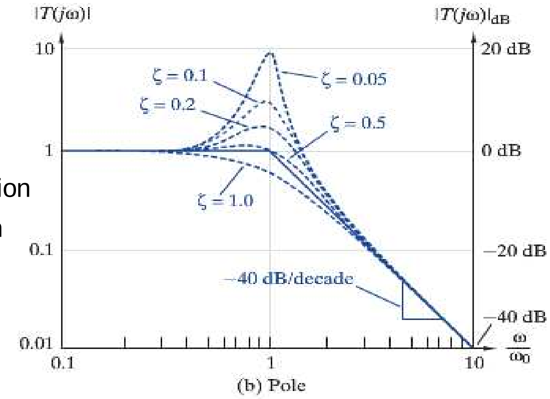
How do overdamped, critically damped, and underdamped bandpass circuits relate to each other?
Is an underdamped bandpass circuit useful?

Underdamped circuit $\zeta<0.5$
Low Pass and High Pass Circuit

Critically damped at extremes

Find value of Gain in dB at ω_0
Last week we plugged $j\omega_0$ into our transfer function and made a substitution for α using α=ζ*ω0 then found $\left|H(j\omega_0)\right|$



Look back at notes and you'll find that

$$\left|H(j\omega_0)\right| = \frac{1}{2\zeta}$$

Finally take $20\log\left(\left|H(j\omega_0)\right|\right)$

i.e. ζ=0.5 gave $\left|H(j\omega_0)\right| = 1$

$\zeta = \frac{1}{\sqrt{2}}$ $\quad\left|H(j\omega_0)\right| = 0.707$

$\zeta = 0.05$ $\quad\left|H(j\omega_0)\right| = 20$

Process Summary

2ND ORDER PROCESS SUMMARY

Overdamped
1) Find Poles
2) Identify Regions
3) Build Straight Line Approximations
4) Add corrections (-3db)

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2ND ORDER PROCESS SUMMARY

Critically Damped
1) Find Poles
2) Identify Regions
3) Build Straight Line Approximations
4) Add corrections (-6db)

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2ND ORDER PROCESS SUMMARY

Underdamped LPF, HPF
1) Start with critically damped case $?_c = ?_o$
2) Sketch Straight Line Approximations away from $?_o$
3) At $?_o$ 20 log abs H(j $?_o$)=20 log(1/(2 ?)) > -6 dB relative to passband

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2ND ORDER PROCESS SUMMARY

Underdamped BPF
1. Asymptotes take the form of inverted V
2. Each side of V has 20 dB rolloff
3. At $?_o$ 20 log abs H(j $?_o$)=0 dB ALWAYS
4. The point of the inverted V is 20 log abs H(j $?_o$) away from 0dB
5. Use 20 log(2 ?) to find this point pulling V up or down relative to 0dB making it narrow or wide

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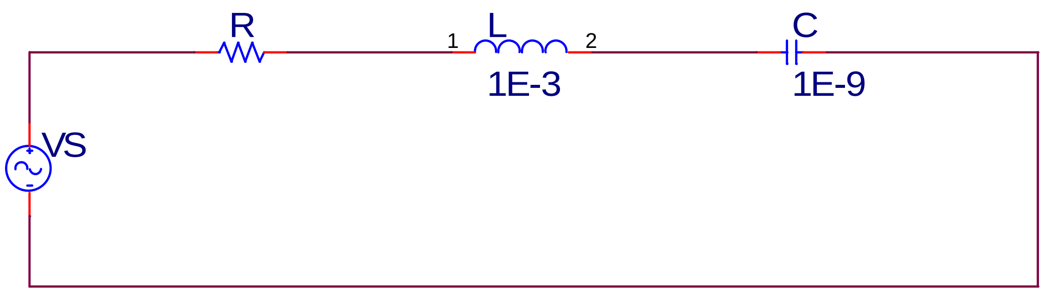
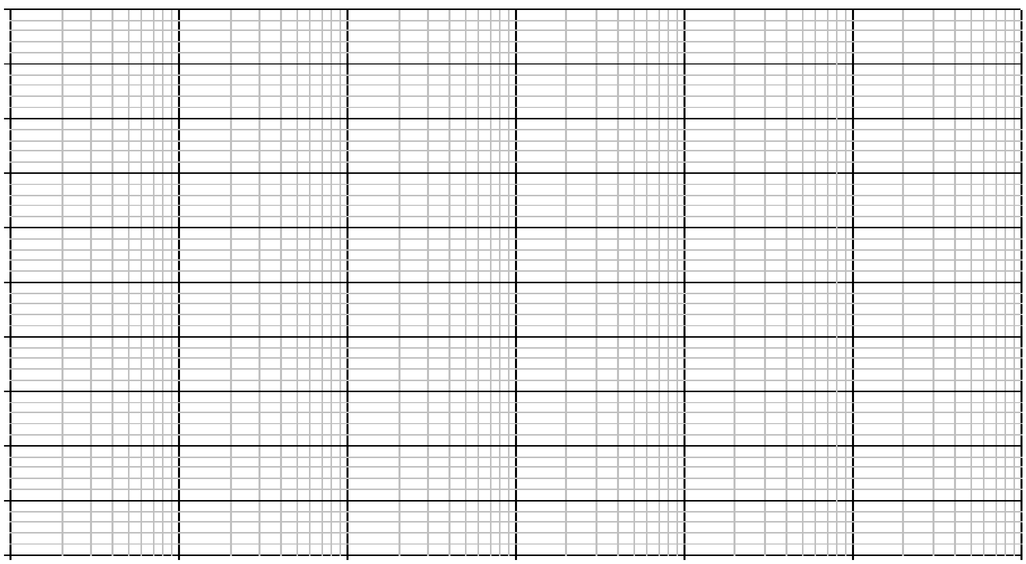
Problem 1)

1) Second order circuits



a) H(s) = VR(s)/V(s) when R=2kΩ



a.) H(s) = VR(s)/V(s) when R = 2kΩ

$\zeta = 1$

$s^2 + 2\cdot 10^6 + 10^{12}$

$C_{1a} := 1\cdot 10^{-9}$

$R_{1a} := 2k\Omega$

$L_{1a} := 1\cdot 10^{-3}H$

$$H(s) = \frac{\frac{R}{L}\cdot s}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = \frac{2\alpha s}{s^2 + 2\alpha s + \omega_o^2}$$

$$\frac{R_{1a}}{L_{1a}} = 2\times 10^6 \frac{1}{s}$$

$$H(s) = \frac{2\cdot 10^6\cdot s}{s^2 + 2\cdot 10^6 s + 1\cdot 10^{12}} = \frac{2\cdot 10^6 s}{\left(s + 1\cdot 10^6\right)^2}$$

$$\frac{1}{L_{1a}\cdot C_{1a}} = 1\times 10^{12}\frac{1}{H}$$

We expected -6dB correction at double pole using the straight line approximation, also 20 dB rolloff

$$\left|H(j\omega_0)\right| = \frac{2\alpha s}{-\omega_0^{\,2} + 2\alpha s + \omega_o^{\,2}} = 1$$

It will always be one no matter what the damping for a critically damped BPF

$20\log(1) = 0$ At ωo 0db point ALWAYS

So we need to meet two rules.
1. 0db at wo
2. -6db correction at double pole

The 20 db asymptote lines meet at 6 dB above ωo!

This is 20 log (2ζ)!

$$20\cdot\log(2\cdot 1) = 6.021$$

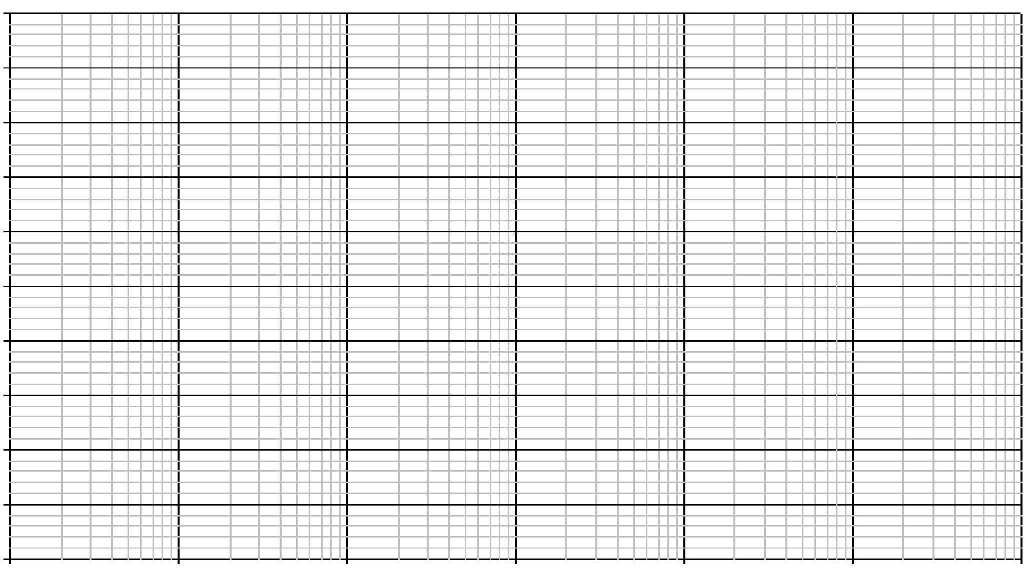
b) H(s) = VR(s)/V(s) when R=10.1kΩ (ζ=5.05)

$R_{1b} := 10.1\text{k}\Omega$ $L_{1b} := L_{1a}$

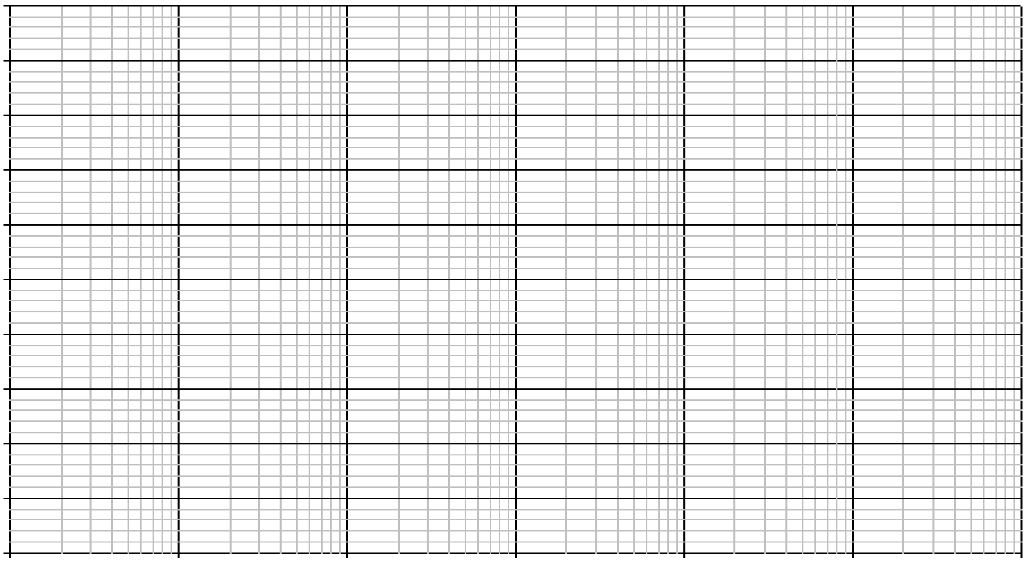
$20\log(2\cdot 5.05) = 20.086$

$\alpha := \dfrac{R_{1b}}{2\cdot L_{1b}}$ $\omega_0 := \sqrt{\dfrac{1}{L_{1b}\cdot C_{1a}}}$ $\dfrac{\alpha}{\omega_0} = 5.05\,\dfrac{\text{m}\cdot\text{kg}^{0.5}}{\text{A}\cdot\text{s}^2}$

Looks exactly like the overdamped case!



b) H(s) = VR(s)/V(s) when R=100Ω



Now the **underdamped** BPF case

$R = 100\Omega$

$\omega_0 = 10^6$

$\alpha = 5 \cdot 10^4$

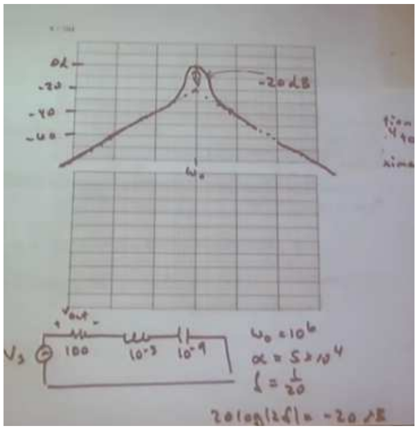
$\zeta = \frac{1}{20}$

At ωo we always have a 0dB point
Where do we put the straight line approximations

$$20\log\left(2\cdot\frac{1}{20}\right) = -20$$

We must converge to the asymptotes that start (point) at -20 db from 0db

Narrow bandpass filter



Introduction to Filter design

Chain rule

$$H(s) = H_1(s) \cdot H_2(s) \cdot H_3(s) \quad .... H_n(s)$$

These are stages and they can be either first or or second order active filters

Real poles are produced using first order building blocks
Complex poles are produced by second order building blocks (damping ratio and ω0)

Design an active filter by controlling the poles introduced by each stage of a cascade connection

In a second order transfer function, the denominator controls the location of poles and thus the critical frequencies

The numerator controls the zeros and thus determines if a transfer function is a low-pass, high-pass or band reject filter

Better filter, high and flat gain, steep rolloff

In general For first order filters cascaded there is a tradeoff between gain (flatness of passband) and steep roll off

For second order circuits

- Critically damping is equivalent to cascaded first order circuits (-6dB corrections)
- Overdamping is equivalent to cascaded first order circuits (2 -3db corrections at two different critical frequencies)
- Underdamping can achieive a flatter passband with the same stopband! Gold star.

From the book:

From Lecture

First order filters

Low pass filter form	$\dfrac{\omega_c}{s + \omega_c}$	1 pole
High pass filter form	$\dfrac{s}{s + \omega_c}$	1 zero at origin 1 pole

Second order filters

Low pass filter form

$$H(s) = \frac{K}{\left(\frac{s}{\omega_0}\right)^2 + 2\cdot\zeta\left(\frac{s}{\omega_0}\right) + 1} \qquad \frac{\omega_0^{\,2}}{s^2 + 2\alpha s + \omega_0^{\,2}}$$

2 poles

High pass filter form

$$H(s) = \frac{K\left(\frac{s}{\omega_0}\right)^2}{\left[\left(\frac{s}{\omega_0}\right)^2 + 2\cdot\zeta\left(\frac{s}{\omega_0}\right) + 1\right]} \qquad \frac{s^2}{\left(s^2 + 2\alpha s + \omega_0^{\,2}\right)}$$

2 zeros at orgin

2 poles

Bandpass filter form

$$H(s) = \frac{K\left(\frac{s}{\omega_0}\right)}{\left[\left(\frac{s}{\omega_0}\right)^2 + 2\cdot\zeta\left(\frac{s}{\omega_0}\right) + 1\right]} \qquad \frac{2\alpha s}{s^2 + 2\alpha s + \omega_0^{\,2}}$$

1 zero

2 poles

Bandstop (notch filter form)

$$H(s) = \frac{K\cdot\left[\left(\frac{s}{\omega_0}\right)^2 + 1\right]}{\left[\left(\frac{s}{\omega_0}\right)^2 + 2\cdot\zeta\left(\frac{s}{\omega_0}\right) + 1\right]} \qquad \frac{s^2 + \omega_0^{\,2}}{s^2 + 2\alpha s + \omega_0^{\,2}}$$

$$T(s) = \frac{\text{numerator}}{(s/\omega_0)^2 + 2\zeta(s/\omega_0) + 1} = \frac{\text{numerator}}{(s/\omega_0)^2 + \frac{B}{\omega_0}(s/\omega_0) + 1} = \frac{\text{numerator}}{(s/\omega_0)^2 + \frac{1}{Q}(s/\omega_0) + 1}$$

where ζ is the damping ratio, B is the bandwidth and Q is the quality factor of the circuit. Different types of filter circuits are specified using different parameters. All

3) Filter Design

Design a filter that meets the specifications below. You need to pick values for any resistors, capacitors or inductors in your circuit.

ω [rad/s]	|H(s)| in dB
10	-25
100	-5
1000	12
1E4	15
1E5	15
1E6	15

What do you notice here. They highest point saturates at 15 db. This must be the passband.

-3db from 15db, looks like a correction at ωc so ωc=1000 rad/s

Then you see -20 db rolloff

This must be a high pass filter with
1. ωc=1000
2. Correction of -3db
3. Passband at 20 log K=15

$$H(s) = K\frac{s}{s+\omega_c} = 5.6\frac{s}{s+1000}$$