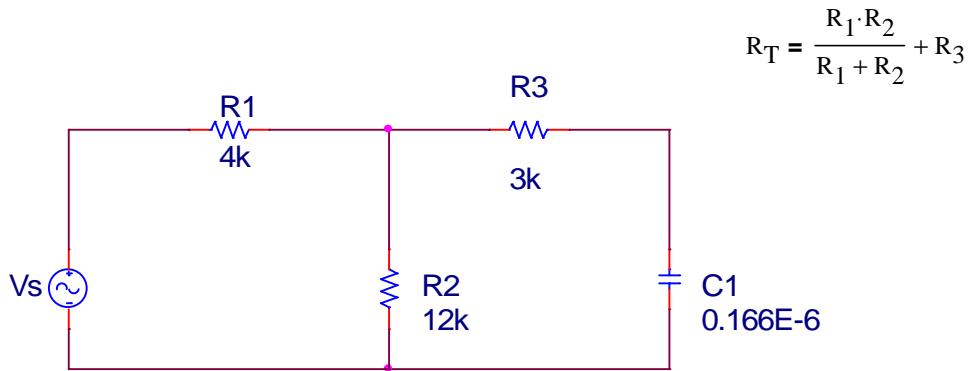


Questions:

- LEC 16: What is phasor notation?
How does phasor notation relate to measurable signals?
How do capacitors behave as a function of frequency?
How do inductors behave as a function of frequency?
What does it mean if we say an impedance is capacitive? inductive?
How do we use transfer functions to determine phasor problems?

- LEC 17: What is admittance?
What are Kirchoff's laws with phasors?
How does a transfer function give you information about a circuit's frequency response?
What is a cascading circuit and how does it make a transfer function?
What is the role of a buffer circuit?
What is a decibel (dB)?

1) Review: RC Circuits with Phasors



The source is a 5V sinusoidal signal with a frequency of 239.7Hz and has a zero phase.

$$f := 239.7\text{Hz}$$

$$\omega := 2 \cdot \pi \cdot f$$

$$\omega = 1.506 \times 10^3 \cdot \frac{\text{rad}}{\text{s}}$$

a. Determine the phasor expression for the voltage source

$$5 < 0\text{deg}$$

$$Z_{R1} := 4 \cdot 10^3 \Omega \quad Z_{R3} := 3 \cdot 10^3 \Omega$$

b. Determine the impedance seen by the source

$$Z_{EQ} = \frac{(Z_{R3} + Z_{C1}) \cdot Z_{R2}}{(Z_{R3} + Z_{C1}) + Z_{R2}} + Z_{R1}$$

$$Z_{R2} := 12 \cdot 10^3 \Omega \quad Z_{C1} := \frac{1}{\omega \cdot 0.166j \cdot 10^{-6}}$$

$$Z_{C1} = -4i \times 10^3 \Omega \quad Z_{C1} = -4 \cdot 10^3 j \Omega$$

$$Z_{EQ} := \frac{12000 \cdot (3000 - 4000j)}{12000 + 3000 - 4000j} + 4000$$

$$12000 \cdot 3000 = 3.6 \times 10^7$$

$$Z_{EQ} = 7.037 \times 10^3 - 2.39i \times 10^3$$

Long way

$$12000 \cdot -4000 = -4.8 \times 10^7$$

$$\frac{3.6 \cdot 10^7 - 4.8 \cdot 10^7 j}{15000 - 4000j} + 4000 + 0j$$

Phasor form

$$\sqrt{(7.037 \cdot 10^3)^2 + (-2.39 \cdot 10^3)^2} = 7.432 \times 10^3$$

$$\text{angle}(7.037 \cdot 10^3, -2.39 \cdot 10^3) = 341.241 \cdot \text{deg}$$

make phasors

$$\sqrt{(3.6 \cdot 10^7)^2 + (-4.8 \cdot 10^7)^2} = 6 \times 10^7$$

$$\text{atan}\left(\frac{-4.8 \cdot 10^7}{3.6 \cdot 10^7}\right) = -53.13 \cdot \text{deg}$$

$$\text{atan}\left(\frac{-2.39 \cdot 10^3}{7.07 \cdot 10^3}\right) = -18.678 \cdot \text{deg}$$

$$\text{note: quadrant 4 } 360 - 18.7 = 341.3$$

$$\sqrt{(15000)^2 + (-4000)^2} = 1.552 \times 10^4$$

$$7432 < -18.7 \text{deg}$$

$$\text{atan}\left(\frac{-4000}{15000}\right) = -14.931 \cdot \text{deg}$$

$$\frac{6 \cdot 10^7}{1.552 \cdot 10^4} = 3.866 \times 10^3$$

$$\frac{6 \cdot 10^7 < -53.13 \text{deg}}{1.55 \cdot 1064 < -14.93 \text{deg}} + 4000$$

$$-53.13 + 14.931 = -38.199$$

$$3.866 \cdot 10^3 < -38.199 + 4000 \quad \text{convert phasor back to rectangular}$$

$$3.866 \cdot 10^3 \cdot \cos(-38.199 \text{deg}) = 3.038 \times 10^3$$

$$3.866 \cdot 10^3 \cdot \sin(-38.199 \text{deg}) = -2.391 \times 10^3$$

$$3.038 \cdot 10^3 - 2.391 \cdot 10^3 j + 4000 + 0j$$

$$3.038 \cdot 10^3 + 4000 = 7.038 \times 10^3$$

$$7.038 \cdot 10^3 - 2.391 \cdot 10^3 j$$

$$\sqrt{(7.038 \cdot 10^3)^2 + (-2.391 \cdot 10^3)^2} = 7.433 \times 10^3$$

$$\text{atan}\left(\frac{-2.391 \cdot 10^3}{7.038 \cdot 10^3}\right) = -18.764 \cdot \text{deg}$$

$$7.433 \cdot 10^3 < -18.7 \text{deg}$$

c. Determine the phasor expression for the current through the source.

$$I_s = \frac{V_s}{Z_{EQ}} = \frac{5 < 0 \text{deg}}{7432 < -18.76 \text{deg}} = (6.73 \cdot 10^{-4} < 18.76 \text{deg}) \text{A}$$

$$\frac{5}{7423} = 6.736 \times 10^{-4}$$

$$6.73 \cdot 10^{-4} \cdot \cos(18.76 \text{deg}) = 6.372 \times 10^{-4}$$

$$6.73 \cdot 10^{-4} \cdot \sin(18.76 \text{deg}) = 2.164 \times 10^{-4}$$

d. Determine the phasor expression for the voltage across resistor R2.
Determine the phasor expression for the voltage across C1.

TEAM ASSIGNMENT!

VR2 strategy, find current through R2 and multiply by ZR2

Using the current divider $Z_{R3} = 3 \times 10^3 \Omega$

$$\frac{(Z_{R3} + Z_{C1}) \cdot Z_{R2}}{Z_{R3} + Z_{C1} + Z_{R2}} \cdot I_s$$

$$Z_{C1} = -4i \times 10^3 \text{ s}$$

$$Z_{R2} = 1.2 \times 10^4 \Omega$$

Easy way

$$\frac{(3000 - 4000j) \cdot 1.2 \cdot 10^4}{3000 - 4000j + 1.2 \cdot 10^4} \cdot (6.372 \times 10^{-4} + 2.164j \times 10^{-4}) = 2.453 - 0.866i$$

$$\sqrt{(2.453)^2 + (-0.866)^2} = 2.601$$

$$\text{atan}\left(\frac{-0.866}{2.453}\right) = -19.445 \cdot \text{deg}$$

$$2.6 < -19.445$$

Hard way

$$\frac{(3000 - 4000j) \cdot 12 \cdot 10^4}{3000 - 4000j + 12 \cdot 10^4} \cdot \left(6.73 \cdot 10^{-4} < 18.76\text{deg} \right)$$

put first part in phasor form then multiply phasors

$$\frac{3.6 \cdot 10^7 - 4.8 \cdot 10^7 j}{15000 - 4000j} \quad \text{from above}$$

$$\frac{6 \cdot 10^7 < -53.13\text{deg}}{1.55 \cdot 1064 < -14.93\text{deg}}$$

$$(3.866 \cdot 10^3 < -38.199) \cdot (6.73 \cdot 10^{-4} < 18.76\text{deg})$$

$$3.866 \cdot 10^3 \cdot 6.73 \cdot 10^{-4} = 2.602$$

$$-38.2 + 18.76 = -19.44$$

$$2.6 < -19.44$$

VC1 strategy: use VR2 and voltage divider

$$V_{C1} = \frac{-Z_{C1}}{Z_{R3} + Z_{C1}} \cdot V_{R2}$$

$$\frac{-4000j}{3000 - 4000j} \cdot (2.6 < -19.44\text{deg})$$

can do this a couple of ways

WAY 1

$$2.6 \cdot \cos(-19.44\text{deg}) = 2.452$$

$$2.6 \cdot \sin(-19.44\text{deg}) = -0.865$$

$$(2.452 - 0.865j) \cdot \frac{-4000j}{3000 - 4000j} = 1.154 - 1.731i$$

$$\sqrt{1.154^2 + (-1.731)^2} = 2.08$$

$$\tan\left(\frac{-1.731}{1.154}\right) = -56.31\text{deg}$$

$$2.08 < -56.31\text{deg}$$

$$\sqrt{3000^2 + (-4000)^2} = 5 \times 10^3$$

$$\tan\left(\frac{-4000}{3000}\right) = -53.13\text{deg}$$

WAY 2

$$\frac{4000 < -90\text{deg}}{5000 < -53.13\text{deg}} \cdot (2.6 < -19.44\text{deg})$$

$$(0.8 < -36.87\text{deg}) \cdot 2.6 < -19.44\text{deg}$$

$$-36.87 - 19.44 = -56.31$$

$$2.08 < -56.31\text{deg}$$

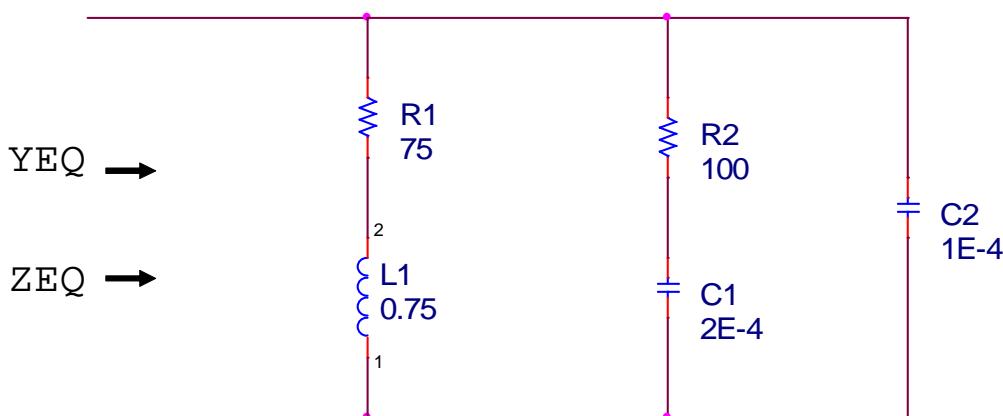
e. Determine the time domain expression for the voltage across C1.

$$V_{C1} = V_A \cdot \cos(\omega t + \phi)$$

$$V_{C1} = 2.08 \cdot \cos(1500t - 56.3\text{deg})$$

2. Admittance

Spring 2014 Homework



- a. Determine the equivalent admittance, Y_{EQ} , for the above circuit for a frequency of 15.9 Hz. Determine the equivalent impedance, Z_{EQ} .

$$Y_{EQ} = Y_1 + Y_2 + Y_3 = \frac{1}{Z_{R1} + Z_{L1}} + \frac{1}{Z_{R2} + Z_{C1}} + \frac{1}{Z_{C2}}$$

$$\omega_2 = 2 \cdot \pi \cdot 15.9\text{Hz}$$

$$\omega_2 := 100 \frac{\text{rad}}{\text{s}}$$

$$Z_{R1} := 75\Omega$$

$$Z_{L1} := 75j\Omega$$

$$Z_{C1} := -50j\Omega$$

$$Z_{R2} := 100\Omega$$

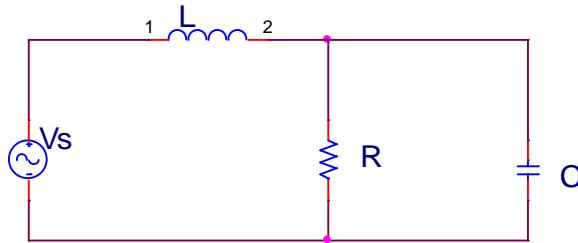
$$Z_{C2} := -100j\Omega$$

$$Y_{EQ} = \frac{1}{75 + j75} + \frac{1}{100 - j50} + \frac{1}{-j100}$$

$$\frac{1}{75 + 75j} + \frac{1}{100 - 50j} + \frac{1}{-100j} = 0.015 + 7.333i \times 10^{-3} \quad \text{mhos}$$

$$\sqrt{0.015^2 + (7.33 \cdot 10^{-3})^2} = 0.017 \quad \text{atan}\left(\frac{7.33 \cdot 10^{-3}}{0.015}\right) = 26.043\text{-deg} \quad 0.017 < 26\text{deg}$$

3. Transfer functions



Determine the transfer functions in the following circuit. Determine the behavior of the transfer function as ω goes to 0 and ω goes to ∞ .

a. $H(s) = \frac{V_C(s)}{V_s(s)}$ voltage across C relative to the source voltage

Us

$$H(s) = \frac{V_C(s)}{V_s(s)} = \frac{\frac{Z_R \cdot Z_C}{Z_R + Z_C}}{\frac{Z_L + \frac{Z_R \cdot Z_C}{Z_R + Z_C}}{sL + \frac{R}{sC}}} = \frac{\frac{R}{sC}}{\frac{R + \frac{1}{sC}}{sL + \frac{R}{sC}}} \quad \text{getting } sC \text{ out of denominator}$$

$$\frac{\frac{R}{sC}}{sCR+1}$$

note sC cancels

$$\frac{\frac{R}{sCR+1}}{sL + \frac{R}{sCR+1}}$$

getting $sCR+1$ out of the denominator

$$\frac{\frac{R}{sCR+1}}{\left(\frac{s^2 RLC + sL + R}{sCR+1} \right)}$$

$(sCR + 1)$ cancels

$$\frac{R}{s^2 \cdot RLC + sL + R}$$

divide by RLC

$$H(s) = \frac{\frac{1}{LC}}{s^2 + s \cdot \frac{1}{RC} + \frac{1}{LC}}$$

b. Determine the magnitude of the transfer function as frequency approaches zero.

Keeping the dominant terms

$$H(s) = \frac{\frac{1}{LC}}{\frac{1}{LC}} = 1$$

consistent with a capacitor becoming an open circuit and inductor becoming a short circuit

c. Determine the magnitude of the transfer function as frequency approaches infinity.

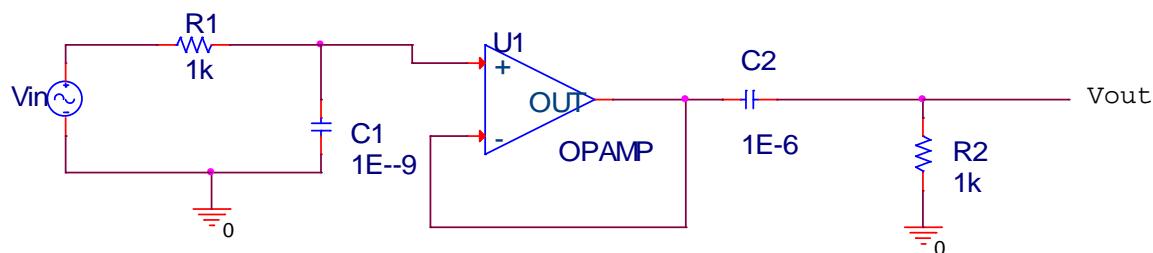
Keeping the dominant terms

$$H(s) = \frac{\frac{1}{LC}}{s^2} = 0$$

consistent with capacitor becoming a short and inductor becoming an open circuit

Spring 2012 HW

4. Transfer functions-multiple stages



- a. How many stages are present in the above circuit.

Three stages

- b. Determine the transfer function, $H(s) = \frac{V_{out}(s)}{V_{in}(s)}$

$$H_1(s) = \frac{\frac{1}{sC1}}{R1 + \frac{1}{sC1}} = \frac{1}{R1C1} \cdot \frac{1}{s + \frac{1}{R1C1}}$$

Voltage divider if point above C1 is Vout

$$H_2(s) = 1$$

Buffer (voltage follower)

$$H_3(s) = \frac{R2}{R2 + \frac{1}{sC2}} = \frac{s}{s + \frac{1}{R2C2}}$$

$$H(s) = H_1(s)H_2(s)H_3(s) = \left(\frac{\frac{1}{R1C1}}{s + \frac{1}{R1C1}} \right) (1) \left(\frac{s}{s + \frac{1}{R2C2}} \right)$$

$$H(s) = \frac{s1E6}{(s+1E6)(s+1E3)}$$

$$H_3(s) = \frac{R2}{R2 + \frac{1}{sC2}} = \frac{s}{s + \frac{1}{R2C2}}$$

Voltage divider again

$$H(s) = H_1(s)H_2(s)H_3(s) = \left(\frac{1}{s + \frac{1}{R1C1}} \right) (1) \left(\frac{s}{s + \frac{1}{R2C2}} \right)$$

$$H(s) = \frac{s1E6}{(s + 1E6)(s + 1E3)}$$

