Questions:

a. What is instantaneous power?
b. What is real power?
c. What is reactive power?
d. What is total power?
e. What is rms voltage? rms current?
f. How do we determine total power produced by a source?
g. What is the power factor?
h. What does it mean if the power factor is ‘leading’? ‘lagging’?

Problem 1)

At 440 V (rms) a two-terminal load draws 3 kVA of apparent power at a lagging power factor of 0.9. Find the following:

a. Irms
b. P
c. Q
d. the load impedance

Draw the power triangle or the load.

a. $I_{rms} := \frac{|S_A|}{V_{rms}}$　　$V_{rms} := 440V$　　$S_A := 3000V\cdot A$

note: apparent power is magnitude of S

$I_{rms} = 6.818\,A$ (rms)

b. $P := V_{rms}\cdot I_{rms}\cdot \cos\theta$　　$\cos\theta := 0.9$

remember cosθ = pf since real power/apparent power

$P = 2.7\cdot kW$

c. For cos θ = 0.9 lagging

Need to find sinθ for Q

Lagging means θ>0 , first quadrant

$\mathrm{acos}(0.9) = 0.451$

$\sin(0.451) = 0.436$

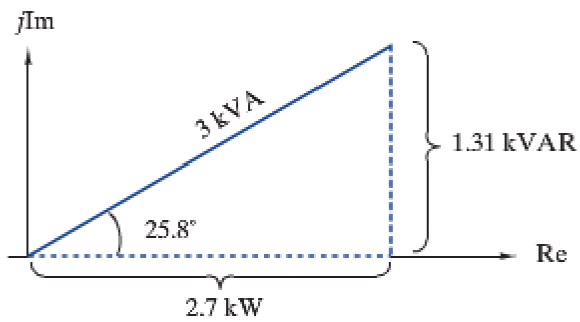
$Q := V_{rms}\cdot I_{rms}\cdot 0.436$

$Q = 1.308 \times 10^{3}$ [VAR]

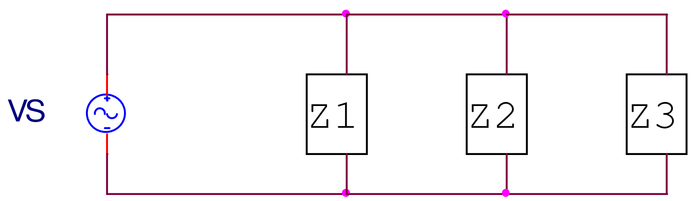
d. Load impedance

$Z_L = \frac{P + jQ}{I_{rms}^{2}}$　　$Z_L := \frac{(2700 + 1310i)W}{I_{rms}^{2}}$　　$Z_L = (58.08 + 28.18i)\,\Omega$

Inductive load proven: lagging



Problem 2)



In the above circuit, the total source power, S, is 60kVA (magnitude) with a 60 Hz, 3kV$_{RMS}$ source voltage. The power factor for the entire parallel load is 0.9 (90%). The loads are described as:

Z1: Purely resistive heating element, 300 Ω with a current of 10A RMS.
Z2: Induction motor with small real loos, R=20Ω, L=0.79 H
Z3: Unknown load

Load 1: Purely resistive heating element 300 Ω with a current of 10A RMS

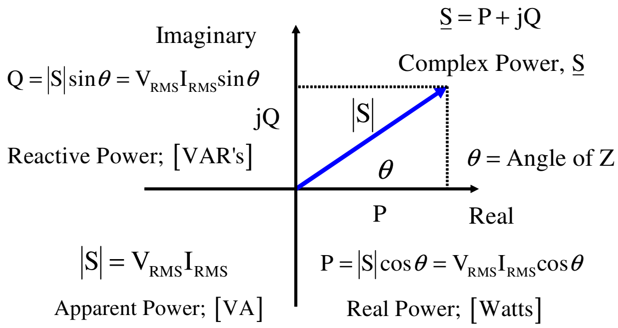
$I_{RMS} = 10$

$Z_1 = 300 = 300\angle 0°$ **note: θ is 0 degrees because it is purely resistive. There is not j or imaginary part in for a purely resistive load.**

$S_1 = I_{RMS}^2 \cdot |Z_{primary}| \cdot \cos\theta_{Zprimary} + j(I_{RMS})^2 \cdot |Z_{primary}| \sin\theta_{Zprimary}$

$S_1 = 10^2 \cdot |300| \cdot \cos(0°) + j \cdot 10^2 \cdot |300| \cdot \sin(0°)$

$S_1 = 30000 + j0$



S1 is the total source complex power for load 1. Also note that S=P + jQ or Complex power =Real power + Reactive power.

Absolute value of S is the magnitude of complex power called Apparent Power.

Power factor is Real Power/Apparent Power
pf = P/abs(S)

$P_1 = 30000W$ $\quad$ $Q_1 = 0VAR$ $\quad$ $|S_1| = 30000$ $\quad$ $pf = \frac{P_1}{|S_1|} = 1$

Load 2: Induction Motor with small real loss, R=20Ω, L=0.79 H

$Z_2 = Z_R + Z_{IND}$ $\quad$ Now there is a real AND imaginary part to the load.

$Z_R = 20\Omega$ $\quad$ $Z_{IND} := 377 \cdot 0.79j$

$Z_{IND} = 297.83i$ $\quad$ Mathcad turns j into i

$Z_2 = 20 + j298 = 298.5\angle 86.1° = |Z_{EQ}| \cdot \angle\theta$

$V_{RMS} = 3000$ $\quad$ Given

$|Z_{EQ}| = 298.5$ $\quad$ $\theta = 86.1°$

$$S_2 = \frac{V_{RMS}^{\ 2}}{|Z_{EQ}|} \cdot \cos\theta_{ZEQ} + j\,\frac{V_{RMS}^{\ 2}}{|Z_{EQ}|} \cdot \sin\theta_{ZEQ}$$

$$S_2 = \frac{(3000^2)}{|298.5|} \cdot \cos(86.1°) + j \cdot \left[\frac{(3000^2)}{|298.5|} \cdot \sin(86.1°)\right]$$

$S_2 = 2000 + j30000$

$P_2 = 2000W$ $\quad$ $Q_2 = 30000VAR$ $\quad$ $|S_2| = 30067$ $\quad$ $pf = \frac{P_1}{|S_1|} = 0.067$

$$\frac{2000}{30067} = 0.067$$

$$\sqrt{30000^2 + 2000^2} = 3.00666 \times 10^4$$

To get the unknown load we need the total power, then we can subtract S1 and S2 from it.

$S_{tot} = |S_{tot}| \cdot \cos(\theta) + j \cdot |S_{tot}| \cdot \sin\theta$

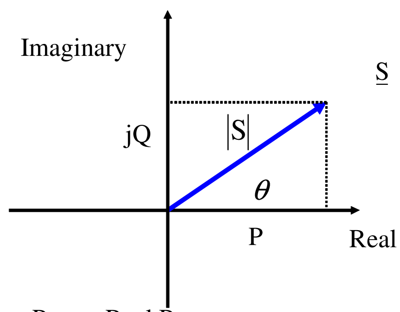
Remember: cos θ= power factor, and we are given the total power factor

cos θ = 0.9

$\theta_{tot} = 25.8°$ $\quad |S_{tot}| = 60000$ Given

$S_{tot} = |60000| \cdot (0.9) + j \cdot |60000| \sin(25.8°)$

$S_{tot} = 54000 + j26153$



$$\cos\theta = \frac{P}{|S|} = \frac{\text{Real Power}}{\text{Apparent Power}} = \text{Power Factor} = \text{pf}$$

$P_{tot} = 54000W$ $\quad Q_{tot} = 26153VAR$ $\quad |S_{tot}| = 60000$ $\quad pf_{tot} = 0.9$

Load 3:

$S_3 = S_{tot} - S_2 - S_3$

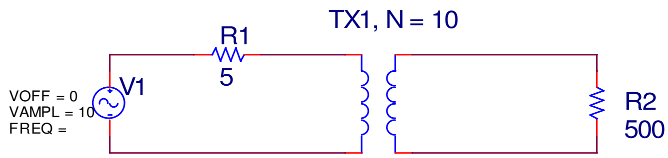
$S_3 = 22000 - j3847$

$P_3 = 22000W$ $\quad Q_3 = -3847VAR$ $\quad |S_3| = 22334$ $\quad pf_3 = 0.9851$

Summary chart

Phase Voltage	P[W]	Q [VAR]	S	lSl [VA]	pf
Load 1	30000	0	30000	30000	1
Load 2	2000	30000	2000+j30000	300067	0.067
Load 3	22000	-3847	22000-j3847	22334	0.9851
Source	54000	26153	54000+j26153	60000	0.9

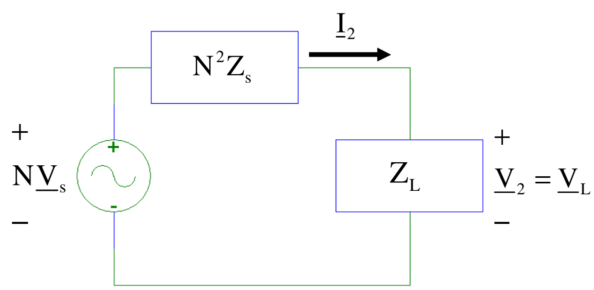
Problem 3)



Determine VR2.

You can do this two ways: Referral to secondary and referal to primary (check to see if they are equivalent)

#1 Refer primary to secondary



$V_{source} = 10 \cdot 10 < 0deg = 100 < 0deg$

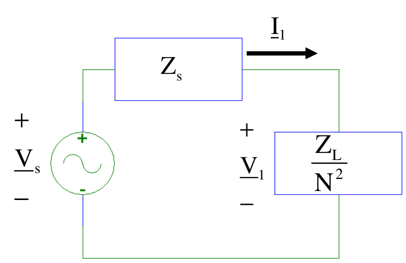
$R_S = 10^2 \cdot 5 = 500$

$R_L = 500$

Voltage divider $V_{R2} = \frac{500}{500 + 500} \cdot 100 < 0deg = 50 < 0deg$

This is V2!!!!

#2 Refer secondary to primary



$V_S = 10 < 0deg$

$Z_S = R_1 = 5$

$Z_{Leq} = \frac{Z_L}{N^2} = \frac{500}{10^2} = 5$

Voltage divider

$V_{RLeq} = \frac{5}{5 + 5} \cdot 10 < 0$

$V_{RLeq} = 5 < 0$

This is V1! So must use $V_2 = N \cdot V_1$ $V_2 = 10 \cdot 5 < 0$ so they are equivalent

Finding voltage across the load, refer primary to secondary.....