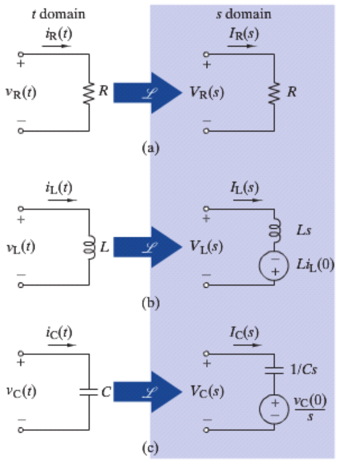
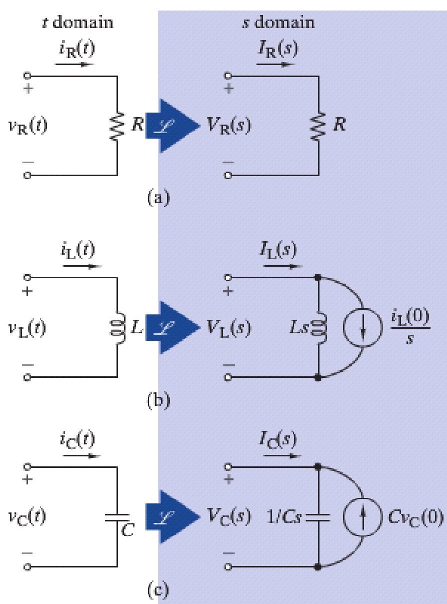
S-domain conversions for current



Voltage coversions, we know and have used.

We have another conversion...



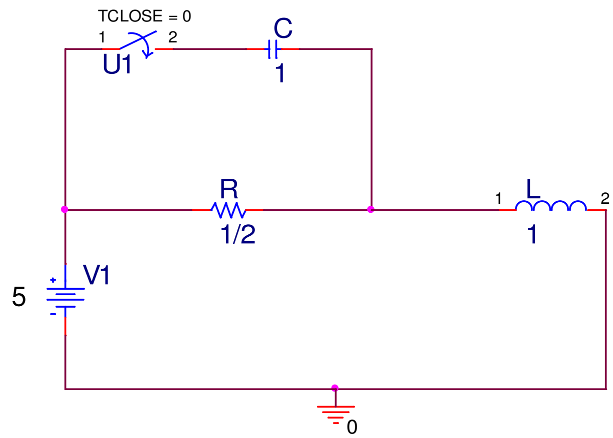
Resistor $I_R(s) = \frac{1}{R} \cdot V_R(s)$

Inductor $I_L(s) = \frac{1}{L \cdot s} \cdot V_L(s) + \frac{i_L(0)}{s}$

Capacitor $I_C(s) = C \cdot s \cdot V_C(s) - C \cdot v_c(0)$

The initial condition sources are in parallel with element impedance (these are just source transformation of the voltage conversions).

Note current direction.

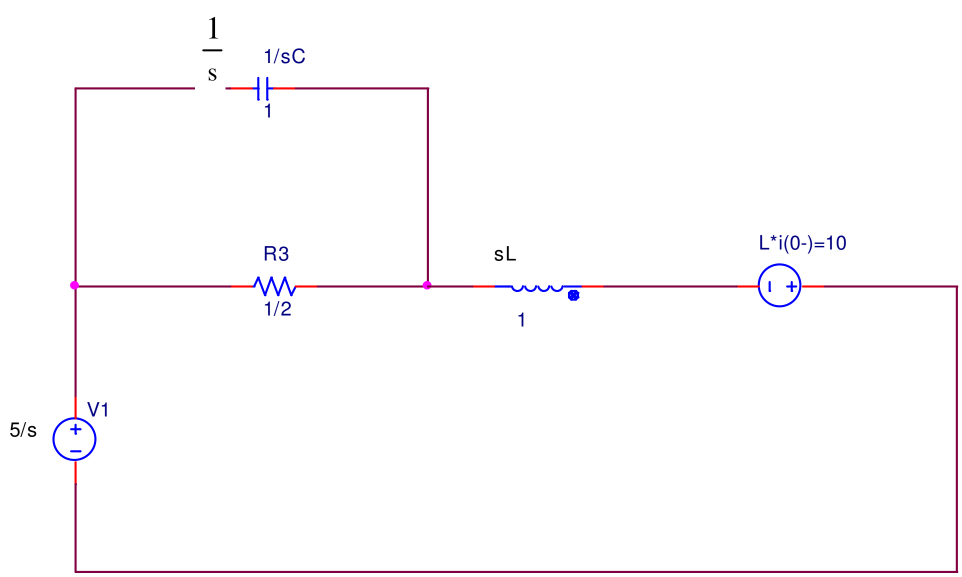


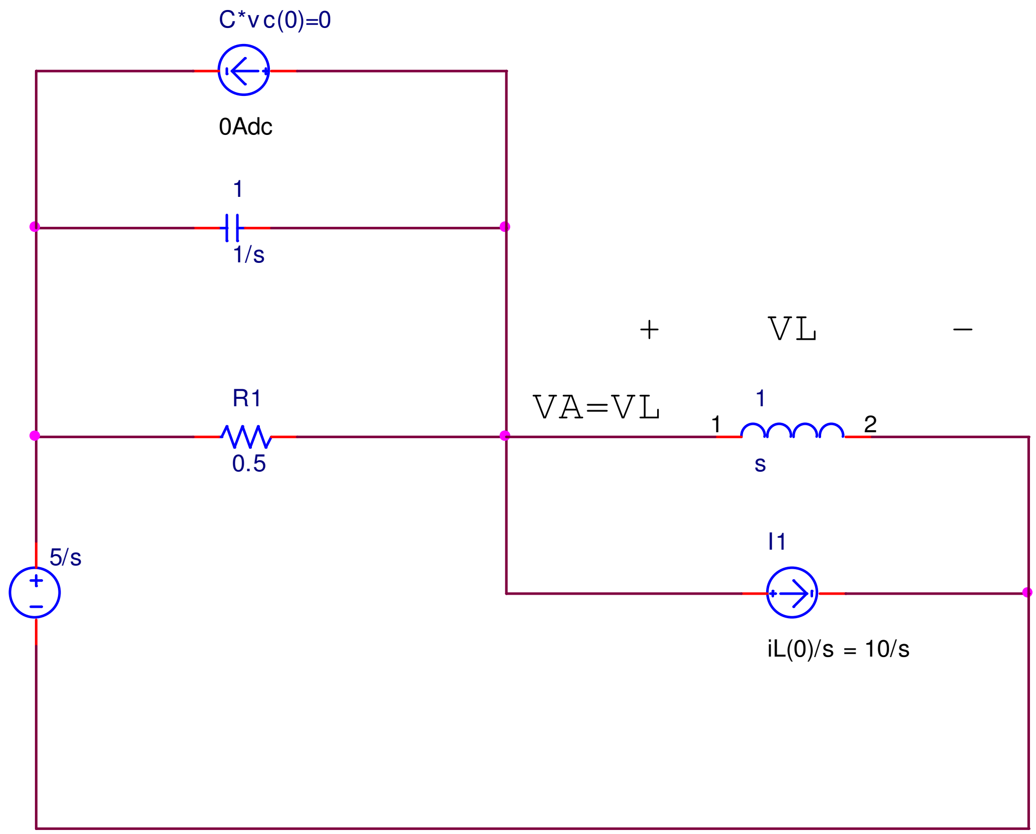
In the above circuit, a 5V DC source is shown. At t = 0, the switch in series with the capacitor is closed.

a) Draw the s-domain equivalent circuit. Include all initial condition source components.

$v_C(0-) = v_C(0+) = 0$ switch closes at t=0 but it must have voltage continuity

$I_L(0-) = I_L(0+) = \frac{5V}{0.5\Omega} = 10$ The inductor must have the same current through the resistor, not voltage parallel is zero (also may consider inductor in DC steady state, which would make it a short).





b) Determine the transfer function for the voltage across the inductor, VL(s)= N(s)/D(s), where N(s) and D(s) are poloynomials.

KCL at node A (between inductor and resistor)

$V_A = V_L$

$$\frac{V_L - \frac{5}{s}}{0.5} + \frac{V_L - \frac{5}{s}}{\frac{1}{s}} + \frac{V_L + 10}{s} = 0$$

these are equiv.

Using s-domain current source conversion

$$\frac{V_L}{s} + \frac{10}{s} + \frac{V_L - \frac{5}{s}}{0.5} + \frac{V_L - \frac{5}{s}}{\frac{1}{s}} = 0$$

$$2V_L - \frac{10}{s} + sV_L - 5 + \frac{V_L}{s} + \frac{10}{s} = 0$$

multiply by s then group

$$2s \cdot V_L - 10 + s^2 \cdot V_L - 5s + V_L + 10 = 0$$

$$V_L\left(2s + s^2 + 1\right) - 10 - 5s + 10 = 0$$

$$V_L\left(2s + s^2 + 1\right) = 5s$$

$$V_L = \frac{5s}{s^2 + 2s + 1}$$

c) Apply partial fraction expansion to your above expression

$$\frac{5s}{s^2 + 2s + 1} = \frac{A_1}{s + 1} + \frac{A_2}{(s + 1)^2}$$

A2 first using cover up rule

$$\frac{5\,s(s + 1)^2}{(s + 1)^2} \qquad \text{at } s = -1$$

$$A_2 = -5$$

A1 using F(0)

$$\frac{5(0)}{(0 + 1)^2} = \frac{A_1}{(0 + 1)} + \frac{-5}{(0 + 1)^2}$$

$$A_1 = 5$$

$$\frac{5s}{s^2 + 2s + 1} = \frac{5}{s + 1} + \frac{-5}{(s + 1)^2}$$

d) Based on your result in part c), determine the voltage across the inductor as a function of time.

$$V_L(t) = 5\cdot e^{-t} - 5\cdot t\cdot e^{-t}$$

Superposition

In the s-domain there are two types of independent sources:
(1) Voltage and current sources representing external driving forces tor t>=0
(2) Initial condition voltage and current sources representing energy stored at t=0.

Superposition principle can be appplied so s-domain response is the sum of two components

1. Zero input response caused by the initial condition sources with external sources turned off
2. Zero state response caused by external inputs wiht the intial conditon sources off

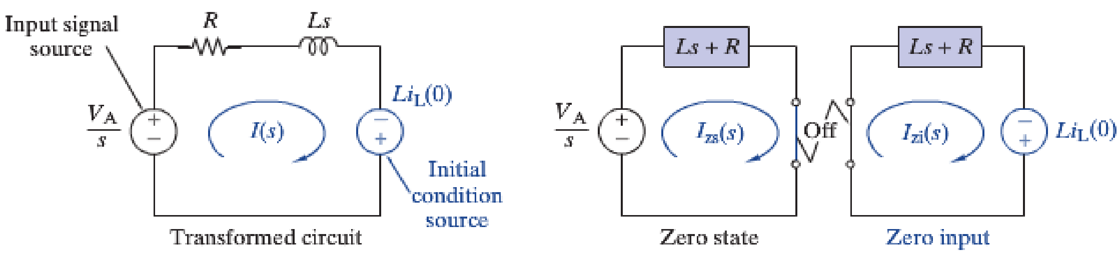
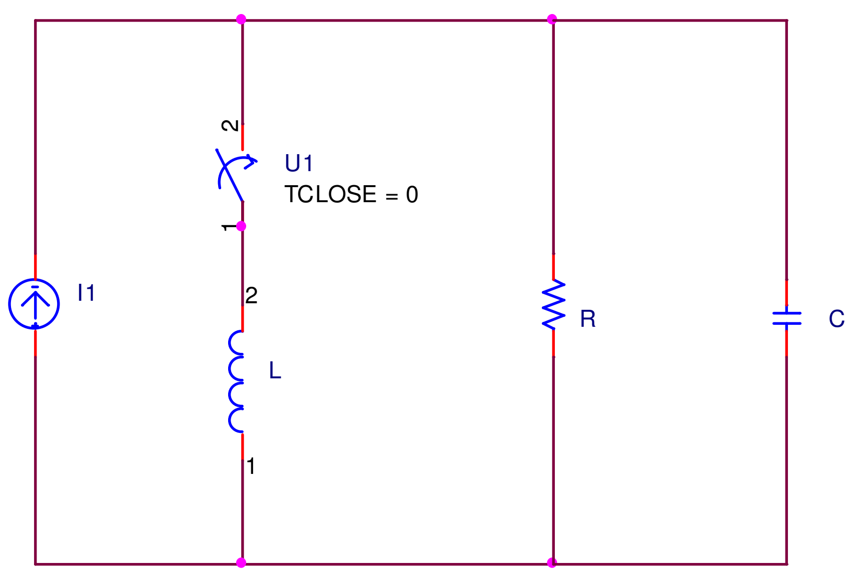


FIGURE 10–20 *Using superposition to find the zero-state and zero-input responses.*

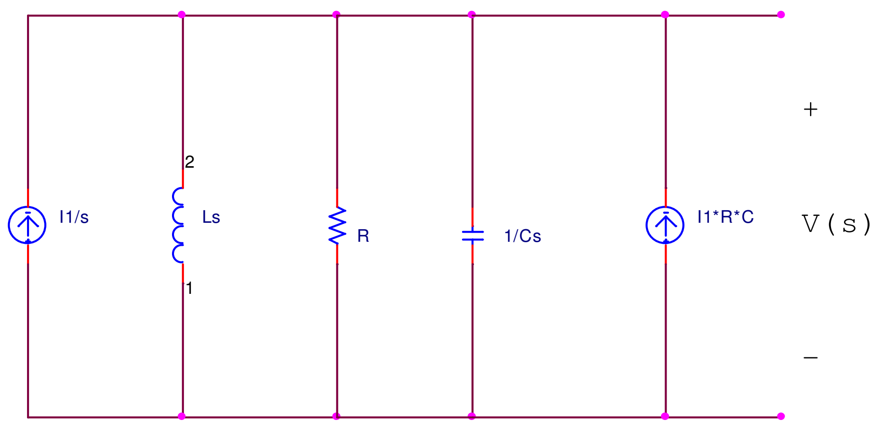
$V(s) = V_{Zs} + V_{Zi}$ $\qquad$ $I(s) = I_{As}(s) + I_{Zi}(s)$

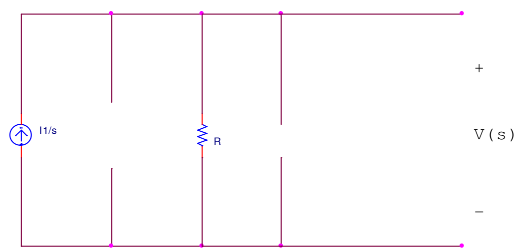


a) Transform the circuit into the s domain
b) Find the zero-state and zero-input components of V(s).
c) Find v(t) for I1 = 1mA, L = 2H, R=1.5kΩ and C = 1/6 μf
With source transformation

$L_1 := 2\mathrm{H} \qquad R_1 := 1.5 \cdot 10^{3}\,\Omega \qquad C_1 := \frac{1}{6} \cdot 10^{-6}\,\mathrm{F} \qquad I_1 := 1\mathrm{mA}$

a)





$I_L(0\text{-}) = I_L(0+) = 0$ this is due to the open switch at t<0

Dc staedy state of capacitor makes it an open circuit.

$v_c(0) = I_1 \cdot R$

switch and constant current source make a step function I1u(t)

b) Replace components with Zequivalent

$$Z_{EQ} = \frac{1}{YEQ} = \frac{1}{\frac{1}{Ls} + \frac{1}{R} + Cs}$$

YEQ is admittance and is the inverse of impedance so each impedance has an admittance value which is 1 over its value...it is convenient in combining parallel components....

multiple by RLs/RLs

$$Z_{EQ} = \frac{RLs}{R + Ls + RLCs^2}$$

First find zero state response

$$V_{Zs} = Z_{EQ}(s)\cdot\frac{I_1}{s}$$

Zero state means kill, or, open circuit initial conditions

$$V_{Zs} = \frac{RLs}{R + Ls + RLCs^2}\cdot\frac{I_1}{s}$$

simplify and put in standard form to find poles,divide top and bottom by RLC

$$V_{Zs} = \frac{\frac{I_A}{C}}{s^2 + \frac{s}{RC} + \frac{1}{LC}}$$

Then find zero input response so I1/s is 0, open circuit

$$V_{Zi}(s) = Z_{EQ}(s)\cdot\left(CRI_1\right) = \frac{RLs}{R + Ls + RLCs^2}\cdot CRI_1$$

simplify and put in stadard form, divide top and bottom by RLC

$$V_{Zi}(s) = \frac{R\cdot I_1\cdot s}{s^2 + \frac{s}{RC} + \frac{1}{LC}}$$

$$\frac{1}{6}\cdot 10^{-6} = 1.667\times 10^{-7}$$

c)

$$V_{Zs} = \frac{6\cdot 10^3}{s^2 + 4\cdot 10^3\cdot s + 3\cdot 10^6}$$

$$\frac{I_1}{C_1} = 6\times 10^3\,\frac{m^2\cdot kg}{A\cdot s^4}$$

$$\frac{1}{R_1\cdot C_1} = 4\times 10^3\,\frac{1}{s}$$

$$V_{Zs} = \frac{6\cdot 10^3}{\left(s + 3\cdot 10^3\right)\cdot\left(s + 1\cdot 10^3\right)}$$

$$\frac{1}{L_1\cdot C_1} = 3\times 10^6\,\frac{1}{s^2}$$

partial fraction expansion

poles $-3\cdot 10^3$ $-1\cdot 10^3$

$$\frac{A_1}{\left(s+3\cdot 10^3\right)}+\frac{A_2}{\left(s+1\cdot 10^3\right)}$$

Find A1 at s = -3000

$$\frac{6\cdot 10^3}{\left(-3\cdot 10^3+1\cdot 10^3\right)}=-3$$

Find A2 at s = -1000

$$\frac{6\cdot 10^3}{\left(-1000+3\cdot 10^3\right)}=3$$

$$\frac{-3}{\left(s+3\cdot 10^3\right)}+\frac{3}{\left(s+1\cdot 10^3\right)}$$

$V_{Zs} = -3\cdot e^{-3000t}+3\cdot e^{-1000t}$ for t>=0

$$V_{Zi} = \frac{1.5s}{\left(s+3\cdot 10^3\right)\cdot\left(s+1\cdot 10^3\right)}$$

$R_1\cdot I_1 = 1.5\,V$

Find A1 at s = -3000

$$\frac{1.5\cdot -3000}{\left(-3\cdot 10^3+1\cdot 10^3\right)}=2.25$$

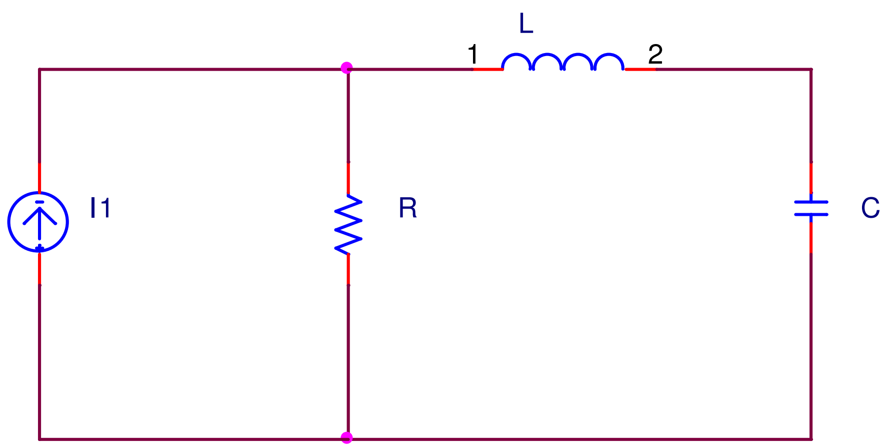
Find A2 at s = -1000

$$\frac{1.5 \cdot -1000}{\left(-1000 + 3 \cdot 10^{3}\right)} = -0.75$$

$$\frac{2.25}{\left(s + 3 \cdot 10^{3}\right)} + \frac{-0.75}{\left(s + 1 \cdot 10^{3}\right)}$$

$$V_{Zs} = 2.5 \cdot e^{-3000t} - 0.75 e^{-1000t}$$ for t>=0

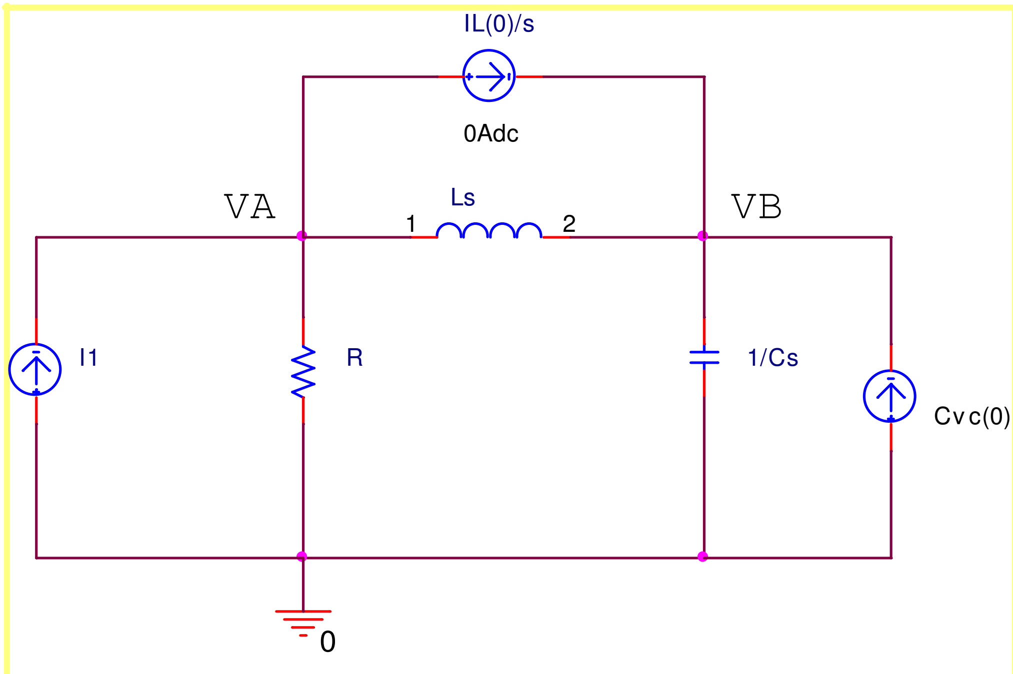
Node Analysis in the s-domain



a. Draw the s-domain circuit
b. Find node equations and put in standard form
c. Solve the node equations and find the zero-state and zero-input reponses
d. Solve for the zero state component of the waveforms VA(t) when

$R_2 := 1\text{k}\Omega \quad C_2 := 0.2\mu\text{F} \quad L_2 := 500\text{mH} \quad I_1(t) = 10\,u(t)\,\text{mA}$

a.



b.

At node A

$$\frac{V_A}{R} + \frac{V_A - V_B}{Ls} - I_1 + \frac{i_L(0)}{s} = 0$$

At node B

$$\frac{V_B}{\frac{1}{CS}} + \frac{V_B - V_A}{Ls} - \frac{i_L(0)}{s} - C \cdot v_c(0) = 0$$

Rearrange

$$V_A \cdot \left(\frac{1}{R} + \frac{1}{Ls}\right) - V_B \cdot \left(\frac{1}{Ls}\right) = I_1 - \frac{i_L(0)}{s}$$ NODEA

$$V_A \left(\frac{-1}{Ls}\right) + V_B \cdot \left(\frac{1}{Ls} + C \cdot s\right) = C \cdot v_c(0) + \frac{i_L(0)}{s}$$

$$M_2 = \begin{bmatrix} \left(\frac{1}{R_2} + \frac{1}{L_2 \cdot s}\right) & -\frac{1}{L_2 \cdot s} \\ \frac{-1}{L_2 \cdot s} & \frac{1}{L_2 \cdot s} + C_2 \cdot s \end{bmatrix}$$

Here are where things get interesting, we need to find the solution to this matrix but we don't have just numbers to plug in.

This is done by hand using Cramers Rule.

$$M_2 = \frac{C_2 \cdot L_2 \cdot s^2 + C_2 \cdot R_2 \cdot s + 1}{L_2 \cdot R_2 \cdot s}$$

$$C_2 = \begin{pmatrix} I_1 - \frac{i_L(0)}{s} \\ C \cdot v_c(0) + \frac{i_L(0)}{s} \end{pmatrix}$$

$$\Delta_S = \frac{C_2 \cdot L_2 \cdot s^2 + C_2 \cdot R_2 \cdot s + 1}{L_2 \cdot R_2 \cdot s}$$

This is the circuits determine without influence of input sources or initial sources

Cramers rule

$$V_A = \frac{\Delta_A}{\Delta_s}$$

where

$$\Delta_A = \begin{pmatrix} I_1 - \frac{i_L(0)}{s} & -\frac{1}{L_2 \cdot s} \\ C \cdot v_c(0) + \frac{i_L(0)}{s} & \frac{1}{L_2 \cdot s} + C_2 \cdot s \end{pmatrix}$$

$$\Delta_A = \frac{C_2 \cdot I_1 \cdot L_2 \cdot s^2 - C_2 \cdot L_2 \cdot i_L(0) \cdot s + I_1 + C \cdot v_c(0)}{L_2 \cdot s}$$

$$V_A = \frac{\dfrac{C_2 \cdot I_1 \cdot L_2 \cdot s^2 - C_2 \cdot L_2 \cdot i_L(0) \cdot s + I_1 + C \cdot v_c(0)}{L_2 \cdot s}}{\dfrac{C_2 \cdot L_2 \cdot s^2 + C_2 \cdot R_2 \cdot s + 1}{L_2 \cdot R_2 \cdot s}}$$

Group together components with initial value sources

$$V_A = \frac{\left(L_2 \cdot C_2 \cdot s^2 + 1\right) \cdot R_2 \cdot I_1}{C_2 \cdot L_2 \cdot s^2 + C_2 \cdot R_2 \cdot s + 1} + \frac{-R_2 \cdot L_2 \cdot C_2 \cdot s \cdot i_L(0) + R_2 \cdot C_2 \cdot v_c(0)}{C_2 \cdot L_2 \cdot s^2 + C_2 \cdot R_2 \cdot s + 1}$$

ZERO STATE

ZERO INPUT

d.

$$V_{AZs} = \frac{\left(L_2 \cdot C_2 \cdot s^2 + 1\right) \cdot R_2 \cdot I_1}{C_2 \cdot L_2 \cdot s^2 + C_2 \cdot R_2 \cdot s + 1}$$

R.2 is divided

$$L_2 \cdot C_2 \cdot \frac{1}{R_2} = 1 \times 10^{-10} \frac{A^2 \cdot s^5}{m^2 \cdot kg}$$

$$R_2 = 1 \times 10^3 \, \Omega \qquad \frac{1}{R_2} = 1 \times 10^{-3} \frac{1}{\Omega}$$

$$C_2 \cdot R_2 \cdot \frac{1}{R_2} = 2 \times 10^{-7} \, F$$

$$V_{AZs} = \frac{1 \cdot 10^{-7} \cdot s^2 + 1}{1 \cdot 10^{-10} \cdot s^2 + 2 \cdot 10^{-7} s + 1 \cdot 10^{-3}} \cdot \frac{10^{-2}}{s}$$

source

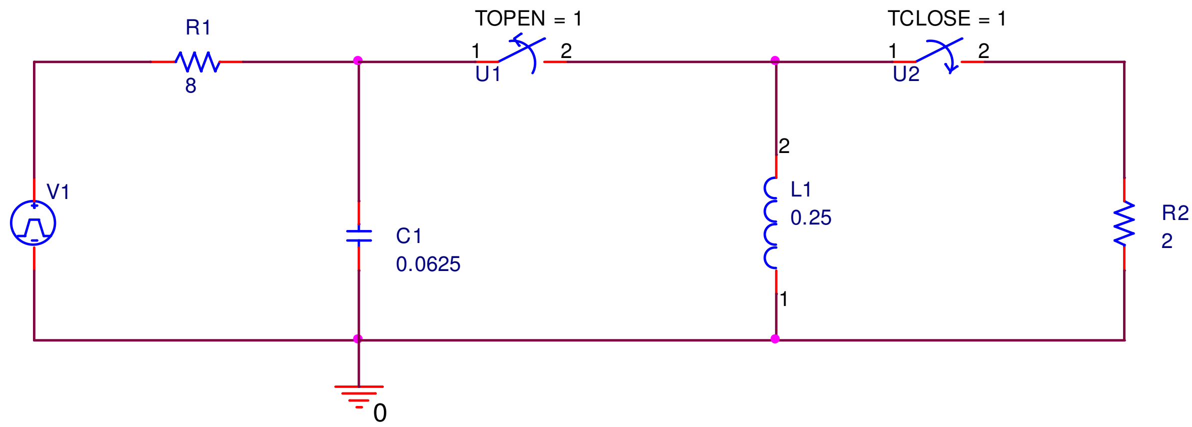
$$V_{AZs} = 10 \cdot \frac{\left(s^2 + 10^7\right)}{s \cdot \left[(s + 1000)^2 + 3000^2\right]}$$

partial fraction expansion

Takes quite a bit to finish but

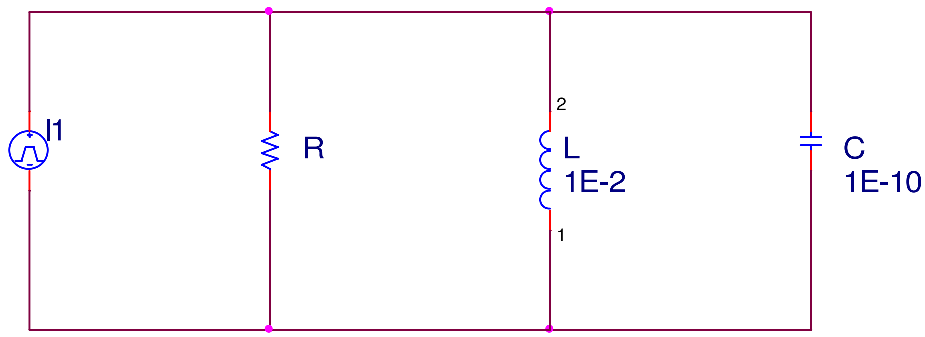
$$V_{Az} = 10\,u(t) - 20 \cdot e^{-1000t} \cdot \left(\frac{1}{3} \cdot \sin(3000t)\right) \cdot u(t)$$

Review Problems for Test



In the above circuit, the source is 8V for t<0 and 16 V for t>0. Additionally, the switch U1 closes are t = 1 s and switch U2 open at t = 1 s (They close and open at the same time).

a. At t=0+, determine the current through L1
b. At t=0+ determine the voltage across L1
c. For t < 1s, determine the differential equation for the current through R1 (Draw the circuit)
d. For t<1s, is the circuit underdamped, overdamped or critically damped?
e. Determine the voltage across L1 as a function of time for 0<t<1s
f. Based on your above expression, dtermine the voltage across the inductor at t =1s
g. Determine the voltage as a function of time across the inductor for t>1.



In the above circuit, the source current is 20mA for t < 0 and 0 for t > 0 (the source turns off at t = 0).

a. What is the initial (t=0+) current through the inductor? What is the (t=0+) voltage across the inductor?

b. What is the DC steady state current through the inductor a t goes to ∞?

c. Symbolically, what is the differential equation defining the current through the inductor?

d. For R1 = 100Ω, determine the current through the inductor as a function of time for t>0.

e. For R1 = 5k Ω?

f. For R1 = 100kΩ?