1) Bode plots/Transfer functions

a. Draw magnitude and phase bode plots for the transfer function

$$H(s) = 0.01 \cdot \frac{s \cdot (s+100)}{(s+1E4)}$$

In your magnitude plot, indicate corrections at the poles and zeros.

Step 1: Find poles, zeros

Zeros: 0, 100
Poles: 1E4

Step 2: Define regions and find either H(jω) or H(s)

$$\frac{100 \cdot 0.01}{1 \cdot 10^4} = 1 \times 10^{-4}$$

$s < 100$

$$H(s) = \frac{0.01 \cdot s \cdot 100}{1 \cdot 10^4} = 1 \cdot 10^{-4} s$$

+20db/dec slope

$$100 \cdot 1 \cdot 10^{-4} = 0.01$$

slope ends at $20 \cdot \log(0.01) = -40$

$100 < s < 10^4$

$$\frac{0.01}{1 \cdot 10^4} = 1 \times 10^{-6}$$

$$H(s) = \frac{0.01 \cdot s \cdot s}{1 \cdot 10^4} = 1 \cdot 10^{-6} s^2$$

+40 db/dec slope

$$20 \log\left[10^{-6} \cdot \left(1 \cdot 10^4\right)^2\right] = 40$$

$s > 10^4$

$$H(s) = \frac{0.01 \cdot s \cdot s}{s} = 0.01 s$$

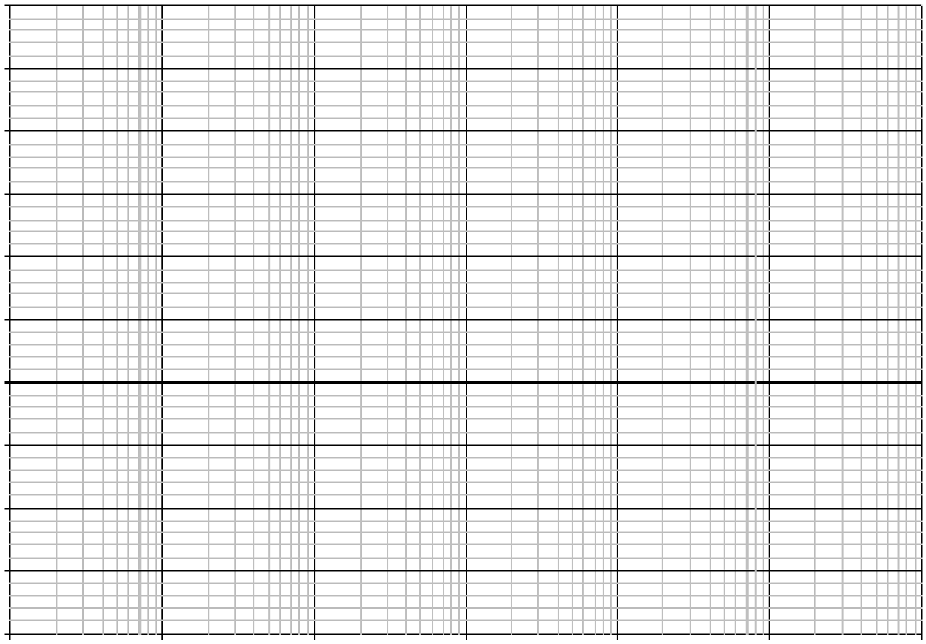
+20db/dec slope

Step 3: Make corrections

Correction at 100 is +3db because it is a ZERO^1!

Correction at 10^4 is -3db because it is a POLE^1!

$$H(s) = 10\frac{s^2}{(s+100)(s+10000)}$$



Phase plots

$0.1\omega_{c1} = 10$

$\omega = 10$

$$\angle H(j10) = 0.01 \cdot \frac{j10 \cdot (j10 + 100)}{(10j + 1E4)} \qquad \frac{\angle 0 \cdot (\angle 90)(\angle 0)}{\angle 0} \qquad \angle 90$$

$$\angle 0 + \angle 90 + \angle 0 - \angle 0$$

$10 \cdot \omega_{c1} = 10^3$

$\omega = 10^3$

$$\angle H\left(j10^3\right) = 0.01 \cdot \frac{j10^3 \cdot \left(j10^3 + 100\right)}{\left(10^3 j + 1E4\right)} \qquad \frac{\angle 0 \cdot (\angle 90)(\angle 90)}{\angle 0} \qquad \angle 180$$

$$\angle 0 + \angle 90 + \angle 90 - \angle 0 \qquad 180 - 45 = 135$$

$\omega = 100$ $\qquad \angle 135$

$0.1\omega_{c2} = 10^3$ already done $\qquad \angle 180$

$10\omega_{c2} = 10^5$

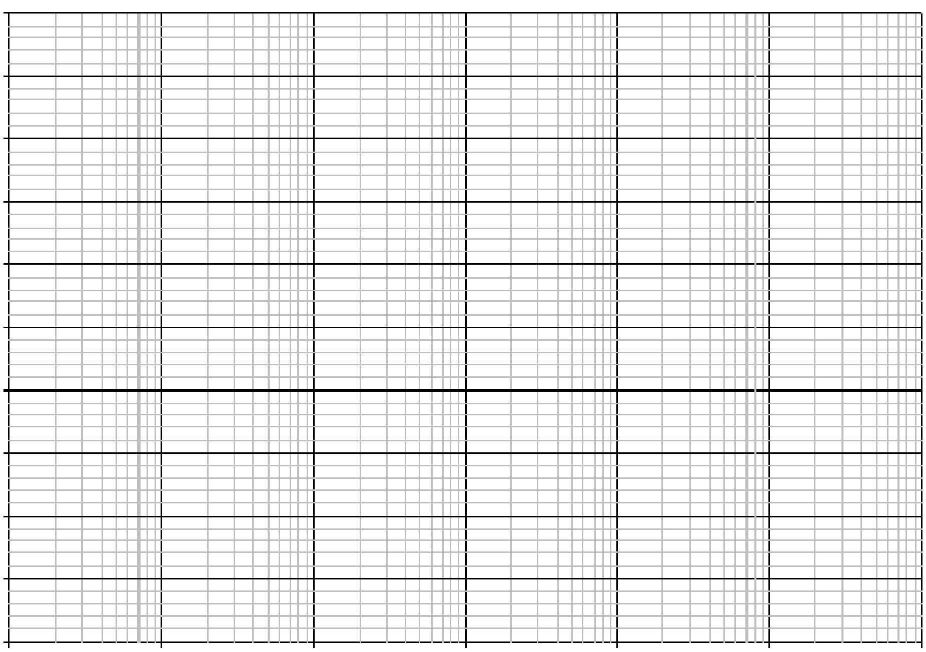
$$\angle H\left(j10^5\right) = 0.01 \cdot \frac{j10^5 \cdot \left(j10^5 + 100\right)}{\left(10^5 j + 1E4\right)} \qquad \frac{\angle 0 \cdot (\angle 90)(\angle 90)}{\angle 90} \qquad \angle 90$$

$$\angle 0 + \angle 90 + \angle 90 - \angle 90$$

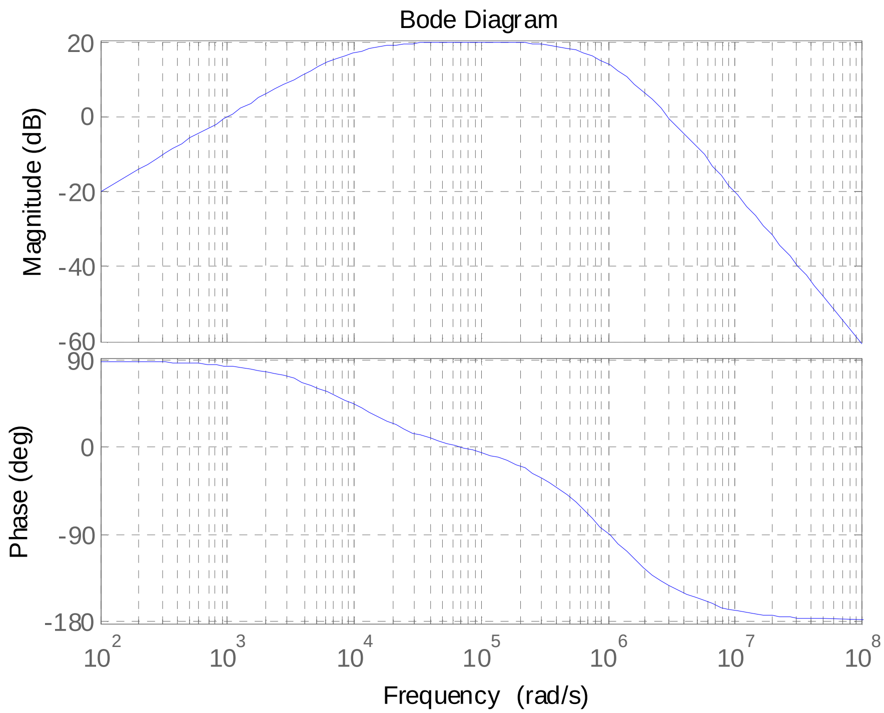
$\omega = 10^4$

$$\angle H\left(j10^4\right) = 0.01 \cdot \frac{j10^4 \cdot \left(j10^4 + 100\right)}{\left(10^4 j + 1E4\right)} \qquad \frac{\angle 0 \cdot (\angle 90)(\angle 90)}{\angle 45} \qquad \angle 135$$

$$H(s) = 10\frac{s^2}{(s+100)(s+10000)} \qquad \angle 0 + \angle 90 + \angle 90 - \angle 45 \qquad 180 - 45 = 135$$



b. Determine the transfer function for the following magnitude and phase Bode plots (the plots are for the same circuit).



1. We have a 20 db pass band so

$$20\log(K) = 20\text{db} \qquad \frac{20}{20} = 1 \qquad 10^1 = 10$$

$$K = 10$$

2. We have 40 db rolloff on the right where ωo=1E6 rad/s, critically damped low pass filter, double pole there

$$\frac{\left(10^6\right)^2}{\left(s + 10^6\right)^2} = \left(\frac{\omega_o}{s + \omega_o}\right)^2$$

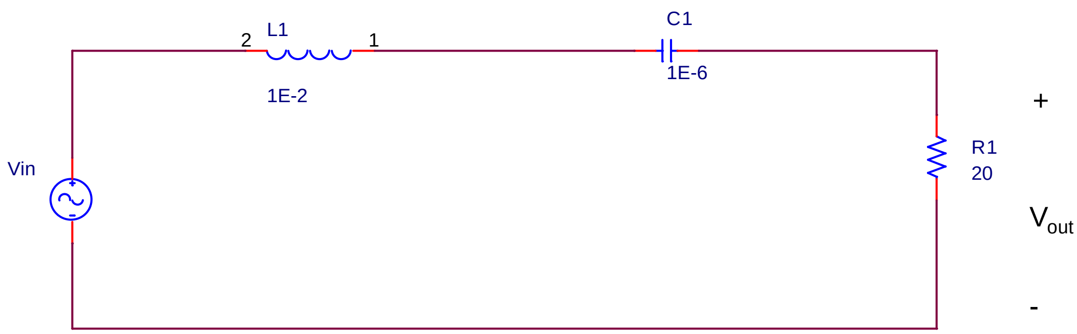
3. 1st order high pass filter, 20db rolloff, ωo=10^4 rad/s

$$\frac{s}{s + 10^4} = \frac{s}{s + \omega_o}$$

Put all together

$$H(s) = 10\cdot\frac{10^{12}}{\left(s+10^6\right)^2}\cdot\frac{s}{\left(s+10^4\right)} = \frac{10\cdot 10^{12}s}{\left(s+10^4\right)\left(s+10^6\right)^2}$$

2) 2nd order Bandpass Bode plots



a. Determine the transfer function for the above circuit $H(s) = \frac{V_R(s)}{V_{in}(s)}$

$$H(s) = \frac{2\alpha s}{s^2 + 2\alpha s + \omega_0^{\,2}} = \frac{\frac{R}{L}\cdot s}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$R_2 := 20$ $\quad L_2 := 1\cdot 10^{-2}$ $\quad C_2 := 1\cdot 10^{-6}$

b. Determine α, ω_0, and ζ (damping ratio).

$\frac{R_2}{L_2} = 2\times 10^3$

b) Band pass filter

1. Asymptotes take the form of an inverted V
2. Each has a 20 db rolloff
3. At ω0, 20 log abs (H(jω)) = 0db always
4. The point of the inverted V is 20 log abs (H(jω0)) away from 0db

$\alpha := 1000$

$\omega_0 := \sqrt{\frac{1}{L_2\cdot C_2}}$

20 log (2ζ) Narrow or wide?

$\omega_0 = 1\times 10^4$

$\zeta := \frac{\alpha}{\omega_0}$ $\qquad \zeta = 0.1$

c. Draw the Bode-magnitude plot. Indicate the magnitude (dB) of the transfer function at the resonant frequency.

Remember peak must always be at 0db at ωo, we have to figure out where the asymptotes will be +,- 20db/dec.

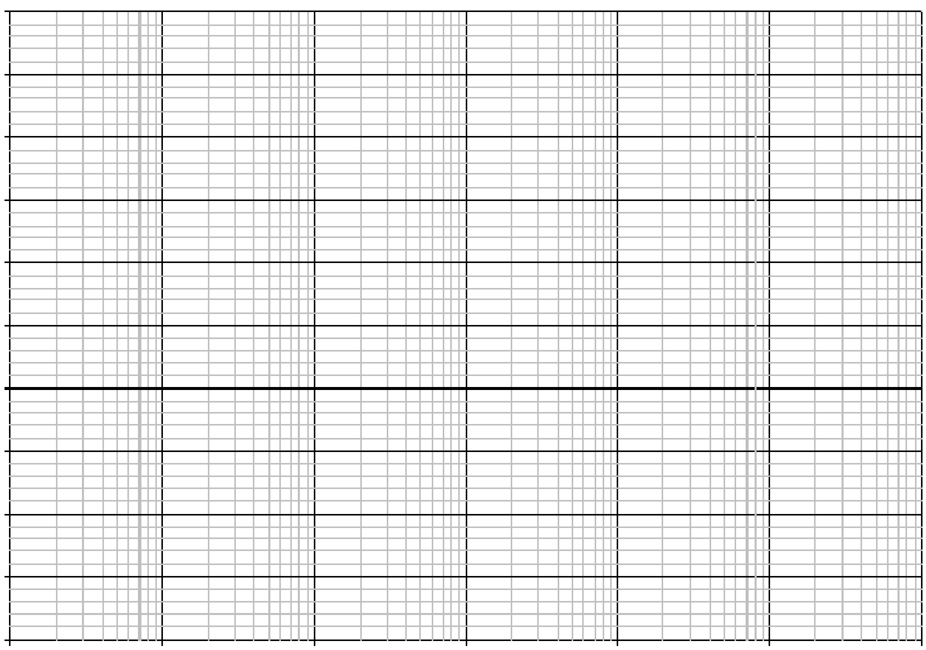
H(s) s goes to 0 $\quad H(s) = \dfrac{2\alpha s}{\frac{1}{LC}}$ +20db/dec

H(s) s goes to ∞ $\quad H(s) = \dfrac{2\alpha s}{s^2}$ -20db/dec

$$H(j\omega_0) = \frac{2\alpha j\omega_0}{-\omega_0^{\;2} + 2\alpha j\omega_0 + \omega_0^{\;2}} = 1$$ always! at ω0 $\quad 2\cdot\zeta\cdot\omega_0^{\;2} j$

$$20\log(2\cdot\zeta) = -13.979$$

$$H(s) = 10\frac{s^2}{(s+100)(s+10000)}$$



4) Design problems-Multiple Stages

Using only first order filters and opamp circuits for each stage, design a filter that meets the specifications below. You need to pick values for any resistors, capacitors or inductors in your circuit. Your circuit must consist of at least one inductor and at least one capacitor.

a. Lowpass filter with a cutoff frequency of 1 MHz (Note: this value is given in Hz).
b. In the passband, the gain must be >10dB
c. The asymptotic slope of the stopbands should be -60dB/decade
d. You circuit must contain at least one inductor and at least one capacitor.

$\omega_c := 2\cdot\pi\cdot 1\text{MHz}$

$\omega_c = 6.283\times 10^6 \cdot \frac{\text{rad}}{\text{s}}$

Gain $\quad 20\log\cdot|K| > 10\text{db}$

Slope indicates a triple pole

$R_1 := 1.59\text{k}\Omega \qquad C_1 := 1\cdot 10^{-10}$

$R_3 := 6.28\text{k}\Omega$

$L_1 := 1\cdot 10^{-3}\text{H}$

$\frac{R_3}{L_1} = 6.28\times 10^6\,\frac{1}{\text{s}}$

Remember $\quad \omega_c = \frac{R}{L}$ for inductor/resistor across resistor

$\omega_c = \frac{1}{RC}$ for capacitor/resistor across capacitor

$\frac{1}{R_1\cdot C_1} = 6.289\times 10^6\,\frac{1}{\Omega}$

$R_4 := 90\text{k}\Omega \qquad R_5 := 10\text{k}\Omega$

Gain

Amplifier $\left(1+\frac{R_4}{R_5}\right) = 10$

Note: Need to overcome the -9 dB loss at corner frequency to get >10 dB to meet spec. So that is ~20dB needed

$20\log(10) = 20$

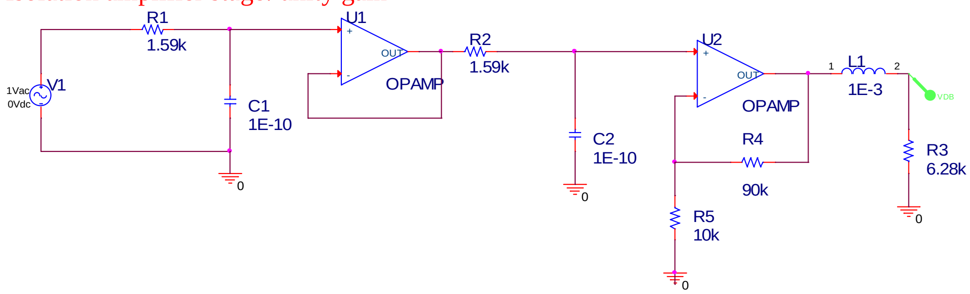
Also need to isolate first order filters

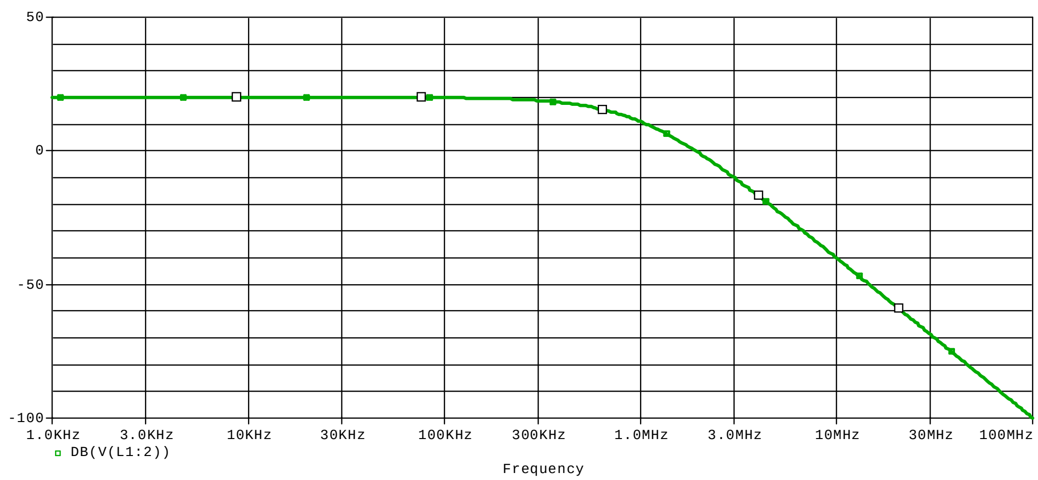
use buffer

Solution 1:
Lowpass filter
Cutoff frequency of 6.28E6 rad/s
Rolloff of 60dB/decade indicates a triple pole.
Three first order filters with amplifier stages for isolation.
-9dB point relative to the passband at 6.28E6 rad/s
Passband gain of 20dB to meet spec, 20log|K|=20 →K=10

$$\boxed{H(s) = K\left(\frac{\omega_c}{s+\omega_c}\right)^3}$$

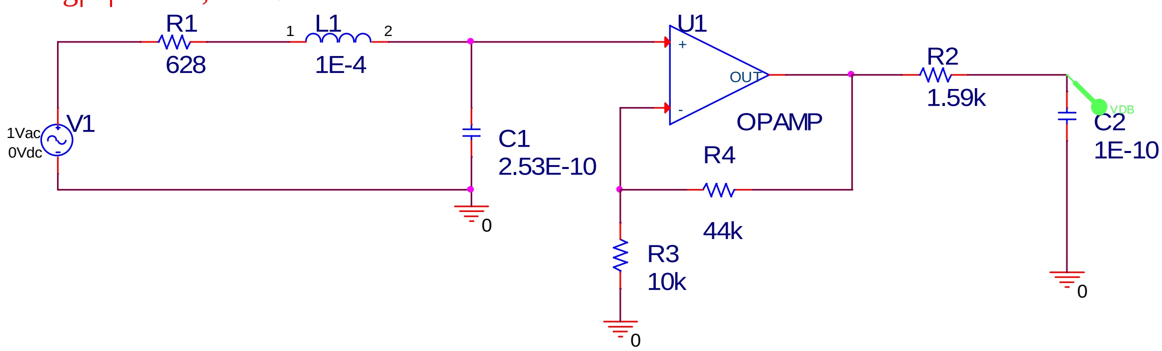
LPF stage, RL circuit: Choose an RL circuit, R = 6.28k, L = 1E-3
LPF stage, RC circuit: Choose an RC circuit, R = 1.59k, C = 1E-10
Amplifier stage: Choose a non-inverting amplifier, R2=90k, R1=10k
Isolation amplifier stage: unity gain





If restriction of only first order filter was lifted....you could also do....

Solution 2:
Second order LPF cascaded with a first order LPF
Use an underdamped circuit with a damping ratio of 0.5.
The gain term than needs to be 13dB to satisfy the passband requirements, giving
20log|K|=13dB, K=4.4\



$$H(s) = \frac{\frac{1}{LC}}{s^2 + \frac{R_T}{L}s + \frac{1}{LC}}$$

$$\omega_0 = \omega_c = 6.283 \times 10^6 \ \frac{rad}{s}$$

$$\omega_0 = \sqrt{\frac{1}{L_1 \cdot C_1}}$$

need ***at least a*** critically damped circuit, should use underdamped

chose L1 first if you like

$$L_{1b} := 1 \cdot 10^{-4}$$

$$\frac{1}{\left(\omega_0{}^2 \cdot L_1\right)} = C_1$$

$$\frac{1}{\left[\left(6.283 \times 10^6\right)^2 \cdot L_{1b}\right]} = 2.533 \times 10^{-10}$$

$$\sqrt{\frac{1}{1 \cdot 10^{-4} \cdot 2.53 \cdot 10^{-10}}} = 6.287 \times 10^6$$

$$\zeta = \frac{\alpha}{\omega_0}$$

For better filter make ζ=0.5

$$\alpha_b := 0.5 \cdot \left(6.283 \times 10^6\right)$$

$$2\alpha = \frac{R}{L}$$

$$R_{1b} := 2 \cdot \alpha_b \cdot L_{1b}$$

$$R_{1b} = 628.3$$

This gives -40 db roll off, with ω0 at 6.23*10^6 and a very flat passband

Still need another -20 db roll off so you can use a first order filter (like previous) which has -3dB rolloff

Gain

For this circuit we need over only overcome -3dB so that means a total of 13 dB

$$20\log\cdot|K| = 13$$

$$\frac{13}{20} = 0.65$$

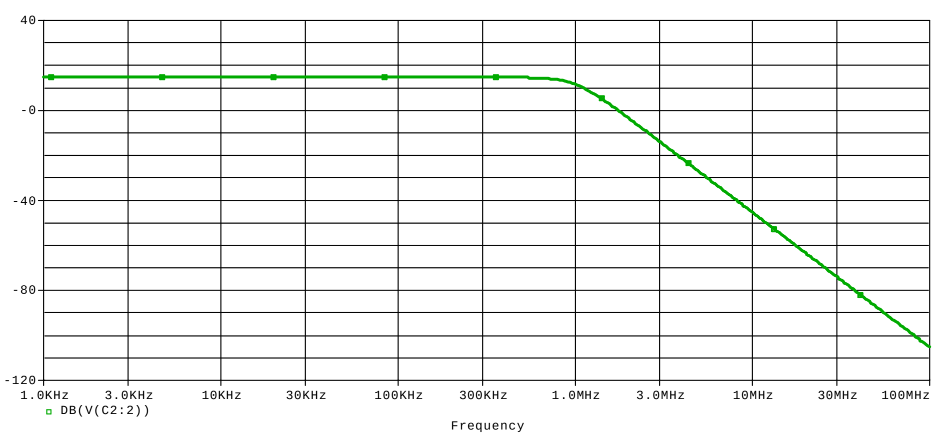
$$10^{0.65} = 4.467$$

Can use a non inverting amplifer with a gain of 4.5

$$R_{4b} := 35k\Omega$$

$$R_{3b} := 10k\Omega$$

$$\left(1 + \frac{R_{4b}}{R_{3b}}\right) = 4.5$$



5) Design Problem

Design a bandpass filter with the following specifications. Show your work and justify your calculations. Include a schematic of your circuit.

a. The passband is 1E4<ω<1E6
b. The passband gain should be 5dB
c. The low frequency stopband rolloff should be 20db/decade
d. The high frequency stopband rolloff shoudl be 60 db/decade
e. The low frequency cutoff frequency, 1E4 [rad/s] should be -3dB relative to the passbnad.
f. The high frequency cutoff frequency, 1E6 [rad/s], should be -3dB relative to the passbnad.
g. You cannot use any first order circuit stages

Build the circuit using BPF LPF

BPF $\omega_{cLow} = 10^4$ $\omega_{cHigh} = 10^6$

2nd order overdamped circuit measured across the resistor

-3corrections -3corrections

R must give ζ > 1

$20\frac{db}{dec}$ $20\frac{db}{dec}$

LPF $\omega_c = 10^6(\text{double})$

need 0dB correction at 10^6 2nd order underdamped ζ=0.5

$40\frac{db}{dec}$ rolloff

pass band gain

$20\log(K) = 5db$

$K = 1.77$

$\left(1 + \frac{0.77k\Omega}{1k\Omega}\right) = 1.77$

Non inverting amp choose R1 $R_1 = 1k$ $R_2 = 0.77k$

$\left(1 + \frac{R_2}{R_1}\right)$

BPF $H(s) = \frac{2\alpha s}{s^2 + 2\alpha s + \omega_0^2}$ you know that denominator must be $\frac{\blacksquare}{\left(s + 10^4\right)\left(s + 10^6\right)}$ expand and 2α in numerator can be given by doing so

$s^2 + 1010000\cdot s + 10000000000$

$$H(s) = \frac{1.01\cdot 10^6 s}{(s+10^4)(s+10^6)}$$

Now find R/L you can pick R or L

$L = 10^{-3}$

$R = 10^{-3}\cdot 1.01\cdot 10^6$

$10^{-3}\cdot 1.01\cdot 10^6 = 1.01\times 10^3$

$R = 1.01k$

$C = 1\cdot 10^{-7}$

$$\omega_0^{\,2} = \frac{1}{LC}$$

$$\frac{1}{10^{10}\cdot 1\cdot 10^{-3}} = 1\times 10^{-7}$$

LPF

$$H(s) = \frac{\omega_0^{\,2}}{s^2 + 2\alpha s + \omega_0^{\,2}}$$

you know that denominator must be

$$\frac{\blacksquare}{(s+10^6)^2}$$

$\omega_c = \omega_0$

$\omega_0 = 10^6$

$\alpha = 0.5\cdot\omega_0$

this is the underdamped condition where correction is 0dB

$\alpha = 5\cdot 10^5$

pick $L = 1\cdot 10^{-3}$

$$\omega_0^{\,2} = \frac{1}{LC}$$

$$C = \frac{1}{10^{12}\cdot 1\cdot 10^{-3}}$$

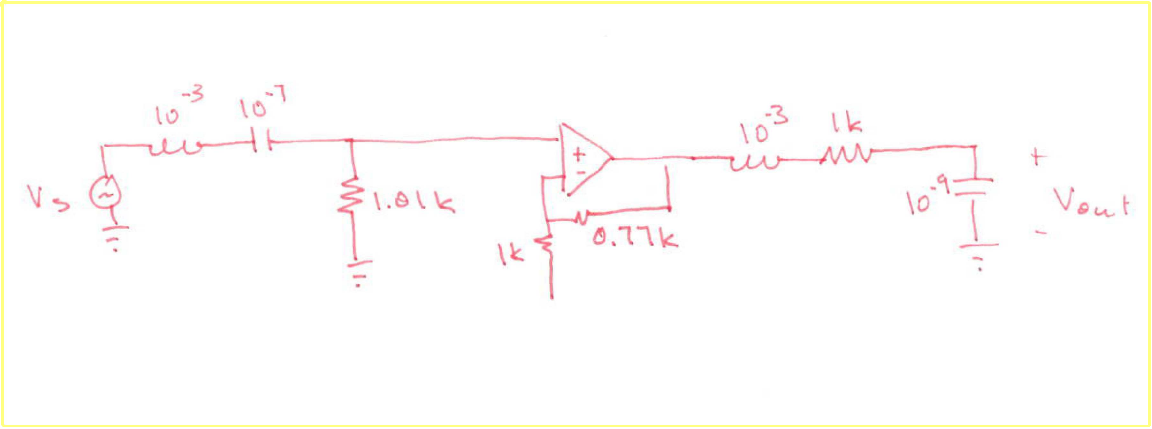
$$\frac{1}{10^{12}\cdot 1\cdot 10^{-3}} = 1\times 10^{-9}$$

$C = 1\cdot 10^{-9}$

$$2\alpha = \frac{R}{L}$$

$2\cdot 5\cdot 10^5\cdot 1\cdot 10^{-3} = 1\times 10^3$

$R = 1k$


10^-3
10^-7
Vs
1.01k
+
-
1k
0.77k
10^-3
1k
10^-9
+
Vout
-