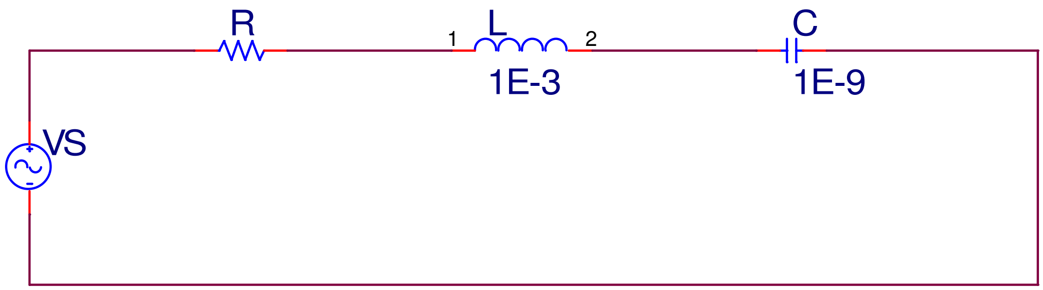
What is the damping ratio, ζ?

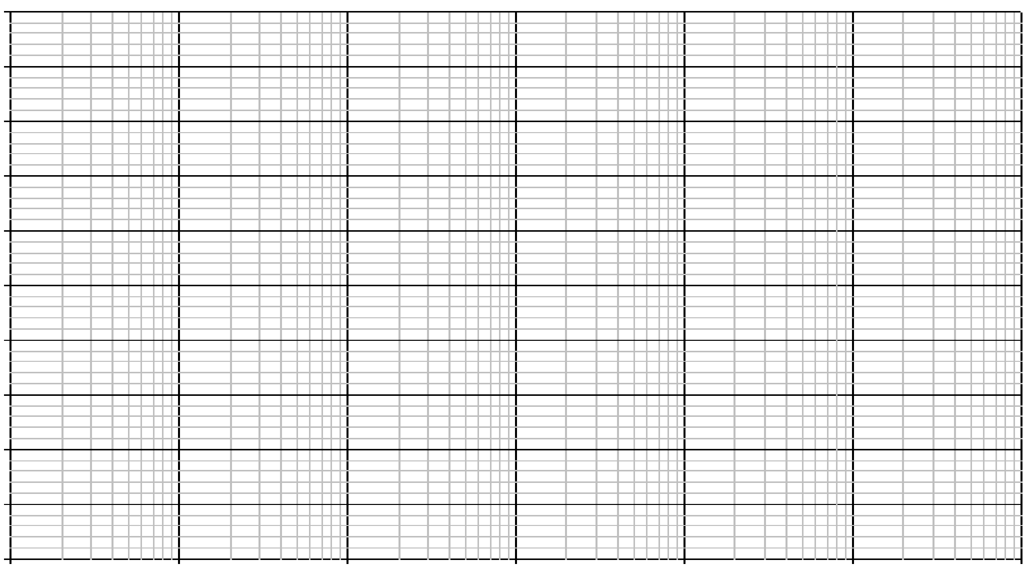
How do overdamped, critically damped, and underdamped circuits behave?

Review

1) Second order circuits



a) H(s) = VC(s)/V(s) when R=10.1kΩ



$$H(s) = \frac{V_c(s)}{V_T(s)} = \frac{\frac{1}{sC}}{R_T + sL + \frac{1}{sC}} = \frac{1}{sCR_T + sC\cdot sL + \frac{sC}{sC}} = \frac{1}{s^2\cdot LC + sCR_T + 1}$$

divide top and bottom by LC

$$H(s) = \frac{\frac{1}{LC}}{s^2 + \frac{R_T}{L}s + \frac{1}{LC}}$$

Also can write as (will use later)

$$H(s) = \frac{\omega_0^2}{s^2 + 2\cdot\zeta\omega_0\cdot s + \omega_0^2}$$

$$2\zeta\omega_0 = \frac{R_T}{L}$$

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

$$\zeta = \frac{R_T}{2}\cdot\sqrt{\frac{C}{L}}$$ damping ratio

$$\zeta = \frac{\alpha}{\omega_0}$$

$\zeta > 1$	overdamped case	real poles	two different real poles	
$\zeta = 1$	critically damped	double pole	-6db at ωc	attenuation of gain!
$\zeta < 1$	underdamped	complex pole	can get resonance!	

$H(s)$ as s goes to 0 $\dfrac{\frac{1}{LC}}{\frac{1}{LC}} = 1$ $H(j\omega) = \dfrac{\frac{1}{LC}}{\frac{1}{LC}} = 1$

$H(s)$ as s goes to ∞ $\dfrac{\frac{1}{LC}}{s^2}$ goes to 0 $H(j\omega) = \dfrac{\frac{1}{LC}}{\omega^2}$ goes to 0

This is a low pass filter and a rolloff of 40db also known as the slope of the stopband at the pole ω^2 or s^2 = 2 *20 log ()

RLC series vout = Vc second order low pass filter

If $R_1 := 10.1k\Omega$ $L_1 := 1mH$ $C_1 := 1nF$ $\frac{1}{L_1\cdot C_1} = 1\times 10^{12}\frac{1}{s^2}$

$$H(s) = \frac{10^{12}}{s^2 + 1.01\cdot 10^7 s + 10^{12}}$$

$$\alpha := \frac{1.01\cdot 10^7}{2} = 5.05\times 10^6$$

$$\omega_0 := \sqrt{\frac{1}{L_1\cdot C_1}} = 1\times 10^6\frac{1}{s}$$

$$\zeta := \frac{\alpha}{\omega_0} = 5.05\,s$$ ***This is > 1 so OVERDAMPED***

$$H(s) = \frac{10^{12}}{\left(s+10^5\right)\cdot\left(s+10^7\right)} \qquad H(j\omega) = \frac{10^{12}}{10^5\cdot 10^7}\cdot\frac{1}{\left(1+\frac{j\omega}{10^5}\right)\cdot\left(1+\frac{j\omega}{10^7}\right)}$$

zero: none
pole 10^5, 10^7

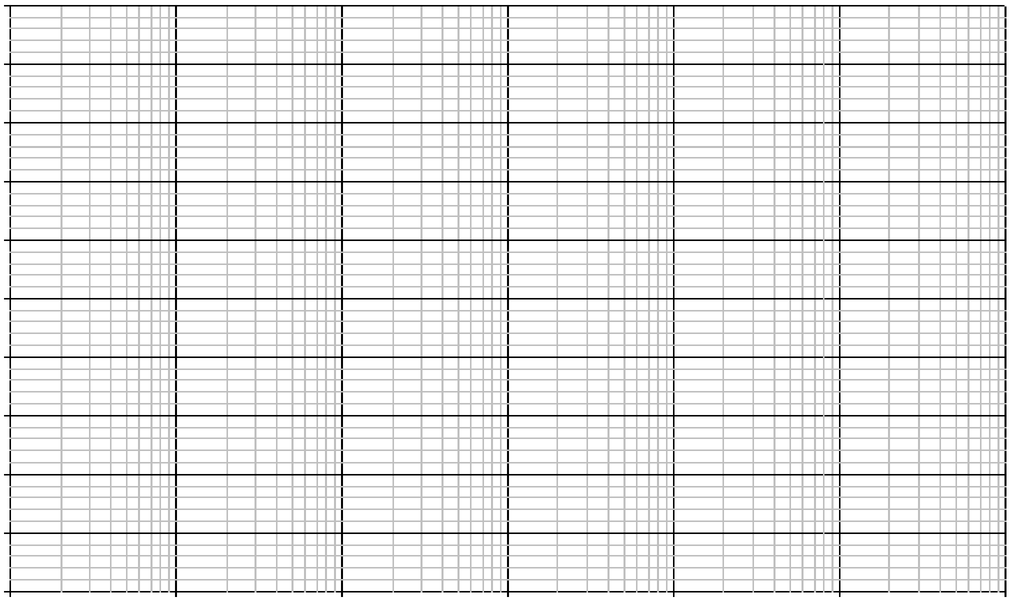
$$s < 10^5 \qquad \frac{10^{12}}{\left(10^5\right)\cdot\left(10^7\right)} = 1 \qquad \omega < 10^5 \qquad 1 \qquad 20\cdot\log(1) = 0$$

$$10^5 < s < 10^7 \qquad \frac{10^{12}}{(s)\cdot\left(10^7\right)} \qquad \frac{10^5}{s} \qquad 10^5 < \omega < 10^7 \qquad \frac{1}{\frac{\omega}{10^5}} \qquad -20\,\frac{db}{dec}$$

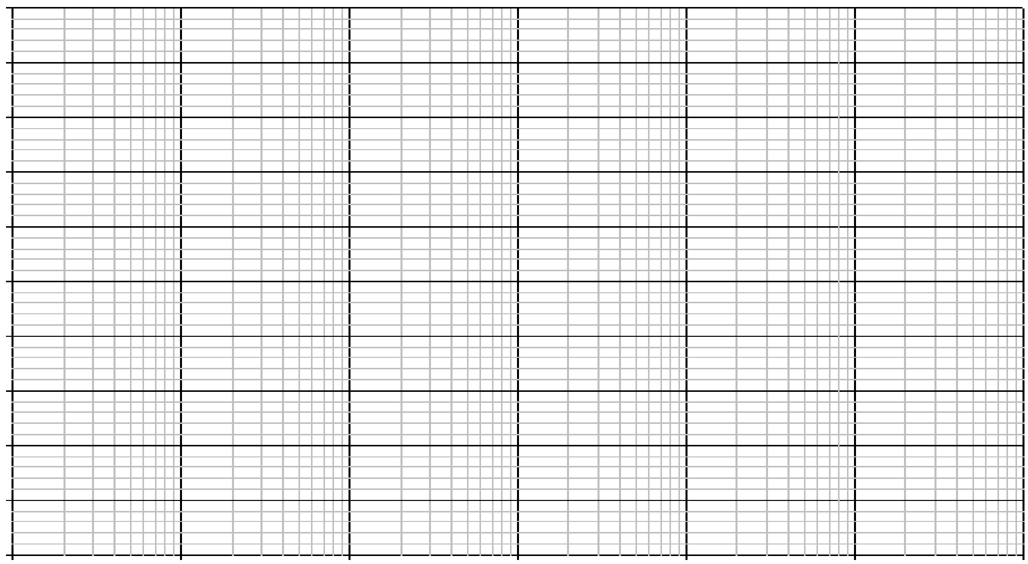
$$s > 10^7 \qquad \frac{10^{12}}{(s)\cdot(s)} \qquad \frac{10^{12}}{s^2} \qquad \omega > 10^7 \qquad \frac{1}{\frac{\omega^2}{10^5\cdot 10^7}} \qquad -40\,\frac{db}{dec}$$

Overdamped case: two different real poles

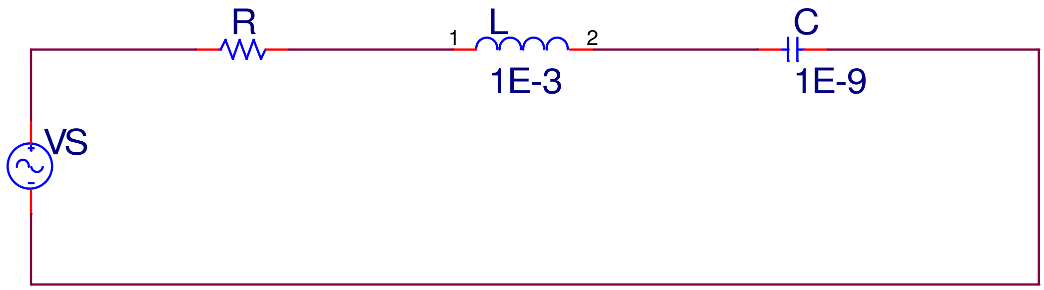
b) $H(s) = VL(s)/V(s)$ when
$R=10.1k\Omega$



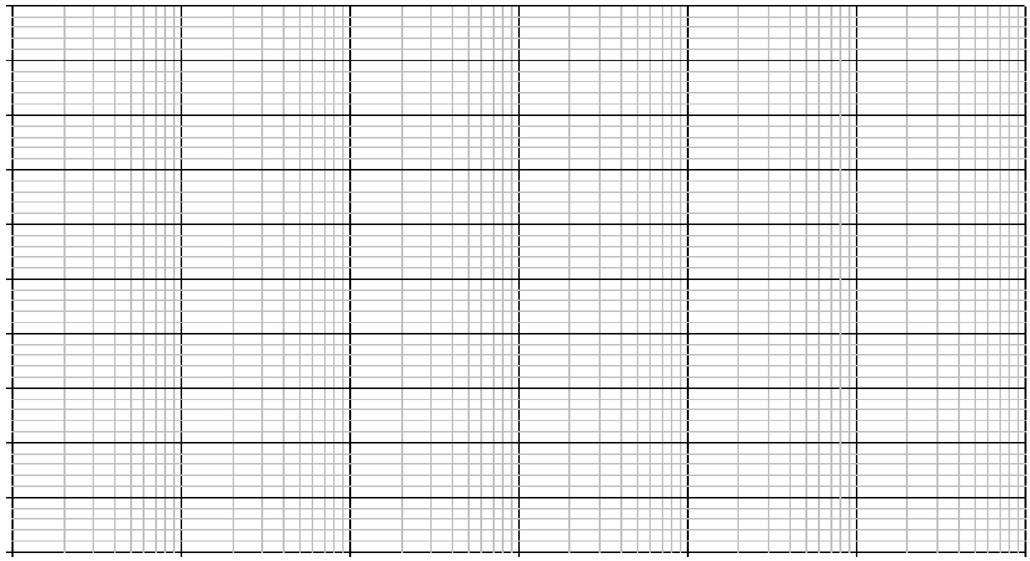
c) $H(s)=V_R(s)/V(s)$ when $R=10.1k\Omega$



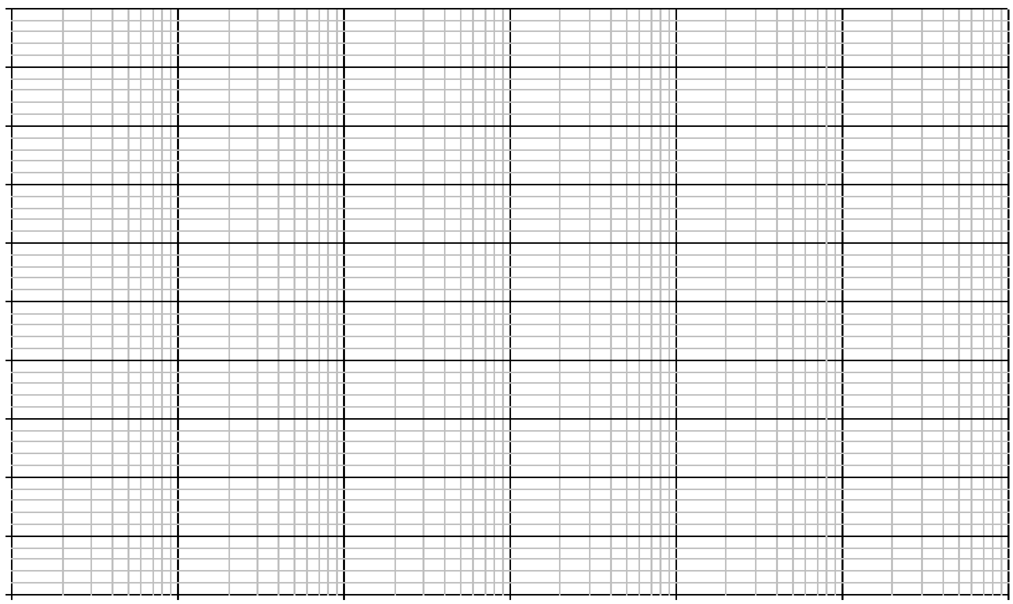
2) Second order circuits TEAM ASSIGNMENT



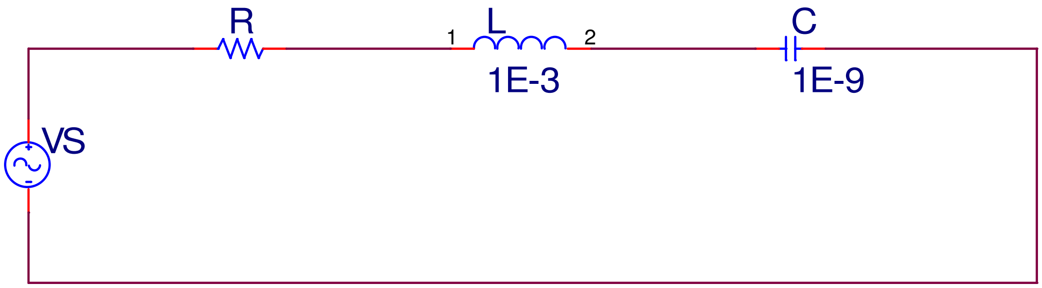
a. H(s) = VC(s)/V(s) ***when R=2kΩ***



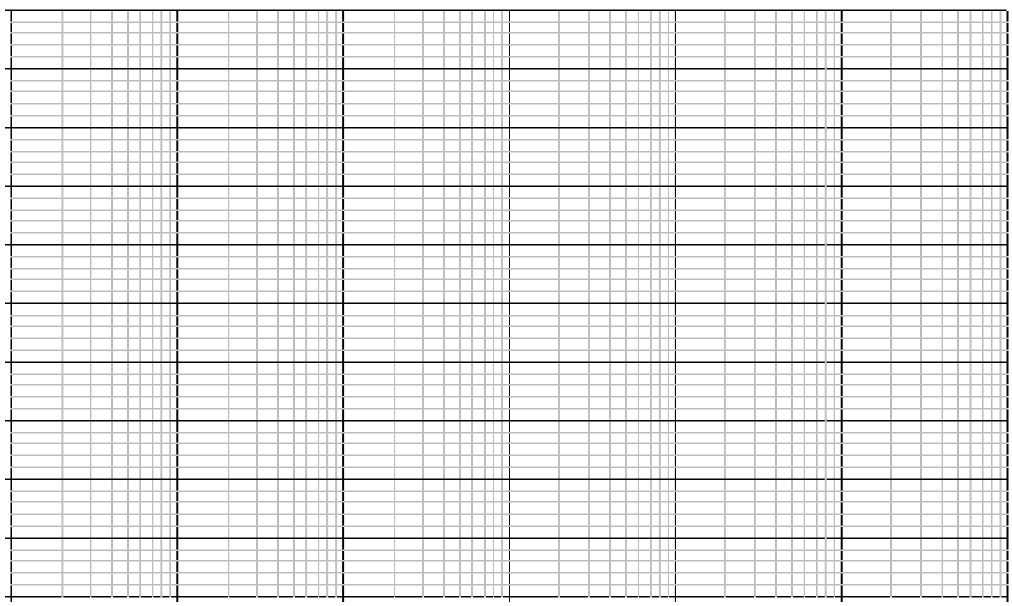
b. H(s) = VL(s)/V(s) when R = 2kΩ



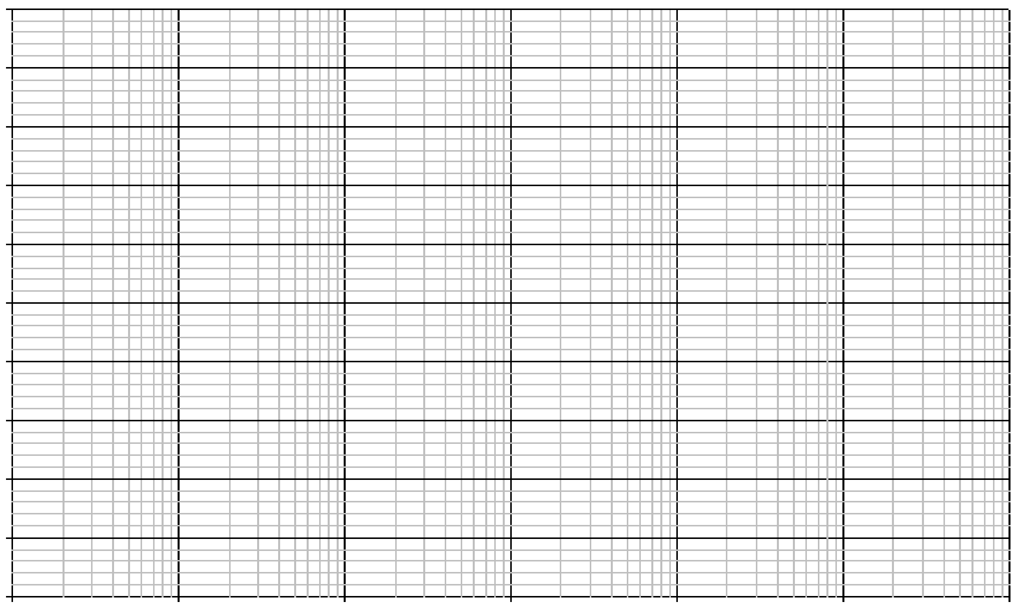
3) Underdamped cases of the RLC series circuit



a.) H(s) = VC(s)/V(s) when R =
1kΩ



b.) H(s) = VC(s)/V(s) when R = 1.41kΩ



c.) $H(s) = V_C(s)/V(s)$ when $R = 100\Omega$