

Questions:

Circuit Analysis: Phasor Math

What is admittance?

How do bridge circuits behave with complex impedances?

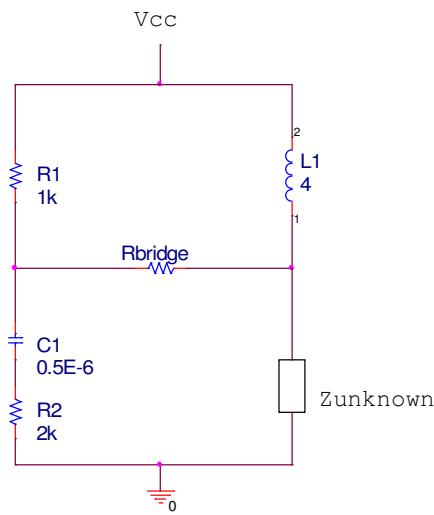
How does the total impedance of a series RLC circuit behave with frequency?

How does the total impedance of a parallel RLC circuit behave with frequency?

Complex Power

- What is instantaneous power?
- What is real power?
- What is reactive power?
- What is total power?
- What is rms voltage? rms current?
- How do we determine total power produced by a source?
- What is the power factor?
- What does it mean if the power factor is ‘leading’? ‘lagging’?

1) Bridge circuits



a) What value of Zunknown balances the bridge (no current through R3) when $\omega=1000 \text{ rad/s}$?

$$C_1 := 0.5 \cdot 10^{-6} \text{ F} \quad R_1 := 1\text{k}\Omega \quad R_2 := 2\text{k}\Omega \quad L_1 := 4\text{H}$$

$$Z_u = \frac{(Z_{C1} + Z_{R2}) \cdot Z_{L1}}{Z_{R1}}$$

$$\omega_1 := 1000 \frac{\text{rad}}{\text{s}}$$

$$\frac{\left(\frac{1}{j\omega C_1} + R_2 \right) \cdot j\omega L_1}{R_1}$$

$$\begin{aligned} (\omega_1 \cdot C_1)^{-1} &= 2 \times 10^3 \Omega & \omega_1 \cdot L_1 &= 4 \times 10^3 \Omega \\ \frac{(-2j \cdot 10^3 + 2 \cdot 10^3) \cdot 4j \cdot 10^3}{1 \times 10^3} &= 8 \times 10^3 + 8i \times 10^3 & 8k + 8kj \end{aligned}$$

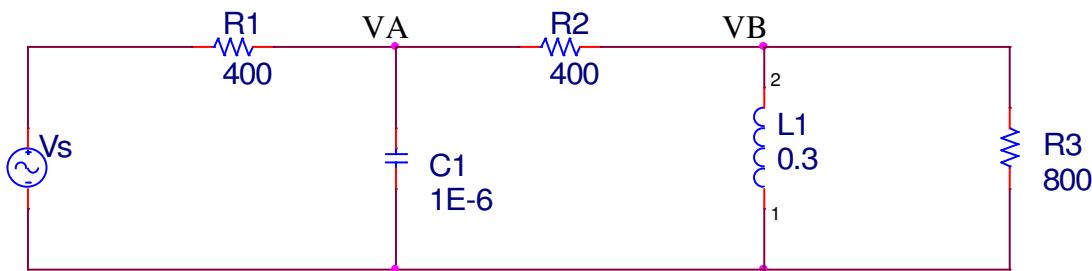
b) What component combination would balance the bridge?

An 8k resistor and 8 H inductor

2) Phasor KCL/KVL

The source in the above circuit is $20\cos(2000t)$

$$\begin{aligned} Z_{R1} &:= 400\Omega & Z_{R2} &:= 400\Omega & Z_{R3} &:= 800\Omega & Z_{C1} &:= -500j\Omega & Z_{L1} &:= 600j\Omega \\ Z_{C1} &= \frac{-j}{2000 \cdot 1 \cdot 10^{-6}} & Z_{L1} &= j \cdot 2000 \cdot 0.3 \end{aligned}$$



a. Determine the voltage across the capacitor using node analysis.

Find VA

At VA:

$$\frac{V_A - V_s}{Z_{R1}} + \frac{V_A}{Z_{C1}} + \frac{V_A - V_B}{Z_{R2}} = 0$$

$$\frac{V_A}{400} + \frac{V_A}{-500j} + \frac{V_A}{400} - \frac{V_B}{400} = \frac{20}{400}$$

$$(1) \quad V_A \left(\frac{1}{400} + \frac{1}{-j \cdot 500} + \frac{1}{400} \right) + V_B \left(\frac{-1}{400} \right) = \frac{20}{400}$$

At VB:

$$\frac{V_B - V_A}{Z_{R2}} + \frac{V_B}{Z_{L1}} + \frac{V_B}{Z_{R3}} = 0$$

$$(2) \quad V_A \left(\frac{-1}{400} \right) + V_B \left(\frac{1}{400} + \frac{1}{j600} + \frac{1}{800} \right) = 0$$

$$M := \begin{bmatrix} \left(\frac{1}{400} + \frac{-1}{500j} + \frac{1}{400} \right) & \left(\frac{-1}{400} \right) \\ \left(\frac{-1}{400} \right) & \left(\frac{1}{400} + \frac{1}{600j} + \frac{1}{800} \right) \end{bmatrix}$$

$$C_2 := \begin{pmatrix} \frac{20}{400} \\ 0 \end{pmatrix}$$

$$M^{-1} \cdot C_2 = \begin{pmatrix} 12.086 - 4.627j \\ 7.873 + 0.414j \end{pmatrix} \quad V_A = 12.086 - 4.627j$$

$$\sqrt{12.086^2 + (-4.627)^2} = 12.941$$

$$\text{atan}\left(\frac{-4.627}{12.086}\right) = -20.949 \cdot \text{deg}$$

$$V_A = 12.94 < -20.95 \text{deg}$$

b. Determine the voltage across the capacitor using mesh analysis.

$$\text{Loop 1: } Z_{R1} \cdot i_1 + Z_{C1} \cdot (i_1 - i_2) - V_S = 0$$

$$(400 - 500j) \cdot i_1 + 500j \cdot i_2 = 20$$

$$\text{Loop 2: } Z_{R2} \cdot i_2 + Z_{L1} \cdot (i_2 - i_3) + Z_{C1} \cdot (i_2 - i_1) = 0$$

$$500j \cdot i_1 + [400 + (600j - 500j)] \cdot i_2 - 600j \cdot i_3 = 0$$

$$\text{Loop 3: } Z_{R3} \cdot i_3 + Z_{L1} \cdot (i_3 - i_2) = 0$$

$$-600j \cdot i_2 + (800 + 600j) \cdot i_3 = 0$$

$$M_2 := \begin{bmatrix} 400 - 500j & 500j & 0 \\ 500j & 400 + 100j & -600j \\ 0 & -600j & (800 + 600j) \end{bmatrix}$$

$$C_3 := \begin{pmatrix} 20 \\ 0 \\ 0 \end{pmatrix}$$

$$M_2^{-1} \cdot C_3 = \begin{pmatrix} 0.0198 + 0.0116i \\ 0.0105 - 0.0126i \\ 0.0098 + 0.0005i \end{pmatrix} \quad i_1 := 0.0198 + 0.0116i$$
$$i_2 := 0.0105 - 0.0126i$$

$$V_{C1} := Z_{2C1} \cdot (i_1 - i_2)$$

$$V_{C1} = (12.1 - 4.65i) \Omega$$

$$\sqrt{(12.1)^2 + (-4.65)^2} = 12.963$$

$$\text{atan}\left(\frac{-4.65}{12.1}\right) = -21.022 \cdot \text{deg}$$