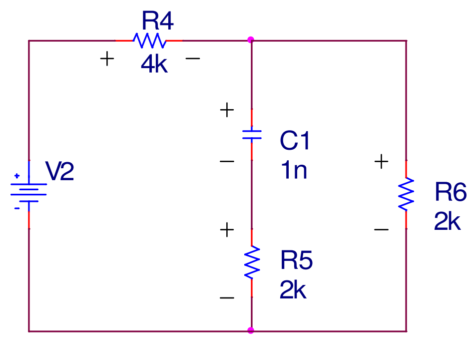
Voltage/Current Continuity

1)



In the above circuit, the voltage is defined as follows:

$$V1 = \begin{cases} 5V & t < 0 \\ 10V & 0 < t \end{cases}$$ (the voltage source changes from 5V to 10V at t = 0)

$R_{41} := 4k\Omega$ $R_{51} := 2k\Omega$ $R_{61} := 2k\Omega$ $C_1 := 1nF$ $V_{1t0-} := 5V$ $V_{1t0+} := 10V$

a. Write V1 in the format ___ + (or -) ____u(t) (for example 2-2u(t))

u(t) is 1 t>0 and 0 when t<0 so at t<0 you need 5V and at t>0 you need 5+5V

Therefore, 5+5u(t)

b. At t = 0- (just before the voltage changes), determine the voltage across each component and the current through each component (use the polarities indicated in the circuit). Draw the circuit.

Component	Voltage	Current
R4		
R5		
R6		
C1		

There is DC steady state here so the capacitor is an open circuit, leaving R5 dangling and a voltage divider for R4 and R6.

$$V_{R41} := \frac{V_{1t0-} \cdot R_{41}}{R_{41} + R_{61}}$$

$V_{R41} = 3.333\,V$

$$I_{R41} := \frac{V_{R41}}{R_{41}}$$

$I_{R41} = 0.833 \cdot mA$

$V_{R51} := 0$

$I_{R51} := 0$

$$V_{R61} := \frac{V_{1t0-} \cdot R_{61}}{R_{41} + R_{61}}$$

$V_{R61} = 1.667\,V$

$$I_{R61} := \frac{V_{R61}}{R_{61}}$$ in series with R4

$I_{R61} = 0.833 \cdot mA$

$V_{C1} := V_{R61}$

voltage divider across R6

$I_{C1} := 0$

open circuit

c. At t = 0+ (just after the voltage changes), determine the voltage across each component and the current through each component for the polarities indicated in the circuit. Draw the circuit.

Component	Voltage	Current
R4		
R5		
R6		
C1		

$v_C(0\text{-}) = v_C(0+) = 1.667\text{V}$ Treat it like a voltage source

Draw below

$V_{C12} := 1.66\text{V}$

Mesh analysis

$$-10 + 4k \cdot i_1 + 1.667 + 2k \cdot i_1 - 2k \cdot i_2 = 0$$

(1) $6k \cdot i_1 - 2k \cdot i_2 = 8.33$

$$-1.667 + 2k \cdot i_2 + 2k \cdot i_2 - 2k \cdot i_1 = 0$$

(2) $-2k \cdot i_1 + 4k \cdot i_2 = 1.667$

$$M_1 := \begin{pmatrix} 6 \cdot 10^3 & -2 \cdot 10^3 \\ -2 \cdot 10^3 & 4 \cdot 10^3 \end{pmatrix} \qquad C_2 := \begin{pmatrix} 8.33 \\ 1.667 \end{pmatrix}$$

$$M_1^{-1} \cdot C_2 = \begin{pmatrix} 1.833 \times 10^{-3} \\ 1.333 \times 10^{-3} \end{pmatrix} \qquad i_1 := 1.833\text{mA} \qquad i_2 := 1.33\text{mA}$$

$I_{R4b} := i_1$ $\qquad I_{R6b} := i_2$

$I_{R4b} = 1.833 \times 10^{-3}\,\text{A}$ $\qquad I_{R6b} = 1.33 \times 10^{-3}\,\text{A}$

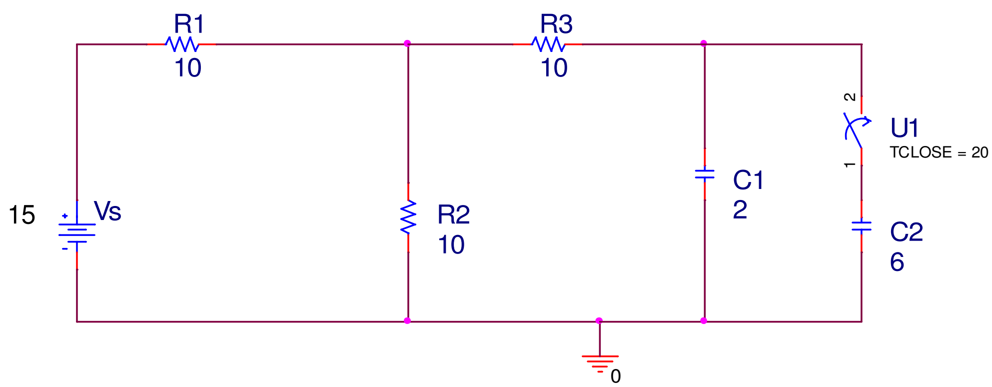
$V_{R4b} := R_{41} \cdot I_{R4b} = 7.332\,\text{V}$ $\qquad V_{R6b} := R_{61} \cdot I_{R6b} = 2.66\,\text{V}$

$V_{R5b} := V_{R6b} - V_{C12}$

In series with R5

$V_{R5b} = 1\,\text{V}$ $\qquad I_{R5b} := \dfrac{V_{R5b}}{R_{51}} = 0.5 \cdot \text{mA}$ $\qquad I_{C12} := I_{R5b} = 0.5 \cdot \text{mA}$

2) First order circuits (Differential Equations)



In the above circuit, the source turns on at t = 0 with a voltage of 15V, Vs=15u(t)V. Additionally, at t = 20s the switch in series with C2 is closed. You can (should) ignore C2 for part a) of this problem.

$V_s := 15V$ $\quad R_{12} := 10\Omega$ $\quad R_{22} := 10\Omega$ $\quad R_{32} := 10\Omega$ $\quad C_{12b} := 2F$ $\quad C_{62} := 6F$

a) For 0<t<20s, determine the voltage across C1 as a function of time, Vc(t). If you do any circuit reduction/transformation, include a drawing of your circuit.

Find thevenin circuit, take off capacitor load and find voltage above R2

$$V_{Th2} := V_s \cdot \frac{R_{22}}{R_{12} + R_{22}} = 7.5\,V$$

Draw equivalent circuit

$$R_{Th} := \frac{R_{12} \cdot R_{22}}{R_{12} + R_{22}} + R_{32}$$

$$R_{Th} = 15\,\Omega$$

$$V_c(t) = A_1 \cdot e^{\frac{-t}{RC}} + A_2$$

final condtion t goes to ∞

$$7.5 = A_2$$

initial condition t goes to 0

$$A_1 + A_2 = 0$$

$$A_1 = -7.5$$

$$V_C(t) = -7.5 \cdot e^{\frac{-t}{30}} + 7.5$$

b) For t>20s, determine the votlage across C1 as a function of time. Use the resistor and capacitor values in your solution. If you do any circuit reduction/transformation, include a drawing of your circuit.

$$V_C(t) = A_1 \cdot e^{\frac{-(t-20)}{RC}} + A_2$$

You have two capacitors in parallel which are easy to combine so do so and use it as a load. The voltage across both of them should be the same.

Draw new circuit below

final condition t goes to infinity

$$7.5 = 0 + A_2$$

initial condition at t = 20

What is Vc(t) when t =0 from part a?

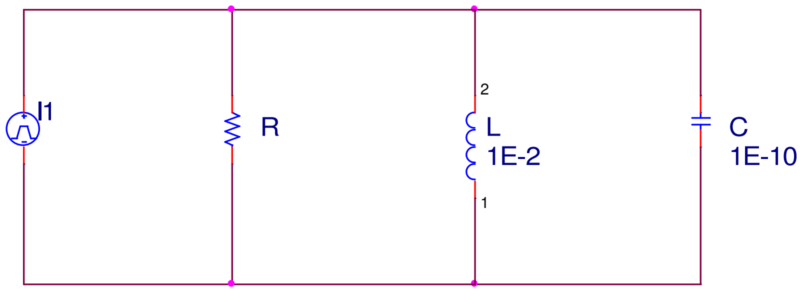
$$-7.5 \cdot e^{\frac{-20}{30}} + 7.5 = 3.649$$

$$A_1 + A_2 = 3.65$$

$$A_1 = -3.85$$

$$V_C(t) = -3.85 \cdot e^{\frac{-(t-20)}{RC}} + 7.5$$

3) Secord order differential equations



In the above circuit, the source current is 20mA for t < 0 and 0 for t > 0 (the source turns off at t = 0).

$R_{13} := 100\Omega$ $\quad L_{13} := 1\cdot 10^{-2}\,H$ $\quad C_{13} := 1\cdot 10^{-10}\,F$

a) What is the initial (t=0+) current through the inductor? What is the (t=0+) voltage across the inductor?

DC steady state short through inductor, open circuit across the capacitor

$I_L(0\text{-}) = I_L(0+) = 20mA$ $\quad v_C(0) = v_L(0) = 0$

b) What is the DC steady state current through the inductor as t ->∞?

The source is off so the current is 0 through the inductor

c) Symbolically, what is the differential equation defining the current through the inductor?

Directly from crib sheet $\quad \frac{d^2 I_L}{dt^2} + \frac{1}{RC}\cdot\frac{dI_L}{dt} + \frac{1}{LC}\cdot I_L = \frac{1}{LC}\cdot I_L$

d) For R1 = 100Ω, determine the current through the inductor as a function of time for t>0.

$\alpha := \frac{1}{2\cdot R_{13}\cdot C_{13}} = 5\times 10^{7}\,\frac{1}{s}$ $\quad \omega_o := \sqrt{\frac{1}{L_{13}\cdot C_{13}}} = 1\times 10^{6}\,\frac{1}{s}$

Circuit is overdamped

$I_L(t) = A_1\cdot e^{-s_1\cdot t} + A_2\cdot e^{-s_2\cdot t} + A_3$

$s_1 := -\alpha + \sqrt{\alpha^2 - \omega_o^2}$ $\quad s_2 := -\alpha - \sqrt{\alpha^2 - \omega_o^2}$

$s_1 = -1\times 10^{4}\,\frac{1}{s}$ $\quad s_2 = -9.999\times 10^{7}\,\frac{1}{s}$

$$I_L(t) = A_1 \cdot e^{-1 \cdot 10^4 t} + A_2 \cdot e^{-10 \cdot 10^7} + A_3$$

Use initial and final conditions

final conditions, dc steady state so A3 must = 0

$I_L(0+)$ initial condition

$$A_1 + A_2 = 0.02$$

$$\frac{dI_L(0+)}{dt} = \frac{V_L(0+)}{L} = 0$$ from defnition of an inductor

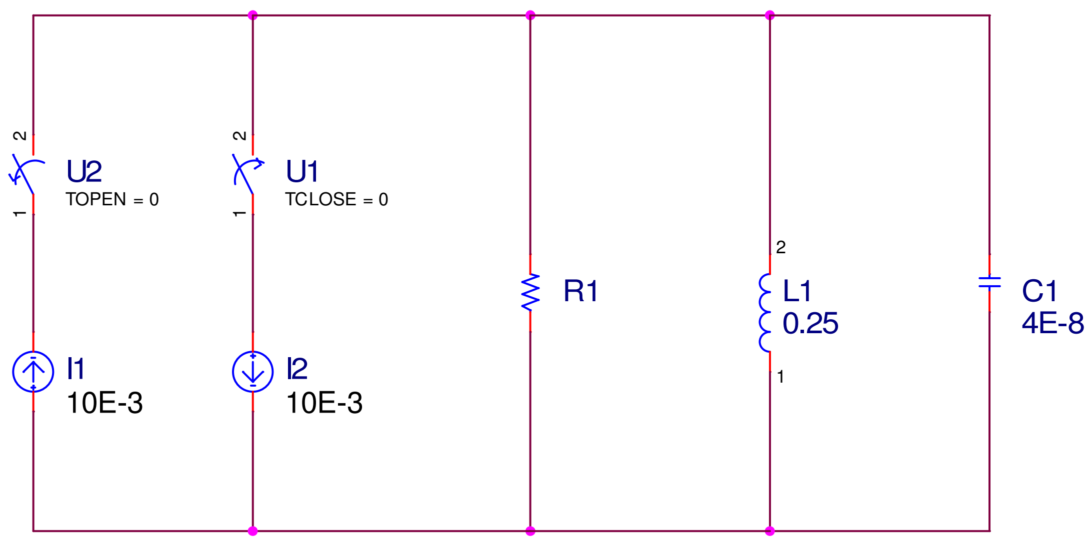
$$-1 \cdot 10^4 \cdot A_1 - 10 \cdot 10^7 A_2 = 0$$

Two equations two unknowns

$A_1 = 0.02$ $A_2 = -2 \cdot 10^6$

$$I_L(t) = 0.02 \cdot e^{-1 \cdot 10^4 t} + -2 \cdot 10^6 \cdot e^{-10 \cdot 10^7}$$

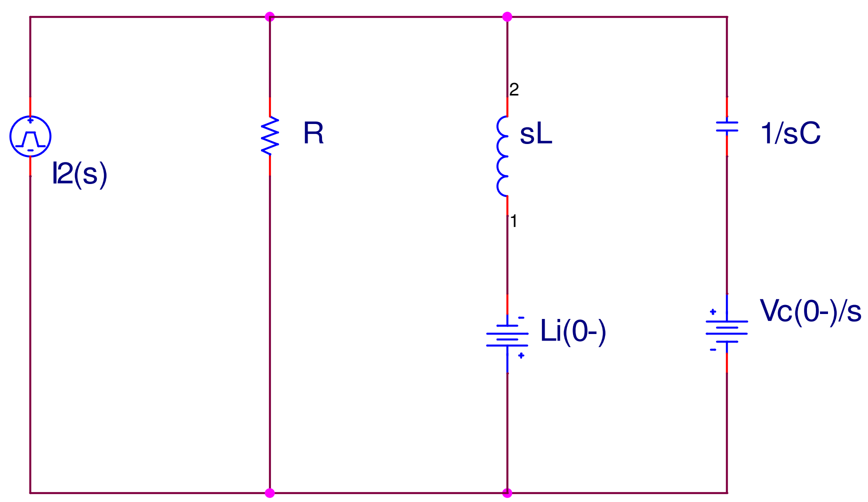
4) Second order, s- domain and Laplace



At t =0, U1 closes and U2 opens

$L_{14} := 0.25\text{H} \qquad C_{14} := 4 \cdot 10^{-8}\text{F}$

a) Draw the s-domain equivalent circuit. Include all intial conditions and label your component values using symbolic notation (i.e. sL1)



$i(0\text{-}) = 10\text{mA} \qquad 0.25\text{H} \cdot 10\text{mA} = 2.5 \times 10^{-3}\text{Wb} \qquad v_C(0\text{-}) = 0$

b) Using impedances, determine the transfer function for the current through the capacitor, C1. Use symbolic values in your expression (R, L, C, I1, I2)

Start with KCL with top node as Vc(s), the voltage across the capacitor (and its intial condition source) I1 just provides initial contions. I2(s) will be used in the equation below.

$$\frac{V_C(s)}{R} + \frac{V_C(s) + L\cdot i(0)}{sL} + \frac{V_C(s) - \frac{V_C(0)}{s}}{\frac{1}{sC}} = I_2(s)$$

$$I_2(s) = \frac{1}{R}\cdot V_C(s) + \frac{1}{sL}\cdot V_C(s) + sC\cdot V_C(s) + \frac{L\cdot i(0)}{sL} - C\cdot V_C(0)$$

$$I_2(s) = V_C(s)\cdot\left(\frac{1}{R} + \frac{1}{sL} + sC\right) + \frac{L\cdot i(0)}{sL} - C\cdot V_C(0)$$

solve for Vc(s) then we'll use ohms law

$$V_C(s) = \frac{1}{\left(\frac{1}{R} + \frac{1}{sL} + sC\right)}\cdot\left(I_2(s) - \frac{L\cdot i(0)}{sL} + C\cdot V_C(0)\right)$$

$$I_C(s) = \frac{V_C(s)}{\frac{1}{sC}} = \frac{sC}{\left(\frac{1}{R} + \frac{1}{sL} + sC\right)}\cdot\left(I_2(s) - \frac{L\cdot i(0)}{sL} + C\cdot V_C(0)\right)$$

$$\frac{sC\cdot RsL}{sL + R + RLCs^2} = \frac{s^2RLC}{RLC\cdot\left(\frac{1}{RC}s + \frac{1}{LC} + s^2\right)}$$

$$I_C(s) = \frac{s^2}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}\cdot\left(I_2(s) - \frac{L\cdot i(0)}{sL} + C\cdot V_C(0)\right)$$

$$I_2(2) = \frac{-0.01}{s}$$

step function!

Apply initial conditions

$$\frac{s^2}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}\cdot\left(\frac{-0.01}{s} - \frac{0.25\cdot 0.01}{s\cdot 0.25} + C\cdot 0\right)$$

$$\frac{s^2}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}\cdot\left(\frac{-0.02}{s}\right)$$

$$I_C(s) = \frac{-0.02s}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$$

SKIP THIS ONE, too long

c) Find the current through the capacitor as a function of time for R1 = 12.5k. Is this circuit underdamped, overdamped or cricially damped.

$$I_C(s) = \frac{-0.02s}{s^2 + 2000s + 1\cdot 10^8}$$

$$\frac{1}{12.5\cdot 10^3 \cdot C_{14}} = 2\times 10^3 \frac{1}{F}$$

$$\frac{1}{L_{14}\cdot C_{14}} = 1\times 10^8 \frac{1}{s^2}$$

$$s^2 + 2000s + 1\cdot 10^8$$

$$\begin{pmatrix} -1000 + 3000i\cdot\sqrt{11} \\ -1000 - 3000i\cdot\sqrt{11} \end{pmatrix} \qquad 3000\cdot\sqrt{11} = 9.95\times 10^3$$

complex conjugate poles $-1000 + 9.95j\cdot 10^3$ Underdamped

$$-1000 - 9.95j\cdot 10^3$$

PFE

$$I_C(s) = \frac{-0.02s}{\left(s + 1000 - 9.995j\cdot 10^3\right)\left(s + 1000 + 9.995j\cdot 10^3\right)} = \frac{A_1}{\left(s + 1000 - 9.995j\cdot 10^3\right)} + \frac{A_2}{\left(s + 1000 + 9.995j\cdot 10^3\right)}$$

For A1

$$\frac{-0.02s\cdot\left(s + 1000 - 9.995j\cdot 10^3\right)}{\left(s + 1000 - 9.995j\cdot 10^3\right)\left(s + 1000 + 9.995j\cdot 10^3\right)}$$ for s = -1000+9.995*10^3

$$\frac{-0.02\cdot\left(-1000 + 9.995j\cdot 10^3\right)}{-1000 + 9.995j\cdot 10^3 + 1000 + 9.995j\cdot 10^3} = -0.01 - 1.001i\times 10^{-3}$$

A2 is its complex conjugate so $-0.01 + 1.001j\cdot 10^3$

$$I_C(t) = -0.01 - 1.001j\cdot 10^{-3}\cdot e^{-\left(-1000+9.995j\cdot 10^3\right)t} - 0.01 + 1.001j\cdot 10^3\cdot e^{-\left(-1000-9.995j\cdot 10^3\right)t}$$

d) Find the current through the capacitor as a function of time for R1 = 0.25k. Is this circuit underdamped, overdamped or cricially damped.

$$I_C(s) = \frac{0.02s}{s^2 + 1\cdot 10^5 s + 1\cdot 10^8}$$

$$\frac{1}{0.25\cdot 10^3\cdot C_{14}} = 1\times 10^5 \frac{1}{F}$$

$$s^2 + 1\cdot 10^5 s + 1\cdot 10^8 = 0$$

$$\frac{1}{L_{14}\cdot C_{14}} = 1\times 10^8 \frac{1}{s^2}$$

$$\begin{pmatrix} 20000\cdot\sqrt{6} - 50000 \\ -20000\cdot\sqrt{6} - 50000 \end{pmatrix}$$

$$20000\cdot\sqrt{6} - 50000 = -1.01\times 10^3$$

$$-20000\cdot\sqrt{6} - 50000 = -9.899\times 10^4$$

Overdamped

PFE

$$I_C(s) = \frac{0.02s}{\left(s + 1.01\cdot 10^3\right)\cdot\left(s + 9.899\cdot 10^4\right)} = \frac{A_1}{s + 1.01\cdot 10^3} + \frac{A_2}{\left(s + 9.89\cdot 10^4\right)}$$

For A1

$$\frac{0.02\left(-1.01\cdot 10^3\right)}{\left(-1.01\cdot 10^3 + 9.899\cdot 10^4\right)} = -2.062\times 10^{-4}$$

For A2

$$\frac{0.02\left(-9.899\cdot 10^4\right)}{\left(-9.899\cdot 10^4 + 1.01\cdot 10^3\right)} = 0.02$$

$$I_C(t) = -2.062\cdot 10^{-4}\cdot e^{-1.01\cdot 10^3 t} + 0.02\cdot e^{-9.899\cdot 10^4 t}$$