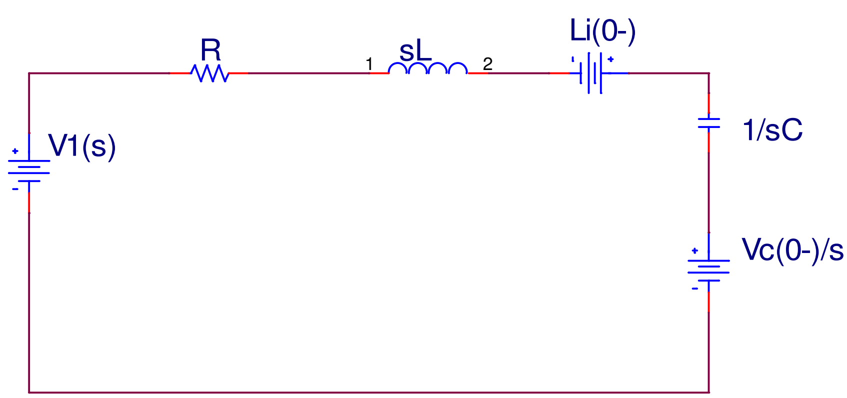
The method of analysis is the same as circuits with no inital conditions. At some point, you'll need to substitute in the correct s-domian values for the intial condition sources. The are DC sources that appear at the time of a switch, like a step function, so they should be N/s. Therefore, you'll need to account for another s (or more than one s) in your ratio. See example below.



$L_1 := 1 \cdot 10^{-6} \mathrm{H}$ $\quad C_1 := 1 \cdot 10^{-12} \mathrm{F}$ $\quad V_{1t0+} := 10\mathrm{V}$ $\quad V_{1t0-} := 5\mathrm{V}$

In the circuit above, the voltage source is 5V for t<0 and 10 V for t >0.

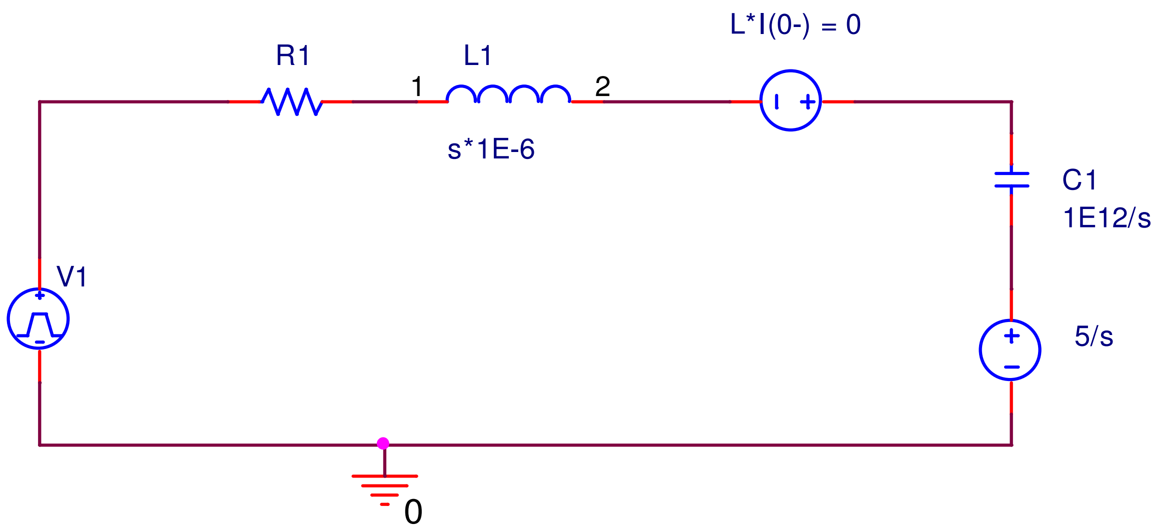
a. Draw the s-domain equivalent circuit. Include the intial conditions in your s-domain circuit. Label your component values using symbolic notation (i.e. sL1).



$I_L(0-) = I_L(0+) = 0$ ***DC steady state for components so capacitor is open so the current in series circuit in 0.***

$$v_C(0\text{-}) = v_C(0+) = 5V$$

The voltage across the capacitor must be 5V, the value of the source at t<0.



b. Using impedances, determine the transfer function for the voltage across C1. Use symbolic numbers in your expression.

$$V_C(s) = \frac{\frac{1}{sC_1}}{R_1 + \frac{1}{sC_1} + \frac{1}{s\cdot L_1}}\cdot\left(V_1(s) + L\cdot I(0\text{-}) - \frac{V_C(0\text{-})}{s}\right) + \frac{V_C(0\text{-})}{s}$$

This is a voltage divider. The voltage sources must be accounted for.

$$V_C(s) = \frac{\frac{1}{L_1\cdot C_1}}{s^2 + \frac{R_1}{L_1}\cdot s + \frac{1}{L_1\cdot C_1}}\cdot\left(V_1(s) + L\cdot I(0\text{-}) - \frac{V_C(0\text{-})}{s}\right) + \frac{V_C(0\text{-})}{s}$$

You can insert intial conditions s-domain values here. If we were looking for the Vc(t), then you'd do partial fraction expansion and the inverse laplace transform after the following.

$$V_C(s) = \frac{\frac{1}{L_1 \cdot C_1}}{s^2 + \frac{R_1}{L_1}\cdot s + \frac{1}{L_1 \cdot C_1}} \cdot \left(\frac{10}{s} + 0 - \frac{5}{s}\right) + \frac{5}{s}$$

$$V_C(s) = \frac{\frac{1}{L_1 \cdot C_1}}{s^2 + \frac{R_1}{L_1}\cdot s + \frac{1}{L_1 \cdot C_1}} \cdot \left(\frac{5}{s}\right) + \frac{5}{s}$$

c. Using your result from part b., determine the transfer function for the current through the capacitor.

$$I_C(s) = \frac{V_C(s) - \frac{V_C(0\text{-})}{s}}{\frac{1}{sC_1}}$$

Using ohm's law where current is related to capacitor impedance Zc=1/sC by Vc =Zc Ic

The voltage source is subtracted from Vc to measure current through capacitor and use ohm's law.

$$I_C(s) = \frac{\frac{s}{L_1}}{s^2 + \frac{R_1}{L_1}\cdot s + \frac{1}{L_1 \cdot C_1}} \cdot \left(V_1(s) + L_1 \cdot I(0\text{-}) - \frac{V_C(0\text{-})}{s}\right)$$

You can insert intial condition values here.

$$I_C(s) = \frac{\frac{s}{L_1}}{s^2 + \frac{R_1}{L_1}\cdot s + \frac{1}{L_1 \cdot C_1}} \left(\frac{10}{s} + 0 - \frac{5}{s}\right)$$

$$I_C(s) = \frac{\frac{s}{L_1}}{s^2 + \frac{R_1}{L_1}\cdot s + \frac{1}{L_1 \cdot C_1}} \left(\frac{5}{s}\right)$$

$$I_C(s) = \frac{\frac{5}{L_1}}{s^2 + \frac{R_1}{L_1}\cdot s + \frac{1}{L_1\cdot C_1}}$$

d. Using the transfer function from part c. for R1 = 5kΩ, determine teh current through the capacitor as a function of time for t>0.

$$I_C(s) = \frac{5\cdot 10^6}{\left(s^2 + 5\cdot 10^9 s + 10^{18}\right)}$$

$$R_1 := 5k\Omega$$

$$\frac{1}{L_1\cdot C_1} = 1\times 10^{18}\,\frac{1}{s^2}$$

$$s^2 + 5\cdot 10^9 s + 10^{18} = 0$$

$$\frac{5}{L_1} = 5\times 10^6\,\frac{1}{H}$$

$$500000000\cdot\sqrt{21} - 2500000000 = -2.087\times 10^8$$

$$-500000000\cdot\sqrt{21} - 2500000000 = -4.791\times 10^9$$

$$\frac{R_1}{L_1} = 5\times 10^9\,\frac{1}{s}$$

Partial fraction expansion

$$\frac{5\cdot 10^6}{\left(s + 2.09\cdot 10^8\right)\cdot\left(s + 4.8\cdot 10^9\right)} = \frac{A_1}{\left(s + 2.09\cdot 10^8\right)} + \frac{A_2}{\left(s + 4.8\cdot 10^9\right)}$$

For A1

$$\frac{5\cdot 10^6}{\left(-2.09\cdot 10^8 + 4.8\cdot 10^9\right)} = 1.089\times 10^{-3}$$

For A2

$$\frac{5\cdot 10^6}{\left(-4.8\cdot 10^9 + 2.09\cdot 10^8\right)} = -1.089\times 10^{-3}$$

$$I_C(s) = \frac{\left(1.1\cdot 10^{-3}\right)}{\left(s + 2.09\cdot 10^8\right)} + \frac{\left(-1.1\times 10^{-3}\right)}{\left(s + 4.8\cdot 10^9\right)}$$

$$I_C(t) = 1.1\cdot 10^{-3}\cdot e^{-2.09\cdot 10^8 t} - 1.1\cdot 10^{-3}\cdot e^{4.8\cdot 10^9 t}$$