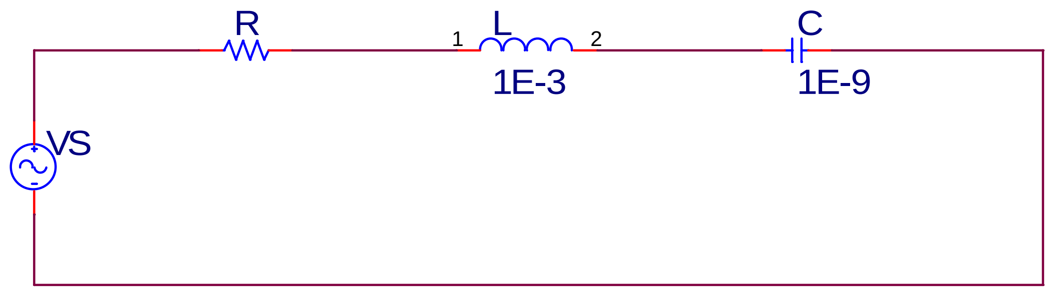
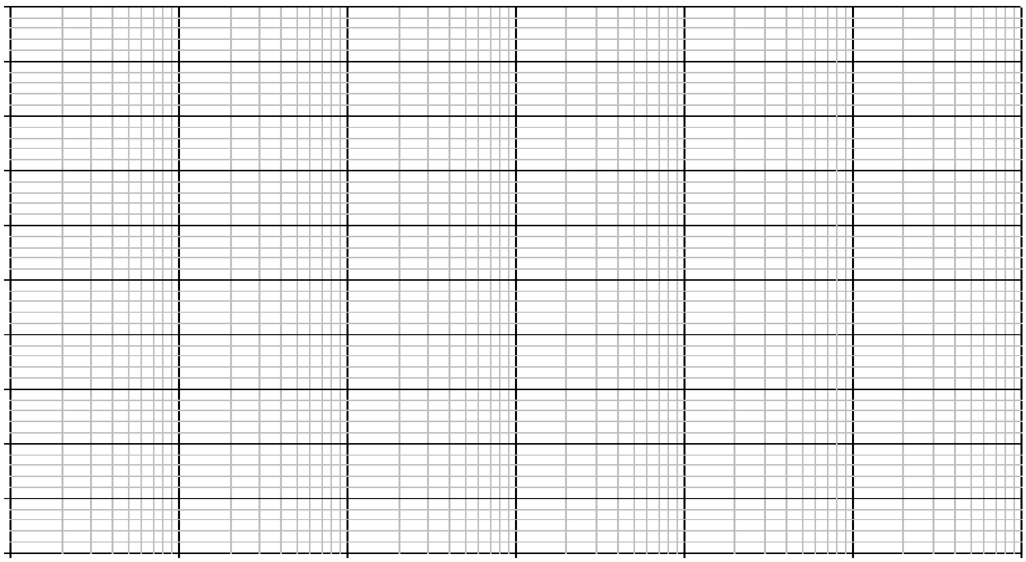
What is the damping ratio, ζ?
How do overdamped, critically damped, and underdamped circuits behave?

Review 1)

1) Second order circuits



a) H(s) = VC(s)/V(s) when R=10.1kΩ



$$H(s) = \frac{V_C(s)}{V_T(s)} = \frac{\frac{1}{sC}}{R_T + sL + \frac{1}{sC}} = \frac{1}{sCR_T + sC\cdot sL + \frac{sC}{sC}} = \frac{1}{s^2\cdot LC + sCR_T + 1}$$

divide top and bottom by LC

$$H(s) = \frac{\frac{1}{LC}}{s^2 + \frac{R_T}{L}s + \frac{1}{LC}}$$

Also can write as (will use later)

$$H(s) = \frac{\omega_0^2}{s^2 + 2\cdot\zeta\omega_0\cdot s + \omega_0^2}$$

$$2\zeta\omega_0 = \frac{R_T}{L} \qquad \omega_0 = \sqrt{\frac{1}{LC}}$$

$$\zeta = \frac{R_T}{2}\cdot\sqrt{\frac{C}{L}}$$ damping ratio $$\zeta = \frac{\alpha}{\omega_0}$$

$\zeta > 1$ overdamped case — real poles — two different real poles

$\zeta = 1$ critically damped — double pole — -6db at ωc attenuation of gain!

$\zeta < 1$ underdamped — complex pole — can get resonance!

$H(s)$ as s goes to 0 $$\frac{\frac{1}{LC}}{\frac{1}{LC}} = 1 \qquad H(j\omega) = \frac{\frac{1}{LC}}{\frac{1}{LC}} = 1$$

$H(s)$ as s goes to ∞ $$\frac{\frac{1}{LC}}{s^2}$$ goes to 0 $$H(j\omega) = \frac{1}{\frac{LC}{\omega^2}}$$ goes to 0

This is a low pass filter and a rolloff of 40db also known as the slope of the stopband at the pole ω^2 or s^2 = 2 *20 log ()

RLC series vout = Vc second order low pass filter

If $R_1 := 10.1k\Omega$ $L_1 := 1mH$ $C_1 := 1nF$ $\frac{1}{L_1\cdot C_1} = 1\times 10^{12}\frac{1}{s^2}$

$$H(s) = \frac{10^{12}}{s^2 + 1.01\cdot 10^7 s + 10^{12}}$$

$$\alpha := \frac{1.01\cdot 10^7}{2} = 5.05\times 10^6$$

$$\omega_0 := \sqrt{\frac{1}{L_1\cdot C_1}} = 1\times 10^6\frac{1}{s}$$

$$\zeta := \frac{\alpha}{\omega_0} = 5.05s$$ ***This is > 1 so OVERDAMPED***

$$H(s) = \frac{10^{12}}{\left(s+10^5\right)\cdot\left(s+10^7\right)}$$

$$H(j\omega) = \frac{10^{12}}{10^5\cdot 10^7}\cdot\frac{1}{\left(1+\frac{j\omega}{10^5}\right)\cdot\left(1+\frac{j\omega}{10^7}\right)}$$

zero: none
pole 10^5, 10^7

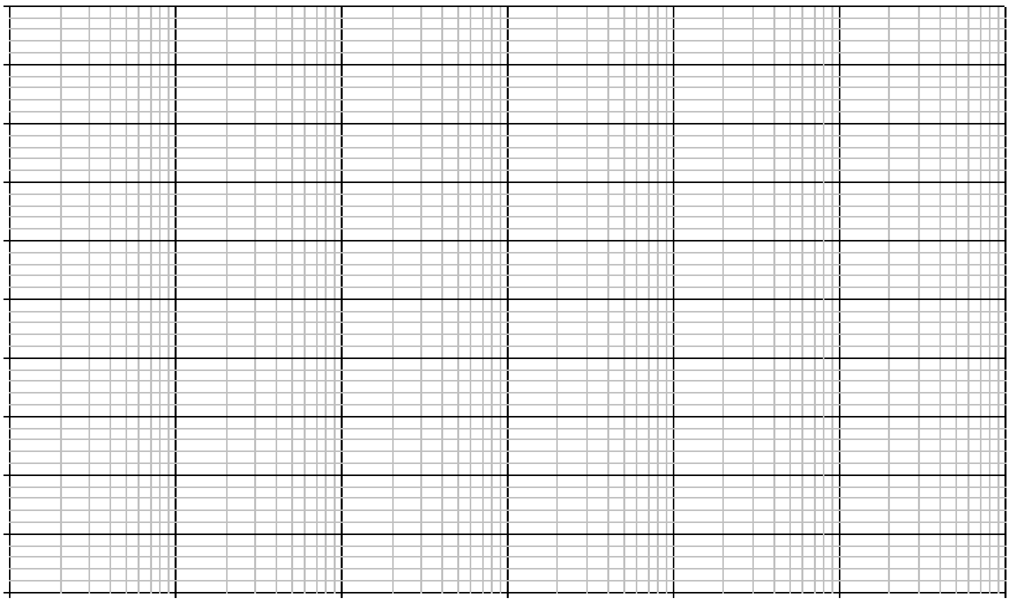
$s < 10^5$ $\quad \frac{10^{12}}{\left(10^5\right)\cdot\left(10^7\right)} = 1$ $\quad\quad \omega < 10^5$ $\quad 1$ $\quad 20\cdot\log(1) = 0$

$10^5 < s < 10^7$ $\quad \frac{10^{12}}{(s)\cdot\left(10^7\right)}$ $\quad \frac{10^5}{s}$ $\quad\quad 10^5 < \omega < 10^7$ $\quad \frac{1}{\frac{\omega}{10^5}}$ $\quad -20\frac{db}{dec}$

$s > 10^7$ $\quad \frac{10^{12}}{(s)\cdot(s)}$ $\quad \frac{10^{12}}{s^2}$ $\quad\quad \omega > 10^7$ $\quad \frac{1}{\frac{\omega^2}{10^5\cdot 10^7}}$ $\quad -40\frac{db}{dec}$

Overdamped case: two different real poles

b) H(s) = VL(s)/V(s) when R=10.1kΩ



$$H(s) = \frac{s^2}{s^2 + \frac{R}{L}\cdot s + \frac{1}{LC}}$$

$$H(j\omega) = \frac{(j\omega)^2}{\left(1 + \frac{j\omega}{10^5}\right)\cdot\left(1 + \frac{j\omega}{10^7}\right)}$$

Hs when s goes to 0 $\quad \frac{s^2}{\frac{1}{LC}} = 0$

Hs when s goes to∞ $\quad \frac{s^2}{s^2} = 1$

This is an overdamped, second order high pass filter

If $R_1 := 10.1k\Omega$ $L_1 := 1mH$ $C_1 := 1nF$ $\frac{1}{L_1 \cdot C_1} = 1 \times 10^{12} \frac{1}{s^2}$

$$H(s) = \frac{s^2}{s^2 + 1.01 \cdot 10^7 s + 10^{12}}$$

$$H(s) = \frac{s^2}{\left(s + 10^5\right) \cdot \left(s + 10^7\right)}$$

$$H(j\omega) = \frac{1}{10^5 \cdot 10^7} \cdot \frac{(j\omega)^2}{\left(1 + \frac{j\omega}{10^5}\right) \cdot \left(1 + \frac{j\omega}{10^7}\right)}$$

zero: double 0
pole 10^5, 10^7

$$\frac{\left(10^5\right)^2}{10^5 \cdot 10^7} = 0.01$$

$$20 \log(0.01) = -40$$

$s < 10^5$ $\frac{s^2}{\left(10^5\right) \cdot \left(10^7\right)}$ $\omega < 10^5$ $\frac{\omega^2}{10^5 \cdot 10^7}$ $40 \frac{db}{dec}$

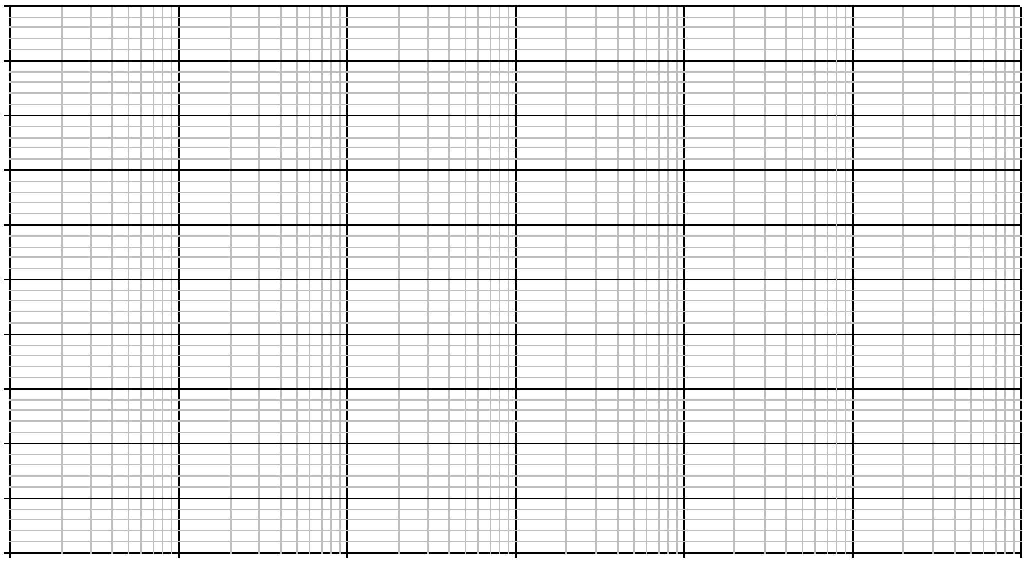
$10^5 < s < 10^7$ $\frac{s^2}{(s) \cdot \left(10^7\right)}$ $\frac{s}{10^7}$ $10^5 < \omega < 10^7$ $\frac{1}{10^5 \cdot 10^7} \cdot \frac{\omega^2}{\frac{\omega}{10^5}} = \frac{\omega}{10^7}$ $20 \frac{db}{dec}$

$\frac{10^5}{10^7} = 0.01$ $20 \log(0.01) = -40$

$s > 10^7$ $\frac{s^2}{(s) \cdot (s)} = 1$ $\omega > 10^7$ $\frac{1}{10^5 \cdot 10^7} \cdot \frac{\omega^2}{\frac{\omega^2}{10^5 \cdot 10^7}}$ 0 db

<u>Overdamped high pass filter, two different real poles</u>

c) H(s)=VR(s)/V(s) when R=10.1kΩ



$$Vout = V_R$$

$$H(s) = \frac{\frac{R}{L}s}{s^2 + \frac{R}{L}\cdot s + \frac{1}{LC}}$$

H(s) as s goes to 0 $$H(s) = \frac{\frac{R}{L}\cdot s}{\frac{1}{LC}} = 0$$

H(s) as s goes to ∞ $$\frac{\frac{R}{L}s}{s^2} = 0$$

Check the "middle" rewrite as $$\frac{2\alpha s}{s^2 + 2\alpha s + \omega_0^2}$$ where

$$2\alpha = \frac{R}{L}$$

Look at resonant frequency $$\omega_0 = \frac{1}{LC}$$

H(s) as s goes to jω0

$$\frac{2\cdot\alpha\cdot\left(j\omega_0\right)}{-\omega_0^2 + 2\alpha\left(j\omega_0\right) + \omega_0^2} = 1$$

At the resonant frequency

Second order bandpass filter where ω0 is the resonant frequency

If $R_1 := 10.1k\Omega$ $L_1 := 1mH$ $C_1 := 1nF$ $\frac{1}{L_1 \cdot C_1} = 1 \times 10^{12} \frac{1}{s^2}$

$$H(s) = \frac{2\alpha s}{s^2 + 2\alpha s + \omega_0^2}$$

$$H(s) = \frac{1.01 \cdot 10^7 s}{s^2 + 1.01 \cdot 10^7 s + 1 \cdot 10^{12}}$$

$$H(s) = \frac{1.01 \cdot 10^7 s}{(s + 10^5)(s + 10^7)}$$

$$H(s) = \frac{1.01 \cdot 10^7 s}{(s + 10^5) \cdot (s + 10^7)} \qquad H(j\omega) = \frac{1.01 \cdot 10^7}{10^5 \cdot 10^7} \cdot \frac{j\omega}{\left(1 + \frac{j\omega}{10^5}\right) \cdot \left(1 + \frac{j\omega}{10^7}\right)}$$

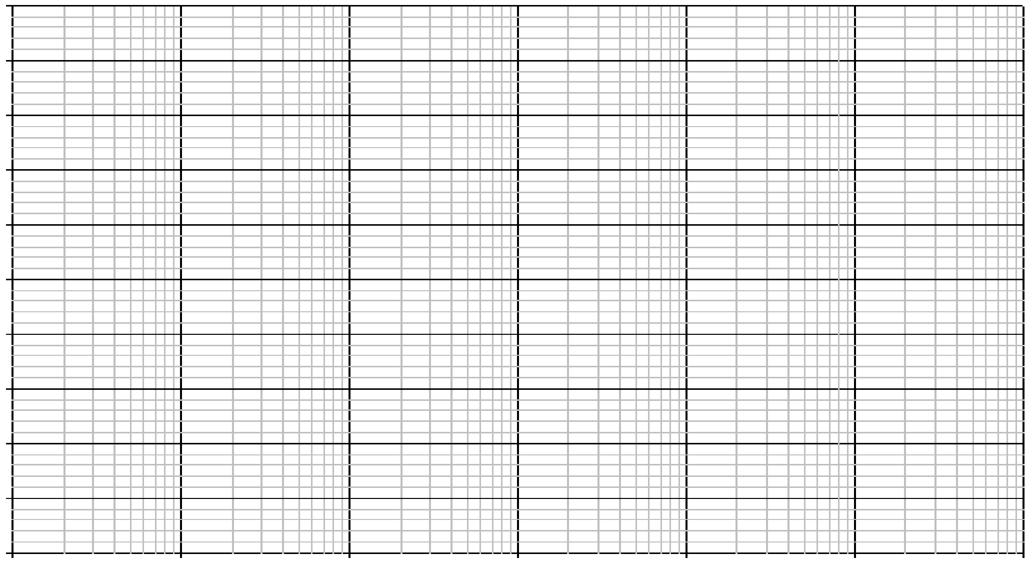
zero: 0
pole 10^5, 10^7

$s < 10^5$ $\quad 1.01 \cdot 10^7 \cdot \frac{s}{(10^5) \cdot (10^7)}$ $\quad \omega < 10^5$ $\quad 1.01 \cdot 10^{-5} \cdot \frac{\omega}{1 \cdot 1}$ $\quad 20 \frac{db}{dec}$ $\quad 20 \log(1) = 0$

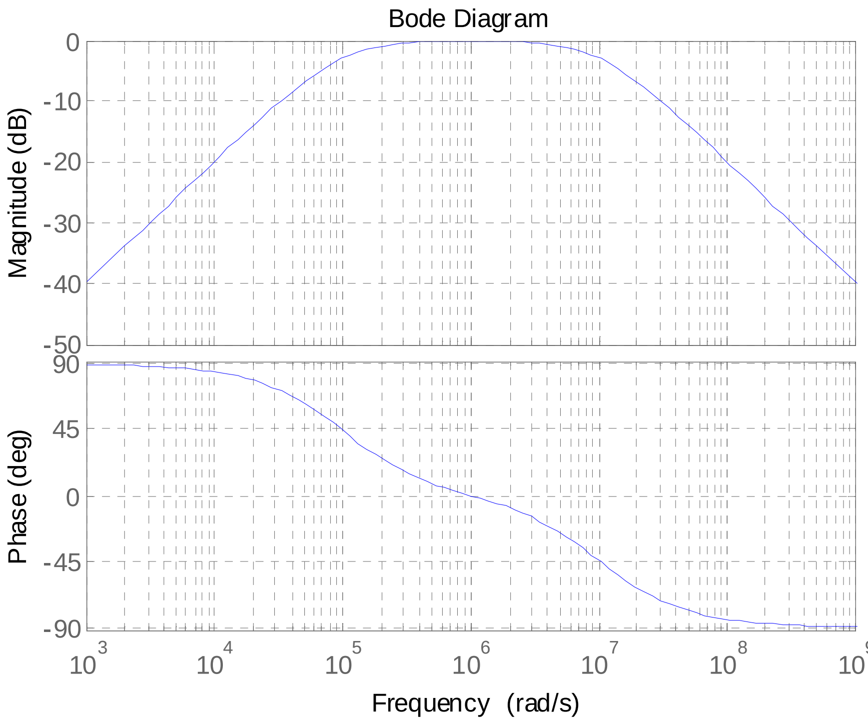
$10^5 < s < 10^7$ $\quad 1.01 \cdot 10^7 \cdot \frac{s}{(s) \cdot (10^7)}$ $\quad \frac{1.01 \cdot 10^7}{10^7}$ $\quad 10^5 < \omega < 10^7$ $\quad 1.01 \cdot 10^{-5} \cdot \frac{\omega}{\frac{\omega}{10^5}} = 1.01 \cdot 10^{-5} \cdot 1.01 \cdot 10^5$

constant

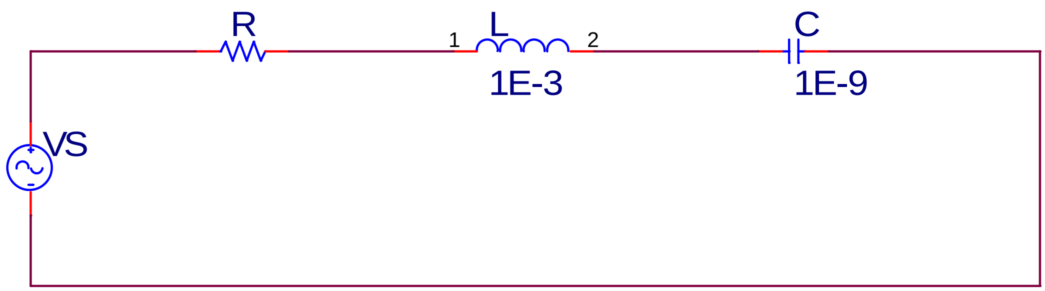
$s > 10^7$ $\quad 1.01 \cdot 10^7 \cdot \frac{s}{(s) \cdot (s)} = \frac{1.01 \cdot 10^7}{s}$ $\quad \omega > 10^7$ $\quad 1.01 \cdot 10^{-5} \cdot \frac{\omega}{\frac{\omega^2}{10^5 \cdot 10^7}}$ $\quad -20 \frac{db}{dec}$



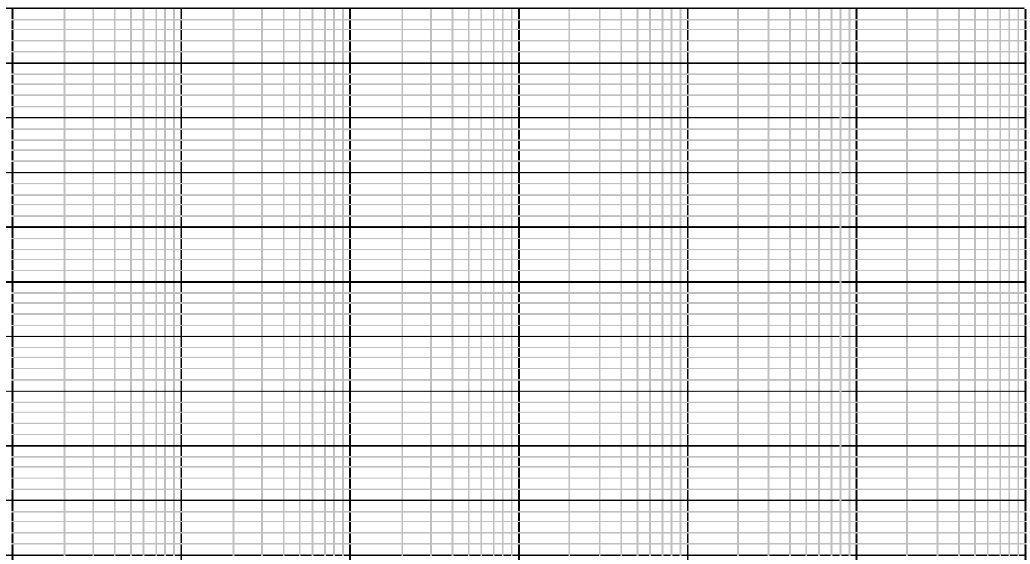
Overdamped, bandpass filter



2) Second order circuits TEAM ASSIGNMENT



a. H(s) = VC(s)/V(s) ***when R=2kΩ***



$R_{2a} := 2k\Omega$

$$H(s) = \frac{\frac{1}{LC}}{s^2 + \frac{R_T}{L}s + \frac{1}{LC}}$$

$$\frac{1}{L_1 \cdot C_1} = 1 \times 10^{12} \frac{1}{s^2}$$

$$\frac{R_{2a}}{L_1} = 2 \times 10^{6} \frac{1}{s}$$

$$H(s) = \frac{10^{12}}{s^2 + 2 \cdot 10^6 + 10^{12}}$$

$$H(s) = \left(\frac{10^6}{s+10^6}\right)^2 \qquad H(s) = \left(\frac{10^6}{s+10^6}\right)\left(\frac{10^6}{s+10^6}\right)$$

equivalent to two cascaded first order filters

Zeros: none
Poles 10^6 double

This is a critically damped second order circuit

critically damped low pass filter

If $R_1 := 2k\Omega$ $\quad L_1 := 1mH$ $\quad C_1 := 1nF$ $\quad \frac{1}{L_1 \cdot C_1} = 1\times10^{12}\frac{1}{s^2}$

$$H(s) = \frac{10^{12}}{s^2 + 2\cdot10^6 s + 10^{12}}$$

$$\alpha := \frac{2\cdot10^6}{2} = 1\times10^6$$

$$\omega_0 := \sqrt{\frac{1}{L_1\cdot C_1}} = 1\times10^6\frac{1}{s}$$

$$\zeta := \frac{\alpha}{\omega_0} = 1s$$

This is = 1 crictically damped

$$H(s) = \frac{10^{12}}{\left(s+10^6\right)^2}$$

$$H(j\omega) = \frac{10^{12}}{10^{12}}\cdot\frac{1}{\left(1+\frac{j\omega}{10^6}\right)^2}$$

zero: none
pole: double 10^6

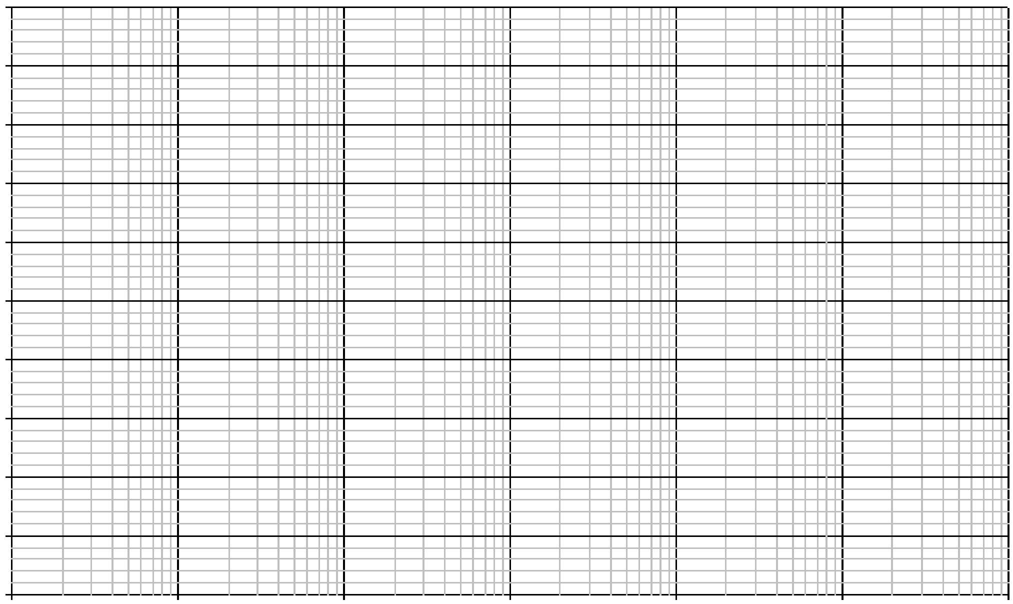
$s < 10^6$ $\quad \frac{10^{12}}{\left(10^6\right)^2} = 1$ $\qquad \omega < 10^6$ $\quad 1$ $\qquad 20\cdot\log(1) = 0$

$s > 10^6$ $\quad \frac{10^{12}}{\left(s^2\right)}$ $\quad \frac{10^{12}}{s^2}$ $\qquad \omega > 10^7$ $\quad \frac{1}{\frac{\omega^2}{10^5\cdot10^7}}$ $\quad -40\frac{db}{dec}$

correction -6 db

Critically damped case: double poles, Low pass filter

b. H(s) = VL(s)/V(s) when R = 2kΩ



$$H(s) = \frac{s^2}{s^2 + \frac{R}{L}\cdot s + \frac{1}{LC}}$$

$$H(j\omega) = \frac{(j\omega)^2}{\left(1 + \frac{j\omega}{10^5}\right)\cdot\left(1 + \frac{j\omega}{10^7}\right)}$$

If $R_1 := 2k\Omega$ $\quad L_1 := 1mH$ $\quad C_1 := 1nF$ $\quad \frac{1}{L_1 \cdot C_1} = 1 \times 10^{12} \frac{1}{s^2}$

$$H(s) = \frac{s^2}{s^2 + 2\cdot 10^6 s + 10^{12}}$$

$$H(s) = \frac{s^2}{\left(s + 10^6\right)^2}$$

$$H(j\omega) = \frac{1}{10^{12}} \cdot \frac{(j\omega)^2}{\left(1 + \frac{j\omega}{10^6}\right)^2}$$

zero: double 0
pole: double pole 10^6

ends at

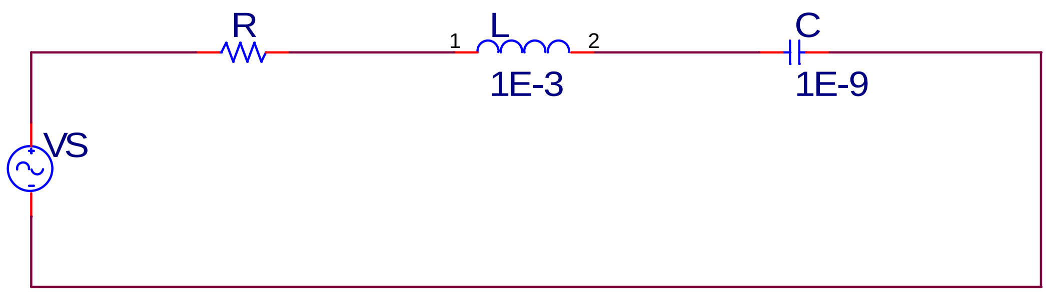
$s < 10^6 \qquad \frac{s^2}{\left(10^6\right)^2} \qquad \omega < 10^6 \qquad \frac{1}{10^{12}} \frac{\omega^2}{1} \qquad 40 \frac{db}{dec} \qquad 20\log(1) = 0$

$s > 10^6 \qquad \frac{s^2}{s^2} = 1 \qquad \omega > 10^6 \qquad \frac{1}{10^{12}} \cdot \left[\frac{\omega^2}{\frac{\omega^2}{\left(10^6\right)^2}}\right]$ consant 0 db

correction -6 db

Critically damped case: double poles, high pass filter

3) Underdamped cases of the RLC series circuit



a.) H(s) = VC(s)/V(s) when R = 1kΩ

$$H(s) = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

If $R_1 := 1k\Omega \qquad L_1 := 1mH \qquad C_1 := 1 \cdot 10^{-9} F$

$$\frac{R_1}{L_1} = 1 \times 10^6 \frac{1}{s} \qquad \frac{1}{L_1 \cdot C_1} = 1 \times 10^{12} \frac{1}{s^2}$$

$$H(s) = \frac{10^{12}}{s^2 + 10^6 s + 10^{12}}$$

$$\alpha := \frac{10^6}{2} = 5 \times 10^5$$

$$\omega_0 := \sqrt{\frac{1}{L_1 \cdot C_1}} = 1 \times 10^6 \frac{1}{s}$$ This is the resonant frequency

$$\zeta := \frac{\alpha}{\omega_0} = 0.5 s$$ ***This is < 1 this is a very specific case of underdamped***

$H(s)$ s goes to 0 goes to 1

$H(s)$ so goes to infinity goes to 0 $\frac{10^{12}}{s^2}$ $-40\frac{db}{dec}$

Still a low pass filter

At the limits you can start like a critically damped bode plot.

Critically damped ωo = ωcutoff (were the same)

At ωo evaluate $\left|H\left(j\omega_o\right)\right|$ so our H(s) convert to H(jω) $H(s) = \frac{\frac{1}{LC}}{s^2+\frac{R}{L}s+\frac{1}{LC}} = \frac{\frac{1}{LC}}{(j\omega)^2 + 2\alpha\cdot j\,\omega + \frac{1}{LC}}$

remember $\omega_o{}^2 = \frac{1}{LC}$

$$\frac{\omega_o{}^2}{-\omega_o{}^2 + 2\alpha j\omega_o + \omega_o{}^2} = \frac{\omega_o{}^2}{2\alpha\omega_o\cdot j}$$

remember $0.5 = \frac{\alpha}{\omega_o}$ so $\alpha = 0.5\omega_o$

$$\left|H\left(j\omega_o\right)\right| = \frac{\omega_o}{2\times 0.5\omega_o\cdot j} = \frac{1}{j}$$

$-j$ $\sqrt{-1^2}$

$$\left|H\left(j\omega_o\right)\right| = 1$$

$20\log\left(\left|H\left(j\omega_o\right)\right|\right) = 0\text{db}$ so we have a known point at ωo and we ELIMINATED the correction or gain loss so we have a better passband. It was -6dB

Damping ratio of 0.5 gives you a flat passband with a 40 db rolloff, better than cascaded first order filte

b.) H(s) = VC(s)/V(s) when R = 1.41kΩ

$$H(s) = \frac{\frac{1}{LC}}{s^2+\frac{R}{L}s+\frac{1}{LC}}$$

If $R_1 := 1.41k\Omega$ $L_1 := 1mH$ $C_1 := 1\cdot 10^{-9}F$

$$\frac{R_1}{L_1} = 1.41\times 10^6\,\frac{1}{s}$$

$$\frac{1}{L_1\cdot C_1} = 1\times 10^{12}\,\frac{1}{s^2}$$

$$H(s) = \frac{10^{12}}{s^2 + 1.41\times 10^6 s + 10^{12}}$$

$$\alpha := \frac{1.41\times 10^6}{2} = 7.05\times 10^5$$

$$\omega_0 := \sqrt{\frac{1}{L_1\cdot C_1}} = 1\times 10^6\,\frac{1}{s}$$ This is the resonant frequency

$$\zeta := \frac{\alpha}{\omega_0} = 0.705\,s \qquad \frac{1}{\sqrt{2}}$$

This is < 1 this is less underdamped than the previous

$H(s)$ s goes to 0 goes to 1

$H(s)$ so goes to infinity goes to 0

Still a low pass filter

$$\left|H(j\omega_o)\right| = \frac{1}{\sqrt{2}}$$

$$\frac{{\omega_o}^2}{-{\omega_o}^2 + 2\alpha j\omega_o + {\omega_o}^2} = \frac{{\omega_o}^2}{2\alpha\omega_o\cdot j}$$

$$20\log\left(\frac{1}{\sqrt{2}}\right) = -3.01 \quad \text{dB}$$

So now this point is -3dB at ωo

now $\alpha = \frac{1}{\sqrt{2}}\omega_o$ $\quad \frac{\omega_o}{2\cdot\frac{1}{\sqrt{2}}\omega_{oj}}$

$$-\frac{\sqrt{2}}{2}j \qquad \left(\sqrt{\frac{\sqrt{2}}{2}}\right)^2 = 0.707 \qquad \frac{1}{\sqrt{2}}$$

c.) H(s) = VC(s)/V(s) when R = 1.41kΩ

$$H(s) = \frac{\frac{1}{LC}}{s^2+\frac{R}{L}s+\frac{1}{LC}}$$

If $R_1 := 100\Omega$ $L_1 := 1mH$ $C_1 := 1\cdot 10^{-9}F$

$$\frac{R_1}{L_1} = 1\times 10^5\,\frac{1}{s}$$

$$\frac{1}{L_1\cdot C_1} = 1\times 10^{12}\,\frac{1}{s^2}$$

$$H(s) = \frac{10^{12}}{s^2 + 10^5 s + 10^{12}}$$

$$\alpha := \frac{\frac{R_1}{L_1}}{2} = 5\times 10^4\,\frac{1}{s}$$

$$\omega_0 := \sqrt{\frac{1}{L_1\cdot C_1}} = 1\times 10^6\,\frac{1}{s}$$ This is the resonant frequency

$\zeta := \dfrac{\alpha}{\omega_0} = 0.05$

This is << 1 this is very underdamped

$H(s)$ s goes to 0 goes to 1

$H(s)$ so goes to infinity goes to 0 $-40\dfrac{db}{dc}$

Still a low pass filter

$|H(j\omega_o)| = 10$

$\dfrac{\omega_o^{\,2}}{-\omega_o^{\,2} + 2\alpha j\omega_o + \omega_o^{\,2}} = \dfrac{\omega_o^{\,2}}{2\alpha\omega_o \cdot j}$

$20\log(10) = 20$ dB

So now this point is 20dB at ωo

now $\alpha = 0.05\omega_o$ $\dfrac{\omega_o}{2\cdot 0.05\omega_{oj}} = \dfrac{1}{0.1j}$

$2\cdot 0.05 = 0.1$

This creates a peak, a large one
and is not necessarily a good low pass filter
A flat passband is ideal remember.

$-10j$ $\sqrt{(-10)^2} = 10$

Basic process summary for second order circuits:

Overdamped

Critically damped

Bode plots
Find real poles, regions, analyze

Underdamped

Use critically damped approximation at the limits

Find correction at ωo

$1 > \zeta > 0.5$ Correction is a -db of some value

$\zeta = 0.5$ Correction is 0db

$\zeta < 0.5$ Correction is positive db (Strongly underdamped which means there is a peak)