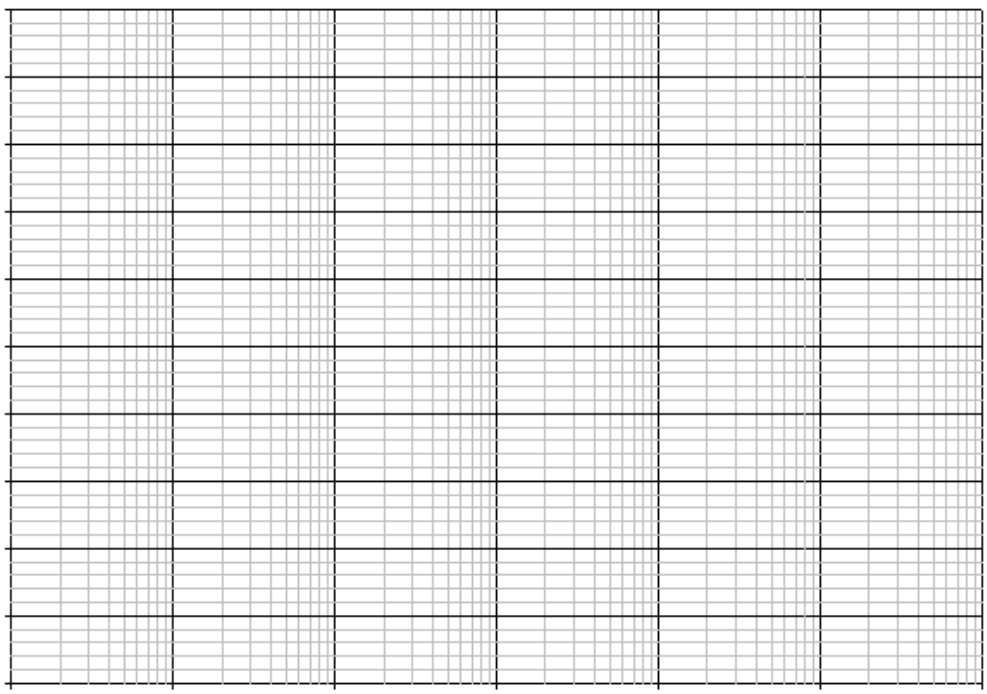
a. What is a high pass filter?
b. What is a low pass filter?
c. What is the cutoff frequency for a first order (RL or RC) circuit?
d. What is a Bode magnitude plot?
e. What is a Bode phase plot?
f. What is a 3dB point?
g. What is rolloff?
h. What does 0dB imply for a transfer function?
i. What is a bandpass filter? a bandstop filter?
j. Can we build a bandpass filter or bandstop filter with a first order circuit?
k. What is gain, K?

1) Plot the Bode plots for the following functions

a) $H(s) = \dfrac{1000}{(s+1000)}$



$H(j\omega) = \dfrac{1000}{j\omega + 1000}$ Three ways to do this. (2 in H(jw) 1 using H(s))

1. Using H(jω) and ratio comparison

factor out 1000 $\dfrac{1}{1 + \dfrac{j\omega}{1000}}$ $\omega_c = 1000$ pole: 1000
zeros: none

Look at what happens before, after and at corner frequency ranges

ω<<<1000 $H(j\omega) = \dfrac{1}{1 + 0} = 1$ $|H(j\omega)| = 1$ $20\cdot\log(1) = 0$

ω>>>1000 $H(j\omega) = \dfrac{1}{\omega j}$ $H(j\omega) \propto \dfrac{1}{\omega^n}$ $-n\cdot 20\,\dfrac{db}{dec}$ slope $n = 1$ $-20\,\dfrac{db}{dec}$

$\omega = \omega_c$ $H(j\omega) = \dfrac{1}{1 + \dfrac{1000j}{1000}} = \dfrac{1}{1 + j}$ $|H(j\omega)| = \dfrac{1}{\sqrt{2}}$ $20\log\left(\dfrac{1}{\sqrt{2}}\right) = -3.01$ this is the correction

2. Using H(jω) directly

Look at what happens before, after and at corner frequency ranges

ω<<<1000 $H(j\omega) = \dfrac{1000}{1000} = 1$ $|H(j\omega)| = 1$ $20\cdot\log(1) = 0$

ω>>>1000 $H(j\omega) = \dfrac{1000}{\omega j}$ $H(j\omega) \propto \dfrac{1}{\omega^n}$ $-n\cdot 20\,\dfrac{db}{dec}$ slope $n = 1$ $-20\,\dfrac{db}{dec}$

$\omega = \omega_c$ $H(j\omega) = \dfrac{1000}{1000j + 1000}$ $|H(j\omega)| = \dfrac{1}{\sqrt{2}}$ $20\log\left(\dfrac{1}{\sqrt{2}}\right) = -3.01$ this is the correction

$$\frac{1000}{\sqrt{1000^2 + 1000^2}} = 0.707$$

2. Using H(s) directly

Look at what happens before, after and at corner frequency ranges

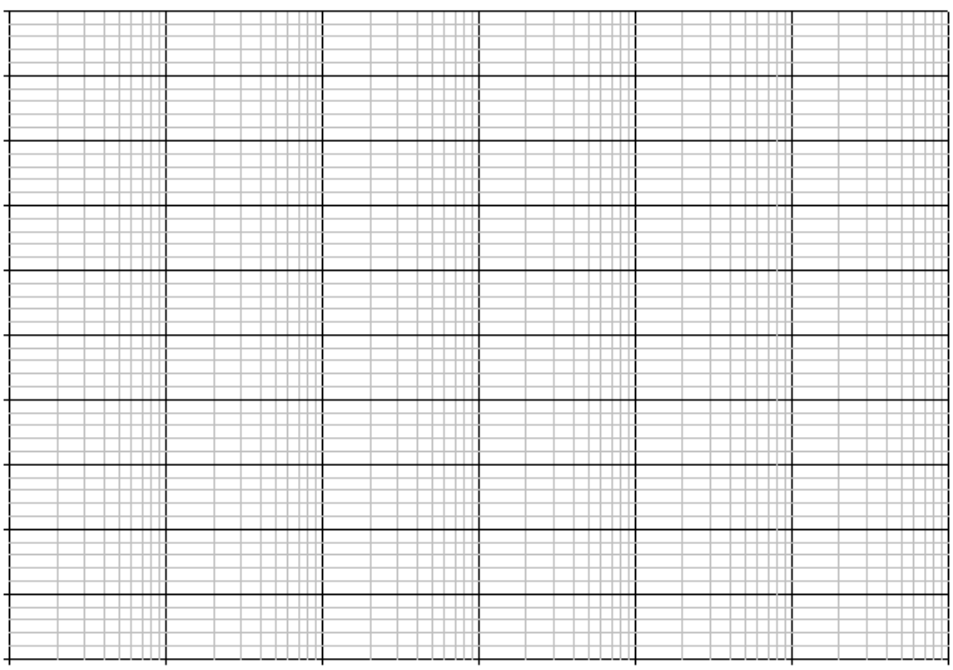
$$H(s) = \frac{1000}{s + 1000}$$

s<<<1000 $\quad \frac{1000}{1000} = 1 \quad |H(j\omega)| = 1 \quad 20\cdot\log(1) = 0$

s>>>1000 $\quad \frac{1000}{s} \quad H(s) \propto \frac{1}{s^n} \quad -n\cdot 20\,\frac{db}{dec}$ slope $\quad n = 1 \quad -20\,\frac{db}{dec}$

$s = \omega_c$ we know there is a -3db correction here

Phase Bode Plot for part a)



$\omega_c := 1000$

pole:
1000

$0.1(\omega_c) = 100$

$10(\omega_c) = 1 \times 10^4$

Slope changes occur here for a total of -90deg

$(\angle)0 - \angle 0.057\text{deg}$

$\angle H(j1) = 0\text{deg}$

$\angle H(j100) = 0\text{deg}$

$\angle H(j1000) = -45\text{deg}$

$\angle H(j10000) = -90\text{deg}$

for ω=1 $\dfrac{1}{1 + \dfrac{j1}{1000}}$ $\quad \text{atan}\left(\dfrac{-0.001}{1}\right) = -0.057 \cdot \text{deg}$

for ω=100 $\dfrac{1}{1 + \dfrac{j100}{1000}}$ $\quad \text{atan}\left(\dfrac{-0.01}{1}\right) = -0.573 \cdot \text{deg}$

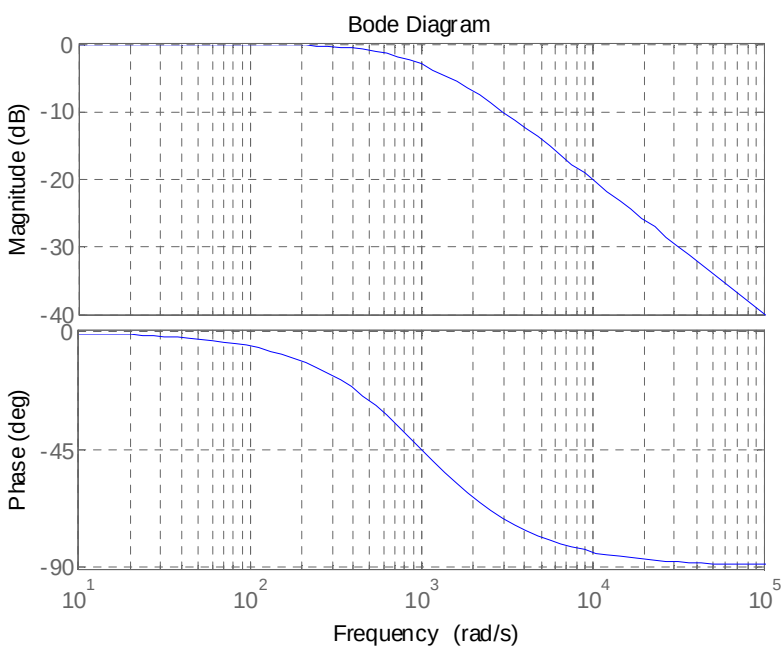
for ω=1000 $\dfrac{1}{1 + \dfrac{j1000}{1000}}$ $\quad \text{atan}\left(\dfrac{-1}{1}\right) = -45 \cdot \text{deg}$

for ω=10000 $\dfrac{1}{1 + \dfrac{j10000}{1000}}$ $\quad \text{atan}\left(\dfrac{-10}{1}\right) = -84.289 \cdot \text{deg}$

Mattlab check

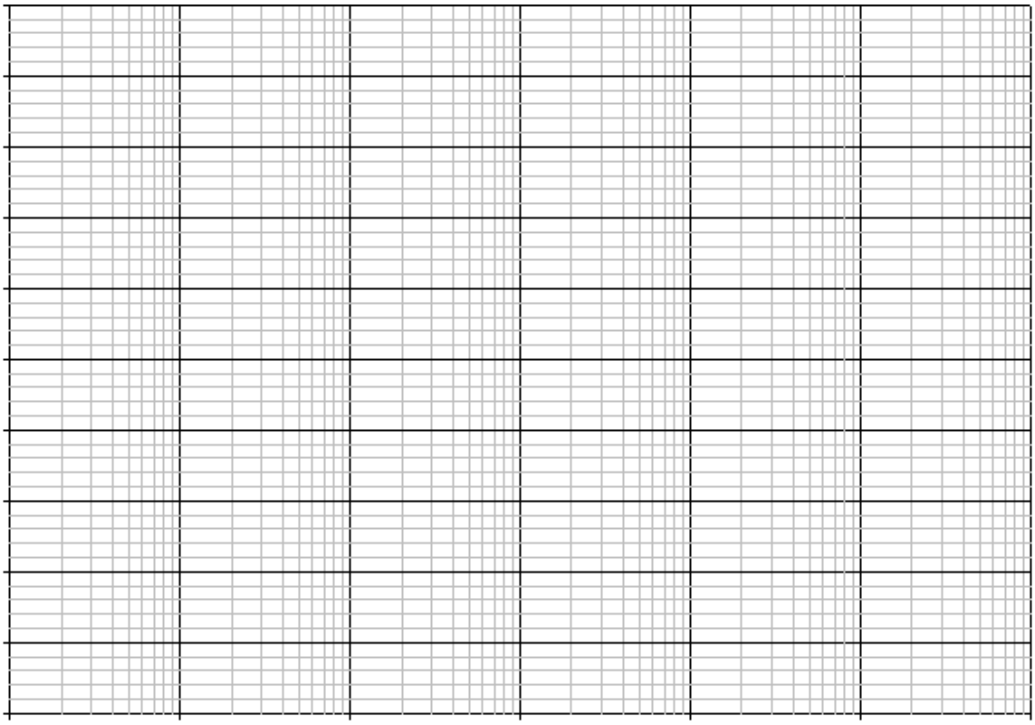
Students, you can check all Bode plot and transfer function with two lines of code in MATLAB!

```
H=tf([1000],[1 1000]);
bode(H);grid
```



b)

$$H(s) = \frac{1000^2}{(s+1000)^2}$$



$$H(j\omega) = \frac{1\cdot 10^6}{\left(j\omega + 1\cdot 10^3\right)^2}$$

pole: 1000
double
zeros: none

when ω is less than ωc, then ωc value dominates
when ω is great than ωc then ω dominates
when ω is = to ωc then it becomes 1/squareroot2

ω<<<<1000

$$H(j\omega) = \frac{1\cdot 10^6}{\left(1\cdot 10^3\right)^2} = 1 \qquad \left|H(j\omega)\right| = 1 \qquad 20\cdot\log(1) = 0$$

ω>>>>1000

$$H(j\omega) = \frac{1\cdot 10^6}{\omega^2} \qquad H(j\omega) \propto \frac{1}{\omega^n} \qquad -n\cdot 20\,\frac{db}{dec} \qquad -2\cdot 20 = -40 \quad \frac{db}{dec}$$

$\omega = \omega_c$

$$H(j\omega) = \frac{1\cdot 10^6}{\left(j1\cdot 10^3 + 1\cdot 10^3\right)^2} \qquad \left|H(j\omega)\right| = \frac{1\cdot 10^6}{\left[\sqrt{\left(1\cdot 10^3\right)^2 + \left(1\cdot 10^3\right)^2}\right]^2}$$

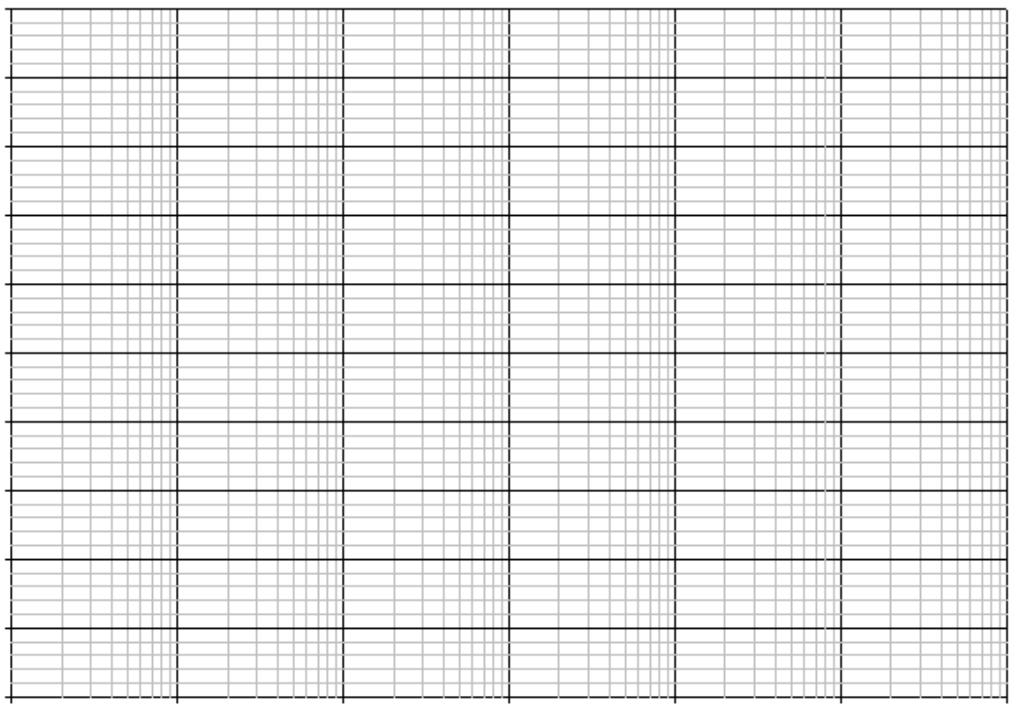
$$\frac{1\cdot 10^6}{\left(\sqrt{2\cdot 10^6}\right)^2} \quad \alpha \quad \frac{1}{\left(\sqrt{2}\right)^2}$$

$$-n\cdot 3db$$

$$20\log\left(\frac{1\cdot 10^6}{2\cdot 10^6}\right) = -6.021 \qquad 20\log\left(\frac{1}{2}\right) = -6.021$$

Double pole gives you a -6dB attenuation which is 2*-3db!

Phase Bode Plot for part b)



$$H(j\omega) = \frac{1\cdot 10^6}{\left(j\omega + 1\cdot 10^3\right)^2}$$

Remember in polar form what we can do with angles

$$\angle H(j1) = 0deg$$

$$\angle H(j100) = 0deg$$

$$\angle H(j1000) = -45deg$$

$$\angle H(j10000) = -90deg$$

for ω=1 $\frac{1\cdot 10^6}{\left(j1 + 1\cdot 10^3\right)^2}$ $\quad \operatorname{atan}\left(\frac{0}{1\cdot 10^6}\right) = 0 \quad \operatorname{atan}\left(\frac{1}{1\cdot 10^3}\right) = 0.057\cdot deg$ $\quad (\angle)0 - (\angle 0.057deg)2 = 0deg$

for ω=100 $\frac{1\cdot 10^6}{\left(j100 + 1\cdot 10^3\right)^2}$ $\quad (\angle)0 - (\angle 0.573deg)2 = 0deg$

for ω=1000 $\frac{1\cdot 10^6}{\left(j1000 + 1\cdot 10^3\right)^2}$ $\quad (\angle)0 - (\angle 45deg)2 = -90deg$

for ω=10000 $\dfrac{1\cdot 10^{6}}{\left(j10000 + 1\cdot 10^{3}\right)^{2}}$ $\operatorname{atan}\left(\dfrac{10000}{1000}\right) = 84.289\cdot \mathrm{deg}$

$(\angle)0 - (\angle 90\mathrm{deg})2 = -180\mathrm{deg}$

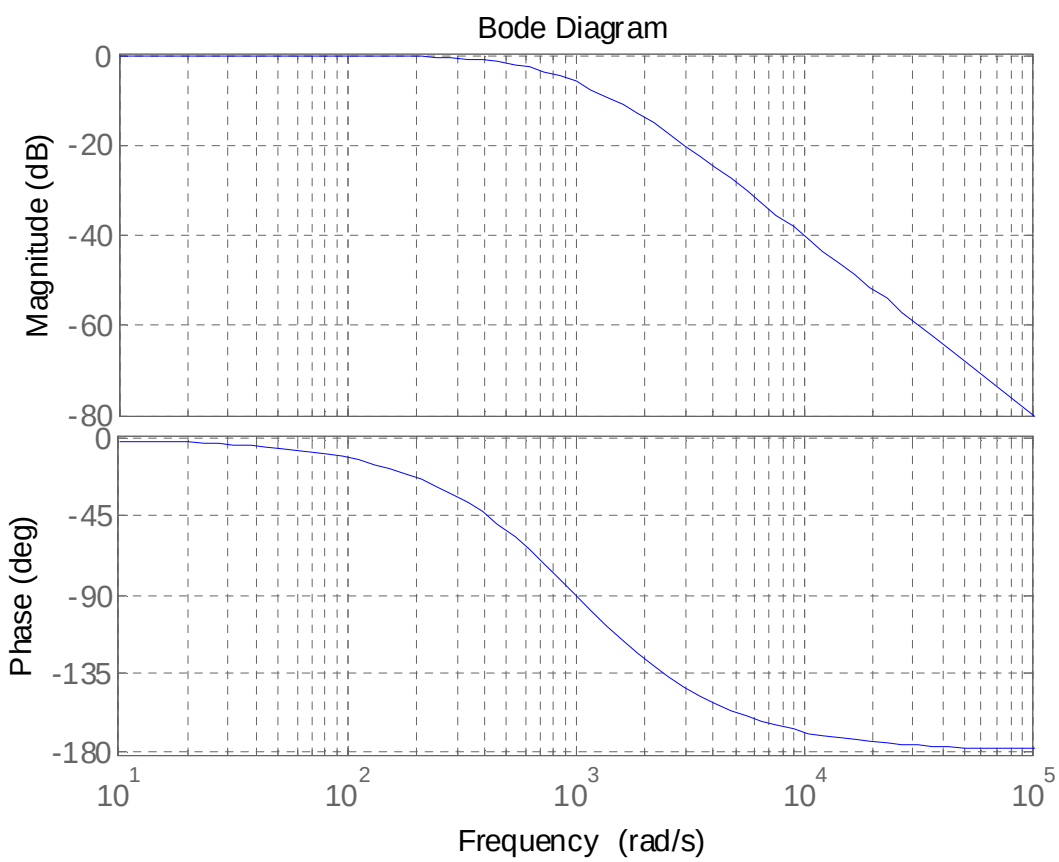
Change is -n*90 degrees or 180 degrees with a -90 deg/dec slope

All of the change happens at 0.1w and 10w, so we can focus on those frequencies

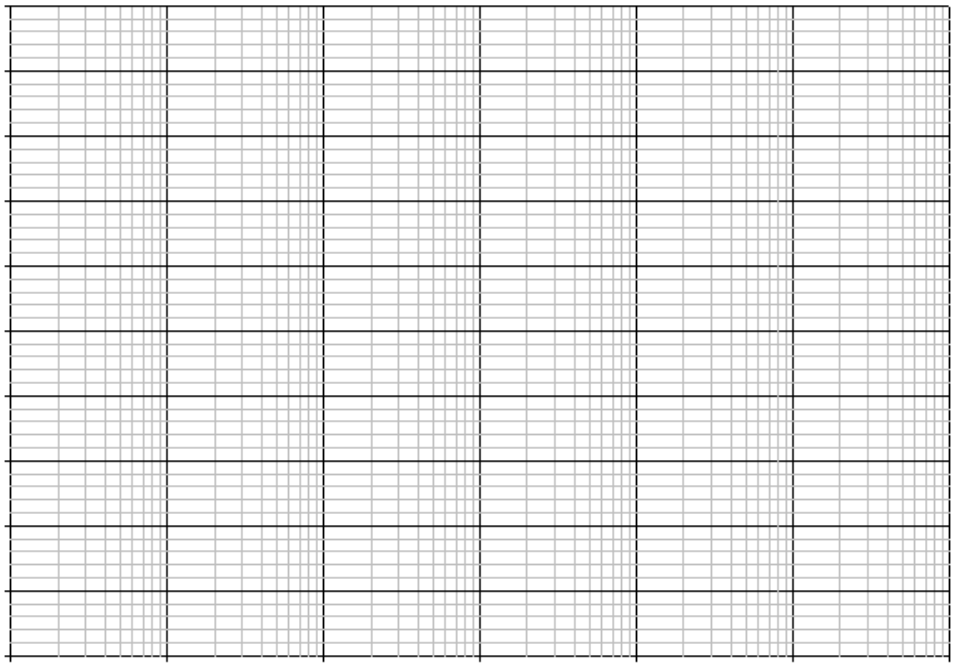
Mattlab check

```
H=tf([1000000],[1 2000 1000000]);

bode(H);grid
```



c) $$H(s) = 10\frac{s^2}{(s+100)(s+10000)}$$



$$H(j\omega) = 10\cdot\frac{(j\omega)^2}{(j\omega + 100)\cdot(j\omega + 10000)}$$

dominant term in numerator will be the same no matter what frequency range.

poles: 100 and 1*10^4
zeros: 0 double

zeros at zero mean a n*20 at the start

expect -40 db/dec at slope

Do sequential but a zero at zero is a dc steady state problem so we don't do this in AC steady state

Regions

$\omega < 100$

take dominant terms

$100 < \omega < 10^4$

$\omega > 10^4$

$\omega < 100$ $\quad 10\cdot\frac{(\omega)^2}{(100)\cdot(10000)}$ $\quad \omega^2$ in the numerator is $+40\frac{db}{dec}$ slope

To find out exactly where we are, we need to plug in ω=100

$10\cdot\frac{(100)^2}{100\cdot 10000} = 0.1 \qquad 20\log(0.1) = -20$

$100 < \omega < 10^4$ $\quad 10\cdot\frac{\omega^2}{(\omega)\cdot(10000)}$ $\quad \frac{10\omega}{1\cdot 10^4}$ $\quad \omega$ in the numerator is $+20\frac{db}{dec}$ slope

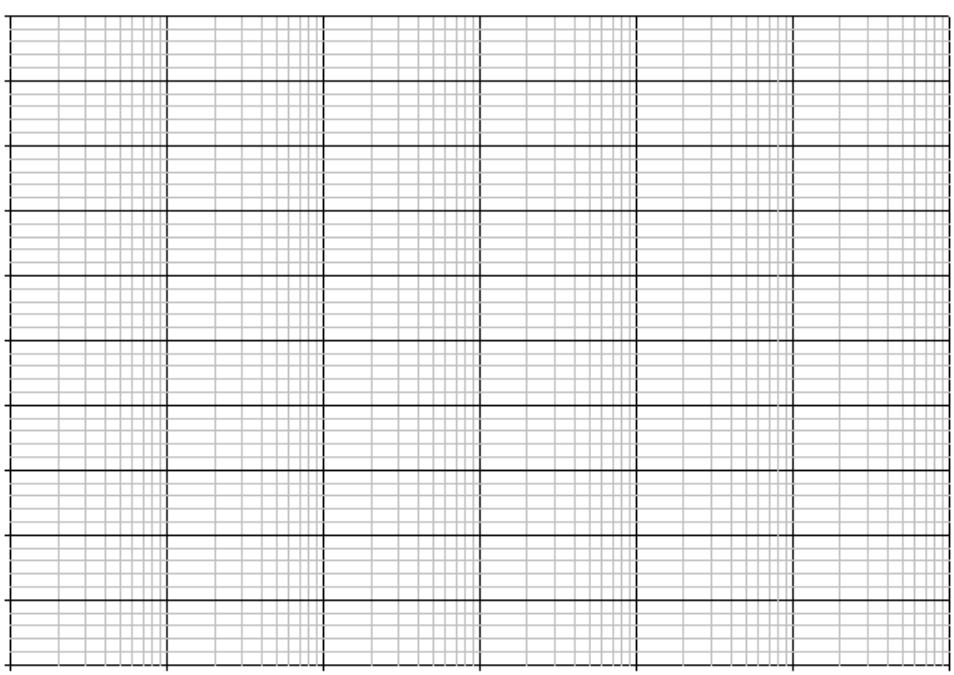
at ω=100 $\quad \frac{10\cdot(100)}{1\cdot 10^4} = 0.1 \qquad 20\log(0.1) = -20$ yep it is the same

at ω=10^4 $\quad \frac{10\cdot 1\cdot 10^4}{1\cdot 10^4} = 10 \qquad 20\cdot\log(10) = 20$ This is where that slope ends

$\omega > 10^4$ $\quad 10\cdot\frac{\omega^2}{(\omega)\cdot(\omega)}$ $\quad 10$ This is a constant

$20\cdot\log(10) = 20$ dB

Phase Bode plot c)



$$H(j\omega) = 10\cdot\frac{(j\omega)^2}{(j\omega + 100)\cdot(j\omega + 10000)}$$

$$\operatorname{atan}\left(\frac{.01}{1}\right) = 0.573\cdot deg$$

$$\operatorname{atan}\left(\frac{1\cdot 10^{-4}}{1}\right) = 5.73\times 10^{-3}\cdot deg$$

$\omega = 1$

$$\angle H(j1) = 10\cdot\frac{(j1)^2}{(j1 + 100)\cdot(j1 + 10000)} \qquad \frac{\angle 0\cdot(\angle 90)^2}{\angle 0(\angle 0)} \qquad \angle 180$$

$$\angle 0 + \angle 180 - \angle 0 - \angle 0$$

$\omega = 10$ this is 0.1 of pole 1

$$\angle H(j10) = 10\cdot\frac{(10\omega)^2}{(j10 + 100)\cdot(j10 + 10000)} \qquad \frac{\angle 0\cdot(\angle 90)^2}{\angle 0(\angle 0)}$$

$$\operatorname{atan}\left(\frac{10}{\frac{100}{1}}\right) = 5.711\cdot deg$$

$$\operatorname{atan}\left(\frac{10}{\frac{10000}{1}}\right) = 0.057\cdot deg$$

$\angle 180$

SKIPPING 100 for a second to see what happens 0.1pole and 10pole

$\omega = 1000$

$$\angle H(j1000) = 10\cdot\frac{(1000\omega)^2}{(j1000 + 100)\cdot(j1000 + 10000)} \qquad \frac{\angle 0\cdot(\angle 90)^2}{\angle 90(\angle 0)}$$

$$\operatorname{atan}\left(\frac{1000}{100}\right) = 84.289\cdot deg$$

$$\operatorname{atan}\left(\frac{1000}{10000}\right) = 5.711\cdot deg$$

$$\angle 0 + \angle 180 - \angle 90 - \angle 0$$

$\angle 90$

Crossed a pole and the angle changed by 90 degrees

$\omega = 100$

$$\angle H(j100) = 10\cdot\frac{(100\omega)^2}{(j100 + 100)\cdot(j100 + 10000)} \qquad \frac{\angle 0\cdot(\angle 90)^2}{\angle 45(\angle 0)}$$

$$\operatorname{atan}\left(\frac{100}{100}\right) = 45\cdot deg$$

$$\operatorname{atan}\left(\frac{100}{10000}\right) = 0.573\cdot deg$$

$$\angle 0 + \angle 180 - \angle 45 - \angle 0 \qquad \angle 135$$

$\omega = 10000$ ***This is a the second pole***

$$\angle H(j10000) = 10\cdot\frac{(10000\omega)^2}{(j10000 + 100)\cdot(j10000 + 10000)}$$

$$\frac{\angle 0\cdot(\angle 90)^2}{\angle 90(\angle 45)}$$

$$\mathrm{atan}\left(\frac{10000}{100}\right) = 89.427\cdot\mathrm{deg}$$

$$\mathrm{atan}\left(\frac{10000}{10000}\right) = 45\cdot\mathrm{deg}$$

$$\angle 0 + \angle 180 - \angle 90 - \angle 45 \qquad \angle 45$$

$$180 - 90 - 45 = 45$$

$\omega = 10^5$

$$\angle H\left(j10^5\right) = 10\cdot\frac{\left(1\cdot 10^5\omega\right)^2}{\left(j\,1\cdot 10^5 + 100\right)\cdot\left(j\,1\cdot 10^5 + 10000\right)}$$

$$\frac{\angle 0\cdot(\angle 90)^2}{\angle 90\,\angle 90}$$

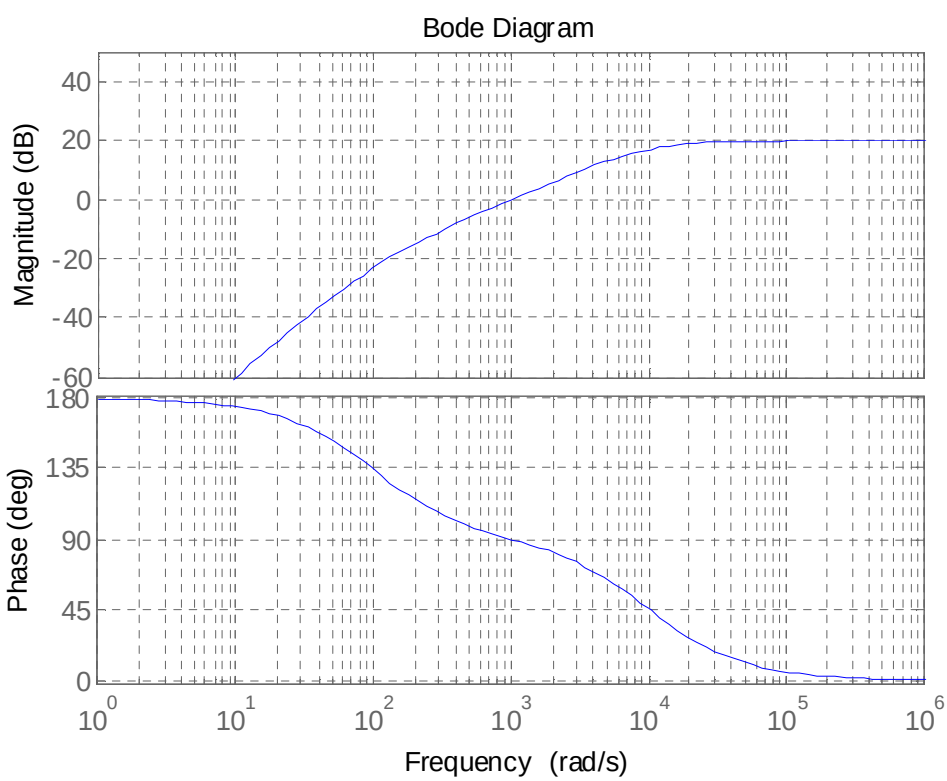
$$\mathrm{atan}\left(\frac{10^5}{100}\right) = 89.943\cdot\mathrm{deg}$$

$$\mathrm{atan}\left(\frac{10^5}{10000}\right) = 84.289\cdot\mathrm{deg}$$

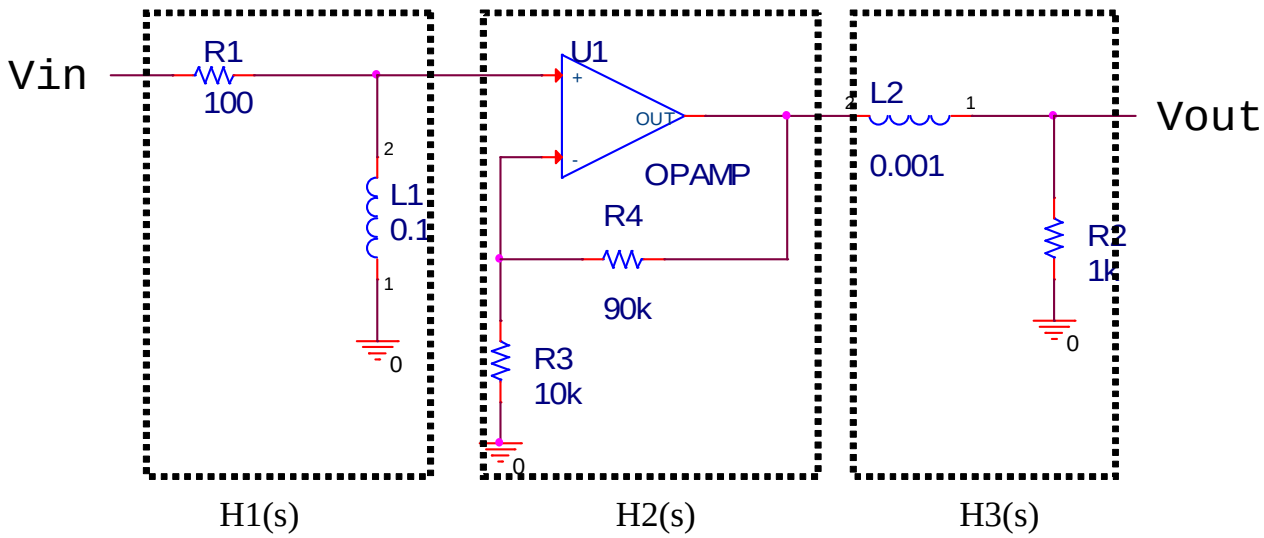
$$\angle 0 + \angle 180 - \angle 90 - \angle 90 \qquad \angle 0$$

Matlab check

```
>> sys=tf([10 0 0],[1 10100 1000000]);
>> h=bodeplot(sys);grid
>> setoptions(h,'MagLowerLimMode','manual','MagLowerLim',-60)
```



2) Bode plot-multiple stages



a. Draw the above circuit as a three stage network. Indicate the transfer function for each stage.

Answer: already done above

b. Determine the transfer function, H(s)=Vout(s)/Vin(s) for the circuit

$R_1 := 100\Omega$

$L_1 := 0.1H$

$R_4 := 90k\Omega$

$R_3 := 10k\Omega$

$L_2 := 0.001H$

$R_2 := 1k\Omega$

$$H_1(s) = \frac{s}{s + \frac{R_1}{L_1}} = \frac{s}{s + 1\cdot 10^3}$$

$$H_2(s) = 1 + \frac{R_4}{R_5} = 10$$

$$H_3 = \frac{\frac{R_2}{L_2}}{s + \frac{R_2}{L_2}} = \frac{1\cdot 10^6}{s + 1\cdot 10^6}$$

$$H(s) = H_1(s)\cdot H_2(s)\cdot H_3(s) = 10\cdot\frac{1\cdot 10^6\cdot s}{\left(s + 1\cdot 10^3\right)\cdot\left(s + 1\cdot 10^6\right)}$$

$$10\cdot 1\cdot 10^6 = 1\times 10^7$$

$$\left(s + 1\cdot 10^3\right)\cdot\left(s + 1\cdot 10^6\right)$$

$$s^2 + 1001000\cdot s + 1000000000$$

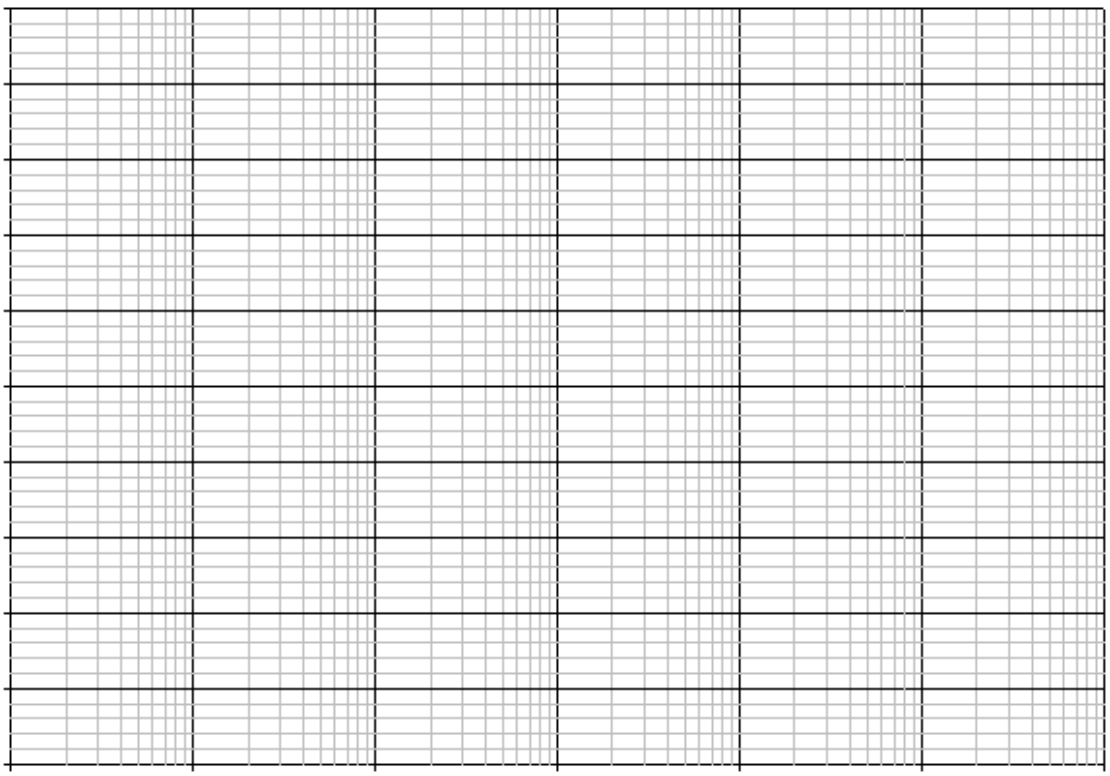
c. What is the gain of the circuit?

The amplifier stage incoporates the gain, K=10.

d. What are the poles and zeros?

Zeros: 0
Poles: $1\cdot 10^{3}, 1\cdot 10^{6}$

e. Sketch an approximate bode dB-log plot fo the magnitude



$$H(s) = 10\cdot\frac{1\cdot 10^{6}\cdot s}{\left(s + 1\cdot 10^{3}\right)\cdot\left(s + 1\cdot 10^{6}\right)} \qquad H(j\omega) = 10\cdot\frac{1\cdot 10^{6}\cdot j\omega}{\left(j\omega + 1\cdot 10^{3}\right)\cdot\left(j\omega + 1\cdot 10^{6}\right)}$$

Three regions

$$10\cdot\frac{1\cdot10^{6}}{1\cdot10^{3}\cdot1\cdot10^{6}} = 0.01$$

where exactly the slope ends

$\omega < 1\cdot10^{3}$ $\quad 10\cdot\frac{1\cdot10^{6}}{1\cdot10^{3}\cdot1\cdot10^{6}}\cdot\omega$ $\quad 20\frac{\text{dB}}{\text{dec}}$ slope $\quad 0.01\cdot1\cdot10^{3} = 10$ $\quad 20\log(10) = 20$

$1\cdot10^{3} < \omega < 1\cdot10^{6}$ $\quad 10\cdot\frac{1\cdot10^{6}}{\omega\cdot1\cdot10^{6}}\cdot\omega = 10$ $\quad$ constant $\quad 20\cdot\log(10) = 20$

$\omega > 1\cdot10^{6}$ $\quad 10\cdot\frac{1\cdot10^{6}}{\omega^{2}}\cdot\omega = \frac{1\cdot10^{7}}{\omega}$ $\quad -20\frac{\text{db}}{\text{dec}}$ slope

For an ending point on the graph $\omega = 1\cdot10^{7}$ $\quad \frac{1\cdot10^{7}}{1\cdot10^{7}} = 1$ $\quad 20\log(1) = 0$ db

Make corrections.

-3db at both corner frequencies

For phase

Zeros:
0
Poles: $1\cdot10^{3}, 1\cdot10^{6}$

Need to find what happens for ωcn, 0.1ωcn and 10ωcn

$\omega_{0.1p1} = 1\cdot10^{2}$

$$\angle H(j100) = 10\cdot\frac{1\cdot10^{6}\cdot j100}{\left(j100 + 1\cdot10^{3}\right)\cdot\left(j100 + 1\cdot10^{6}\right)}$$

$$\frac{\angle 0\cdot(\angle 90)}{\angle 0(\angle 0)}$$

$\angle 90$

$\omega_{10p1} = 1\cdot 10^4$

$$\angle H(j100) = 10\cdot\frac{1\cdot 10^6\cdot j\cdot 1\cdot 10^4}{\left[j\cdot\left(1\cdot 10^4\right) + 1\cdot 10^3\right]\cdot\left[j\cdot\left(1\cdot 10^4\right) + 1\cdot 10^6\right]}\ \frac{\angle 0\cdot(\angle 90)}{90(\angle 0)}$$

$$\angle 0$$

$$\operatorname{atan}\left(\frac{1\cdot 10^4}{\frac{1\cdot 10^3}{1}}\right) = 84.289\cdot deg$$

$$\operatorname{atan}\left(\frac{1\cdot 10^4}{\frac{1\cdot 10^6}{1}}\right) = 0.573\cdot deg$$

$\omega_{c1} = 1\cdot 10^3$ ***Can just draw line between them for now shoudl be <45 degrees but can calculate***

$\omega_{0.1p2} = 1\cdot 10^5$

$$\angle H\left(1\cdot 10^5\right) = 10\cdot\frac{1\cdot 10^6\cdot j\cdot\left(1\cdot 10^5\right)}{\left[j\cdot\left(1\cdot 10^5\right) + 1\cdot 10^3\right]\cdot\left[j\cdot\left(1\cdot 10^5\right) + 1\cdot 10^6\right]}$$

$$\frac{\angle 0\cdot(\angle 90)}{\angle 90(\angle 0)}$$

$$\angle 0$$

$$\operatorname{atan}\left(\frac{1\cdot 10^5}{\frac{1\cdot 10^3}{1}}\right) = 89.427\cdot deg$$

$$\operatorname{atan}\left(\frac{1\cdot 10^5}{\frac{1\cdot 10^6}{1}}\right) = 5.711\cdot deg$$

$\omega_{10p2} = 1\cdot 10^7$

$$\angle H\left[j\cdot\left(1\cdot 10^7\right)\right] = 10\cdot\frac{1\cdot 10^6\cdot j\cdot\left(1\cdot 10^7\right)}{\left[j\cdot\left(1\cdot 10^7\right) + 1\cdot 10^3\right]\cdot\left[j\cdot\left(1\cdot 10^7\right) + 1\cdot 10^6\right]}$$

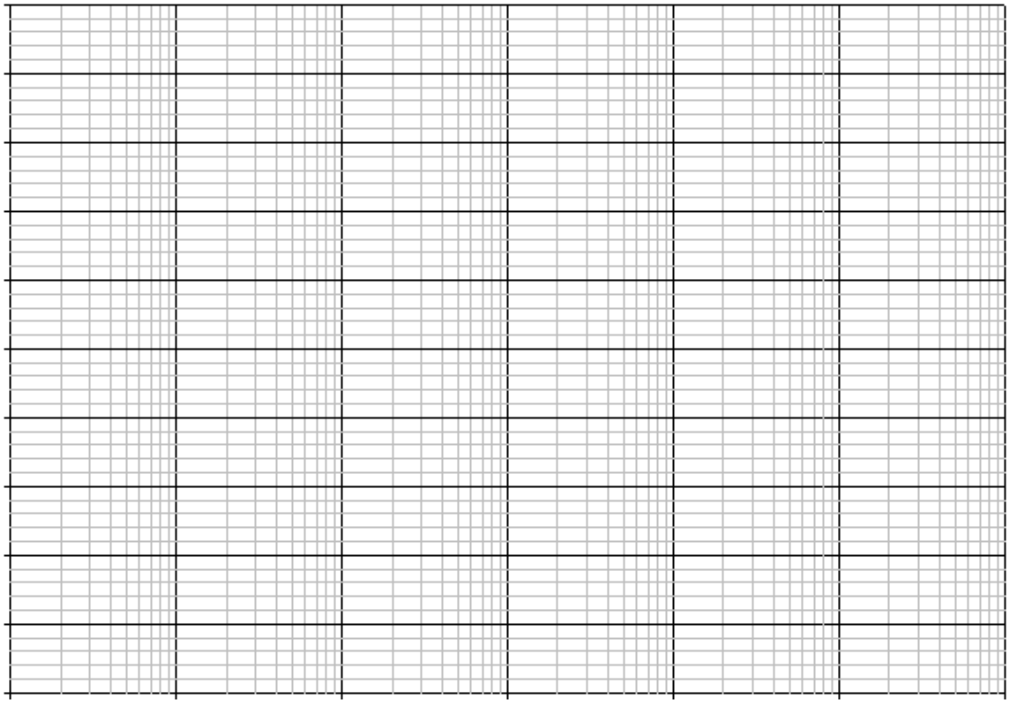
$$\frac{\angle 0\cdot(\angle 90)}{\angle 90(\angle 90)}$$

$$\angle -90$$

$$\operatorname{atan}\left(\frac{1\cdot 10^7}{\frac{1\cdot 10^3}{1}}\right) = 89.994\cdot deg$$

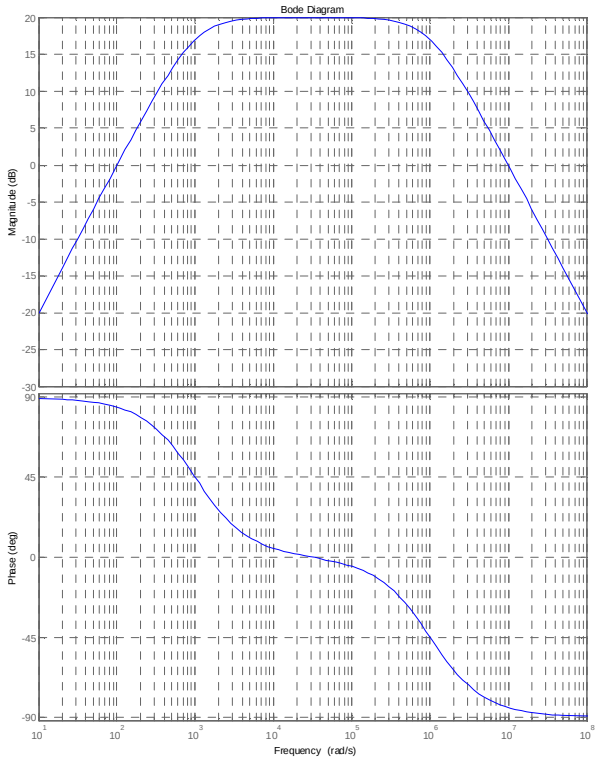
$$\operatorname{atan}\left(\frac{1\cdot 10^7}{\frac{1\cdot 10^6}{1}}\right) = 84.289\cdot deg$$

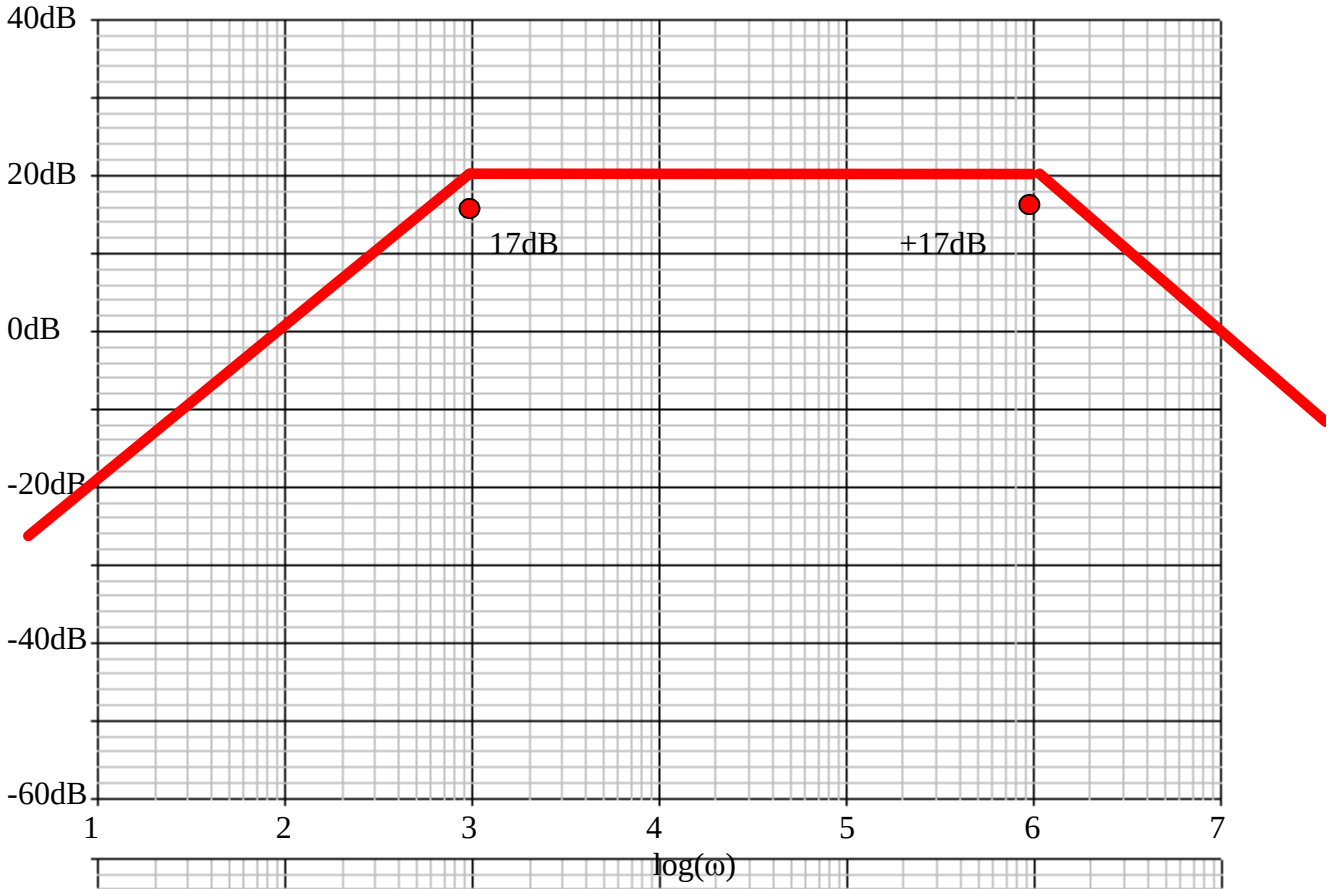
$\omega_{c2} = 1\cdot 10^6$ Can just draw line between them for now shoudl be <-45 degrees but can calculate

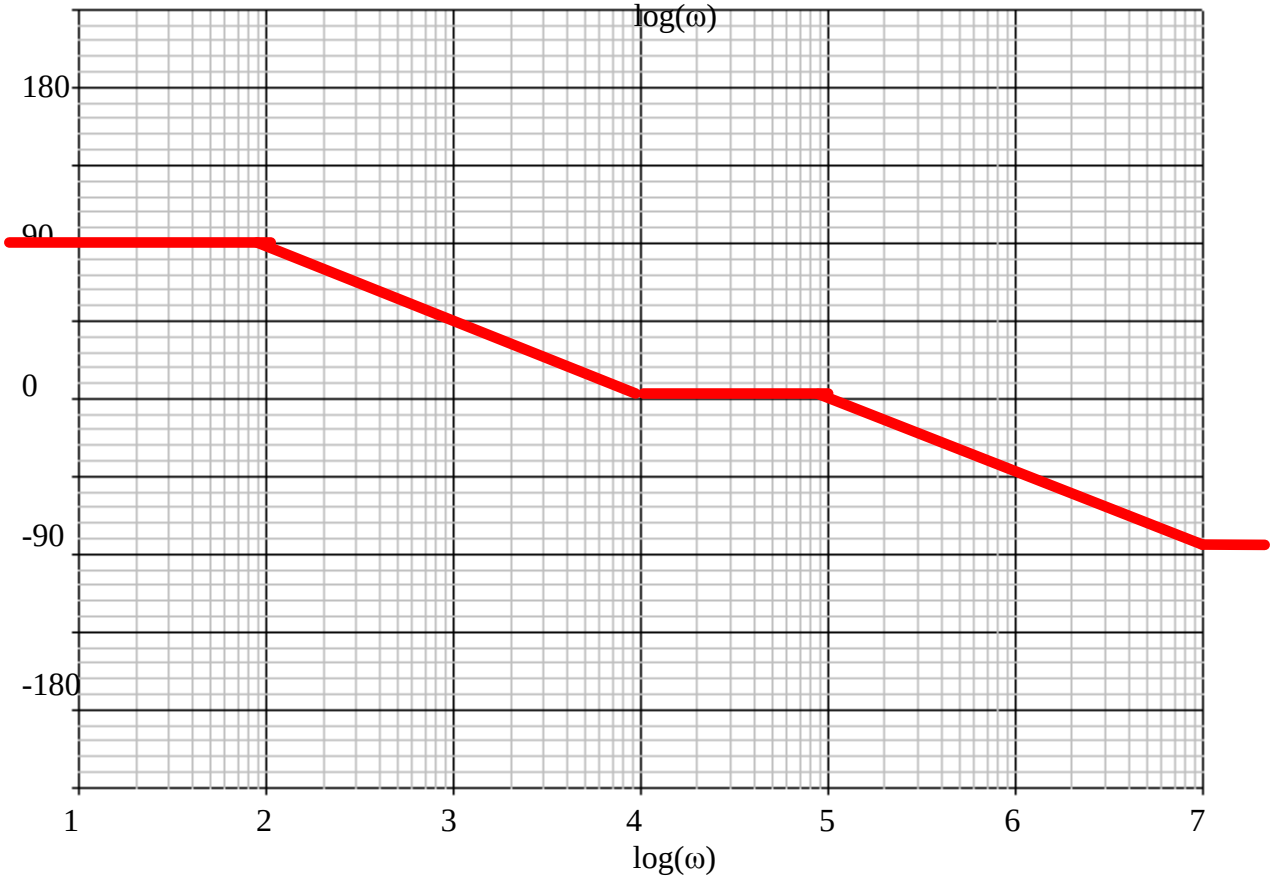


Matlab check

```
>> sys=tf([10000000 0],[1 1001000 1000000000]);
>> h=bodeplot(sys);grid
```




40dB
20dB
0dB
-20dB
-40dB
-60dB
17dB
+17dB
1
2
3
4
5
6
7
log(ω)


180
90
0
-90
-180
1
2
3
4
5
6
7
log(ω)