# Exam 1 Crib Sheet

## Ohm’s Law

- **Linear relationship** between voltage and current in a resistor

\[ V = I \cdot R \]

- **V** – Voltage, Volts [V]
- **I** – Current, Amps [A]
- **R** – Resistance, Ohms [Ω]

## Power

\[ P = V \cdot I \]

- **P** – Power, Watts [W]

Using the above polarities (which may or be correct)

For **P > 0**, the component consumes power

For **P < 0**, the component produces power

## Node

- a connection between two or more components

## Loop

- a closed path through which current can flow

## KCL – Kirchoff’s Current Law

\[ \sum_{n=1}^{N} I_n = 0 \]

The sum of the currents leaving a node is zero (signs determined by polarity).

\[ I_1 - I_2 + I_3 = 0 \]

## KVL – Kirchoff’s Voltage Law

\[ \sum_{n=1}^{N} V_n = 0 \]

The sum of the voltages around any closed loop is zero (signs determined by polarity).

\[ V_1 + V_2 - V_3 = 0 \]

## Resistors in series

\[ R_{EQ} = R_1 + R_2 \]

## Resistors in parallel

\[ R_{EQ} = \left( \frac{1}{R_1} + \frac{1}{R_2} \right)^{-1} \]

## Source transformation

**Superposition** – For each **independent** source, turn off all other **independent** sources and find the contribution from that source. Sum the contribution from each source to get the parameter of interest.
Exam 1 Crib Sheet

Node Analysis

\[
\frac{V_A}{R1} + \frac{V_A - V_B}{R3} = 0
\]
\[
\frac{V_B}{R2} + \frac{V_B - V_A}{R3} - I_1 + \frac{V_C}{R4} = 0
\]
\[
V_C - V_B = 2000I_x
\]
\[
\frac{V_B}{R2} = I_x
\]

Mesh Analysis

\[
(i_1)R1 + (i_1)R3 + (i_1 - i_2)R2 = 0
\]
\[
-2000I_x + (i_1)R4 + (i_2 - i_1)R2 = 0
\]
\[
i_3 - i_2 = I_1
\]
\[
i_1 - i_2 = I_x
\]

Example includes a Current Controlled Voltage Source (CCVS) as a dependent source and I1 as an independent source.

Thevenin voltage (V_{TH}) – Open circuit the load, find the voltage across the load nodes

Norton current (I_N) – Short circuit the load, find the current through that short circuit

Thevenin resistance (R_{TH}) – Turn off all independent sources, replace the load with a test voltage (V_{test}), find the current (I_{test}) through the test voltage, R_{TH} = V_{test}/I_{test}.

\[
V_{TH} = I_N R_{TH} \quad \text{(Ohm’s Law relationship)}
\]

Comparator

If \( V_1 < V_2 \), \( V_{out} = V^{+}_{\text{saturation}} \)
If \( V_1 > V_2 \), \( V_{out} = V^{-}_{\text{saturation}} \)

Inverting amplifier circuit

\[
V_{out} = -\frac{R2}{R1} V_{\text{inn}}
\]

Non-inverting amplifier circuit

\[
V_{out} = \left(1 + \frac{R2}{R1}\right) V_{\text{inn}}
\]

Summing amplifier circuit

\[
V_{out} = -\frac{Rf}{R1} V1 - \frac{Rf}{R2} V2
\]
# Exam 2 Crib Sheet

## IV Characteristics – Time domain

<table>
<thead>
<tr>
<th>Resistors –</th>
<th>Inductors –</th>
<th>Capacitors –</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V(t) = I(t)R )</td>
<td>( V_L(t) = L \frac{dI_L}{dt} )</td>
<td>( I_C(t) = C \frac{dV_C}{dt} )</td>
</tr>
<tr>
<td>( IR(t) )</td>
<td>( IL(t) )</td>
<td>( IC(t) )</td>
</tr>
<tr>
<td>+ [ \text{R} ]</td>
<td>+ [ \text{L} ]</td>
<td>+ [ \text{C} ]</td>
</tr>
<tr>
<td>[ V_R(t) ]</td>
<td>[ V_L(t) ]</td>
<td>[ V_C(t) ]</td>
</tr>
</tbody>
</table>

### Continuity conditions

\[
I_L(t_o^-) = I_L(t_o^+) \quad \quad V_C(t_o^-) = V_C(t_o^+)
\]

## IV Characteristics – Laplace domain

<table>
<thead>
<tr>
<th>( Z_R = R )</th>
<th>( Z_L = sL )</th>
<th>( Z_C = \frac{1}{sC} )</th>
</tr>
</thead>
</table>

### Resistors –

\[
V(s) = Z_R I(s)
\]

\[
IR(s) \quad + \quad VR(s)
\]

### Inductors –

\[
V_L(s) = Z_L I_L(s) - LI(0^+)
\]

\[
VL(s) \quad + \quad sL \quad IL(0+) \quad - \quad + \quad IL(s)
\]

### Capacitors –

\[
V_C(s) = Z_C I_C(s) + V_C(0^+)
\]

\[
VC(s) \quad + \quad 1/sC \quad VC(0+)/s \quad - \quad + \quad IC(s)
\]

### Impedance, \( Z [\Omega] \), properties have the same characteristics as resistance

**Impedances in series add**, \( Z_{EQ} = Z_1 + Z_2 \)

**Impedances in parallel have an inverse relationship**, \( Z_{EQ} = \left( \frac{1}{Z_1} + \frac{1}{Z_2} \right)^{-1} = \frac{Z_1Z_2}{Z_1 + Z_2} \)

### Initial Value Theorem

\[
\lim_{s \to \infty} \{sF(s)\} = f(t = 0^+)
\]

### Final Value Theorem

\[
\lim_{s \to 0} \{sF(s)\} = f(t \to \infty)
\]
First order circuits

Differential equation: \( \tau \frac{dy}{dt} + y = f(t) \), with solution \( y(t) = y_h(t) + y_p(t) \)

- \( f(t) \) represents a source function or \( n \)th derivative of the source function, with appropriate coefficients
- \( y_h(t) \) represents the homogeneous/transient part of the solution
- \( y_p(t) \) represents the particular/forced part of the solution.

For first order circuits, the homogeneous solution always takes the form \( y_h(t) = A e^{-\frac{t}{\tau}} \)

\( \tau \) is the time constant
- For RC circuits, \( \tau = RC \)
- For RL circuits, \( \tau = \frac{L}{R} \)

Second order circuits

Differential equation: \( \frac{d^2y}{dt^2} + 2\alpha \frac{dy}{dt} + \omega_o^2 y = f(t) \), with solution \( y(t) = y_h(t) + y_p(t) \)

- \( s\)-domain \( s^2 Y(s) + 2\alpha s Y(s) + \omega_o^2 Y(s) = F(s) \)
- \( y_h(t) \) represents the homogeneous/transient part of the solution
- \( y_p(t) \) represents the particular/forced part of the solution.

The particular solution is always the same type of function as the source.
- \( f(t) \) represents a source function or \( n \)th derivative of the source function
- \( F(s) \) represents the Laplace transform of the function \( f(t) \)

<table>
<thead>
<tr>
<th>Overdamped ((\alpha &gt; \omega_o))</th>
<th>( y_h(t) = A_1 e^{-\alpha_1 t} + A_2 e^{-\alpha_2 t} )</th>
<th>(-\alpha_1, -\alpha_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y(0^+) = A_1 + A_2 + y_p(0^+) )</td>
<td>( \frac{dy(0^+)}{dt} = -\alpha_1 A_1 - \alpha_2 A_2 + \frac{dy_p(0^+)}{dt} )</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Critically Damped ((\alpha = \omega_o))</th>
<th>( y_h(t) = A_1 e^{-\alpha t} + A_2 te^{-\alpha t} )</th>
<th>( \alpha ) from the differential equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y(0^+) = A_1 + y_p(0^+) )</td>
<td>( \frac{dy(0^+)}{dt} = -\alpha A_1 + A_2 + \frac{dy_p(0^+)}{dt} )</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Underdamped ((\alpha &lt; \omega_o))</th>
<th>( y_h(t) = e^{-\alpha t} \left[ A_1 \cos(\beta t) + A_2 \sin(\beta t) \right] )</th>
<th>( \alpha ) from the differential equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta = \sqrt{\omega_o^2 - \alpha^2} )</td>
<td>( \frac{dy(0^+)}{dt} = -\alpha A_1 + \beta A_2 + \frac{dy_p(0^+)}{dt} )</td>
<td></td>
</tr>
</tbody>
</table>

| RLC series circuit \( \alpha = \frac{1}{2} \frac{R}{L} \) \( \omega_o = \frac{1}{\sqrt{LC}} \) | RLC parallel circuit \( \alpha = \frac{1}{2} \frac{1}{RC} \) \( \omega_o = \frac{1}{\sqrt{LC}} \) |
Partial Fraction Expansion

Simple Real Poles:

In General:

Expand: \( F(s) = \frac{A_1}{s-p_1} + \frac{A_2}{s-p_2} + \frac{A_3}{s-p_3} + \ldots \)

\( A_n = \left[ (s-p_n)F(s) \right]_{s=p_n}; \quad \text{Cover-Up Rule} \)

\[ \Rightarrow f(t) = A_1 e^{p_1 t} + A_2 e^{p_2 t} + A_3 e^{p_3 t} + \ldots \quad t \geq 0 \]

Real, Equal Poles – Double Pole:

- Real, Equal Poles – Double Pole:

Expand \( F(s) = \frac{A_1}{s-p_1} + \left[ \frac{A_{n1}}{s-p_{n1}} + \frac{A_{n2}}{s-p_{n2}} \right] \)

\( A_{n2} = \left[ (s-p_{n2})^2 F(s) \right]_{s=p_{n2}}; \quad \text{Cover-Up Rule} \)

Usually Find \( A_{n1} \) from evaluating \( F(0) \) or \( F(1) \)

\[ \Rightarrow f(t) = (A_1 e^{p_1 t} + \ldots + A_{n1} e^{p_{n1} t} + A_{n2} e^{p_{n2} t}) \quad t \geq 0 \]

Simple Poles       Repeated Poles

Complex Conjugate Poles

In General:

Expand \( F(s) = \frac{A_1}{s-p_1} + \ldots + \frac{A}{s+\alpha-j\beta} + \frac{A^*}{s+\alpha+j\beta} \)

Find \( A_1 \) and \( A = |A|e^{j\phi} \) from Cover-Up Rule

\[ \Rightarrow f(t) = A_1 e^{p_1 t} + \ldots + 2|A|e^{-\alpha t}\cos(\beta t + \phi) \quad t \geq 0 \]

Simple Poles       Complex Poles

LAPLACE TRANSFORMS

<table>
<thead>
<tr>
<th>Signal</th>
<th>f(t)</th>
<th>F(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impulse</td>
<td>( \delta(t) )</td>
<td>1</td>
</tr>
<tr>
<td>Step</td>
<td>( u(t) )</td>
<td>( \frac{1}{s} )</td>
</tr>
<tr>
<td>Constant</td>
<td>( Au(t) )</td>
<td>( \frac{A}{s} )</td>
</tr>
<tr>
<td>Ramp</td>
<td>( tu(t) )</td>
<td>( \frac{1}{s^2} )</td>
</tr>
</tbody>
</table>

LAPLACE TRANSFORMS

<table>
<thead>
<tr>
<th>Signal</th>
<th>f(t)</th>
<th>F(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential</td>
<td>( e^{-\alpha t} u(t) )</td>
<td>( \frac{1}{s+\alpha} )</td>
</tr>
<tr>
<td>Damped Ramp</td>
<td>( [e^{-\alpha t}] u(t) )</td>
<td>( \frac{1}{(s+\alpha)^2} )</td>
</tr>
<tr>
<td>Cosine Wave</td>
<td>( \frac{\cos(\beta t) u(t)}{s} )</td>
<td>( \frac{s+\alpha}{s^2+\beta^2} )</td>
</tr>
<tr>
<td>Damped Cosine</td>
<td>( [e^{-\alpha t}\cos(\beta t)] u(t) )</td>
<td>( \frac{s+\alpha}{(s+\alpha)^2+\beta^2} )</td>
</tr>
</tbody>
</table>

LAPLACE TRANSFORMS

<table>
<thead>
<tr>
<th>Time Domain</th>
<th>s-Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \int_0^t f(t) , dt )</td>
<td>( \frac{F(s)}{s} )</td>
</tr>
<tr>
<td>( \frac{d}{dt} f(t) )</td>
<td>( sF(s) - f(0^-) )</td>
</tr>
<tr>
<td>( e^{-\alpha t} f(t) )</td>
<td>( F(s+\alpha) )</td>
</tr>
<tr>
<td>( t f(t) )</td>
<td>( -\frac{dF(s)}{ds} )</td>
</tr>
<tr>
<td>( f(t-a)u(t-a) )</td>
<td>( e^{-\alpha t} F(s) )</td>
</tr>
</tbody>
</table>
## LAPLACE TRANSFORMS

<table>
<thead>
<tr>
<th>Signal</th>
<th>Time Domain</th>
<th>S Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impulse</td>
<td>$\delta(t)$</td>
<td>1</td>
</tr>
<tr>
<td>Step</td>
<td>$u(t)$</td>
<td>$s^{-1}$</td>
</tr>
<tr>
<td>Constant</td>
<td>$Au(t)$</td>
<td>$As^{-1}$</td>
</tr>
<tr>
<td>Ramp</td>
<td>$tu(t)$</td>
<td>$s^{-2}$</td>
</tr>
<tr>
<td>Exponential</td>
<td>$e^{-at}u(t)$</td>
<td>$(s + \alpha)^{-1}$</td>
</tr>
<tr>
<td>Damped ramp</td>
<td>$te^{-at}u(t)$</td>
<td>$(s + \alpha)^{-2}$</td>
</tr>
<tr>
<td>Cosine</td>
<td>$\cos(\beta t)u(t)$</td>
<td>$\frac{s}{s^2 + \beta^2}$</td>
</tr>
<tr>
<td>Damped cosine</td>
<td>$e^{-at}\cos(\beta t)u(t)$</td>
<td>$\frac{s + \alpha}{(s + \alpha)^2 + \beta^2}$</td>
</tr>
<tr>
<td>Sum</td>
<td>$Af_1(t) + Bf_2(t)$</td>
<td>$Af_1(s) + Bf_2(s)$</td>
</tr>
<tr>
<td>Integral</td>
<td>$\int_0^t f(\tau)d\tau$</td>
<td>$s^{-1}f(s)$</td>
</tr>
<tr>
<td>Derivative</td>
<td>$\frac{df(t)}{dt}$</td>
<td>$sf(s) - f(0^-)$</td>
</tr>
<tr>
<td>Exponential × function</td>
<td>$e^{-at}f(t)$</td>
<td>$f(s + \alpha)$</td>
</tr>
<tr>
<td>t × function</td>
<td>$tf(t)$</td>
<td>$-\frac{df(s)}{ds}$</td>
</tr>
<tr>
<td>Shifted function</td>
<td>$f(t - a)u(t - a)$</td>
<td>$e^{-as}f(s)$</td>
</tr>
</tbody>
</table>

**NOTATION:** $\mathcal{L}\{f(t)\}(s) = f(s)$ and $\mathcal{L}^{-1}\{f(s)\}(t) = f(t)$
### Complex Numbers

**Rectangular form:**

\[ A = A_R + jA_I \]

**Polar form:**

\[ |A| \angle \varphi_A \]

**Rectangular to polar**

\[ |A| = \sqrt{(A_R)^2 + (A_I)^2} \]

\[ \varphi_A = \tan^{-1}\left(\frac{A_I}{A_R}\right) \]

**Polar to rectangular**

\[ A_R = |A| \cos(\varphi_A) \]

\[ A_I = |A| \sin(\varphi_A) \]

---

### Euler’s Law:

\[ e^{j\vartheta} = \cos(\vartheta) + j \sin(\vartheta) \]

---

### Mathematics with complex number

**Addition/Subtraction – Rectangular form**

\[ A + B = (A_R + B_R) + j(A_I + B_I) \]

\[ A - B = (A_R - B_R) + j(A_I - B_I) \]

**Complex conjugate**

\[ A = A_R + jA_I \quad A^* = A_R - jA_I \]

**Multiplication/Division – Rectangular form**

\[ AB = |A||B| \angle (\varphi_A + \varphi_B) \]

\[ \frac{A}{B} = \frac{|A|}{|B|} \angle (\varphi_A - \varphi_B) \]

**Complex conjugate**

\[ A = |A| \angle \varphi_A \quad A^* = |A| \angle -\varphi_A \]

---

### AC Steady State signals

**Time domain signals**

\[ F(t) = A_o \sin(\omega t + \Theta) \]

\[ A_o – \text{amplitude} \]

\[ \omega – \text{radial frequency, } 2\pi f \]

\[ \Theta – \text{phase} \]

**Phasor signals**

\[ \tilde{F} = A_o \angle \theta \]

\[ A_o – \text{amplitude} \]

\[ \theta – \text{phase} \]

**Rectangular form**

\[ F(t) = A_o \sin(\omega t + \Theta) \leftrightarrow A_o \left\{ e^{j(\omega t + \Theta)} \right\} \leftrightarrow A_o e^{j\vartheta} \leftrightarrow A_o \angle \theta \]  

**(Phasor form)**

---

### Impedances – Laplace domain (zero initial conditions)

\[ Z_R = R \]

\[ Z_L = sL \]

\[ Z_C = \frac{1}{sC} \]

---

### Impedances – AC steady state

\[ Z_R = R \]

\[ Z_R = R \angle 0^\circ \]

\[ Z_L = j\omega L \]

\[ Z_L = \omega L \angle 90^\circ \]

\[ Z_C = \frac{1}{j\omega C} \]

\[ Z_C = \frac{1}{\omega C} \angle -90^\circ \]
### Exam 3 Crib Sheet

**Impedance, Z [Ω]**, properties have the same characteristics as resistance

\[
\text{In series add, } Z_{\text{EQ}} = Z_1 + Z_2 \\
\text{In parallel, inverse relationship, } Z_{\text{EQ}} = \left(\frac{1}{Z_1} + \frac{1}{Z_2}\right)^{-1} = \frac{Z_1Z_2}{Z_1 + Z_2}
\]

**Admittance, Y [mho]**, properties have characteristics that are the ‘inverse’ of impedance

\[
\text{In parallel, add, } Y_{\text{EQ}} = Y_1 + Y_2 \\
\text{In series, inverse relationship, } Y_{\text{EQ}} = \left(\frac{1}{Y_1} + \frac{1}{Y_2}\right)^{-1} = \frac{Y_1Y_2}{Y_1 + Y_2}
\]

### AC Steady State Power

\[
S = P + jQ \\
P - \text{Real power, [W]} \\
Q - \text{Reactive power, [VAR]}
\]

\[
|S| - \text{Total power, [VA]}
\]

Using Ohm’s Law relationships for impedances (Z)

\[
\text{Complex Power } S = \frac{1}{2}V_oI_o^* = \left(\frac{1}{2}\right)\left|V_o\right|^2 Z \\
\text{Total Power } |S| = \frac{1}{2}\left|V_o\right|^2 Z^\ast \text{ where } V_{\text{RMS}} = \frac{V_o}{\sqrt{2}}
\]

### POWER TRIANGLE

\[
Q = |S| \sin \theta = V_{\text{RMS}} I_{\text{RMS}} \sin \theta \\
\text{Reactive Power; [VAR's]}
\]

\[
|S| = V_{\text{RMS}} I_{\text{RMS}} \\
\text{Apparent Power; [VA]}
\]

- Capacitive reactance is negative (Q < 0)
- Inductive reactance is positive (Q > 0)

Power produced by the source(s) is equal to the sum of the power produced/stored for each impedance in the circuit

Power factor – a metric over how efficient power consumption/production appears to be

\[
0 < \text{power factor} < 1 \\
\text{Power factor } = \frac{P}{|S|} = \cos(\varphi_S)
\]

### Ideal Transformers

\[
\frac{I_p}{I_s} = \frac{N_p}{N_s} \\
\text{Primary: source side of the transformer} \\
\text{Secondary: load side of the transformer}
\]

\[
N_p : \text{number of windings on the primary} \\
N_s : \text{number of windings on the secondary}
\]

The winding ratio, \( N = \frac{N_s}{N_p} \)

Voltage relationship, \( V_s = NV_p \)

Current relationship, \( I_s = \frac{I_p}{N} \)
Referring the primary to the secondary (voltage source):
\[ V_{o_{eq}} = NV_o \quad Z_{s_{eq}} = N^2 Z_s \]
Referring the primary to the secondary (current source):
\[ I_{o_{eq}} = \frac{I_o}{N} \quad Z_{s_{eq}} = N^2 Z_{s_{eq}} \]
Referring the secondary to the primary:
\[ Z_{L_{eq}} = \frac{Z_L}{N^2} \]

Mutual Inductance

The Tee model for coupled inductors represents an equivalent circuit.

\[ M = k \sqrt{L_1 L_2} \]
where \( k \) is the coupling coefficient \( 0 < k < 1 \)

Student Add-ons

**Referral to Primary**

\[ P = I_{2 \text{RMS}}^2 |Z| \cos \theta \]
\[ Q = I_{2 \text{RMS}}^2 |Z| \sin \theta \]

\[ P = I_{2 \text{RMS}}^2 R(\omega) \]
\[ Q = I_{2 \text{RMS}}^2 X(\omega) \]

\[ P = V_{2 \text{RMS}} I_{2 \text{RMS}} \cos \theta \]
\[ Q = V_{2 \text{RMS}} I_{2 \text{RMS}} \sin \theta \]

**Notes**

\[ R(\omega) = Z_{\text{REAL}} \]
\[ X(\omega) = Z_{\text{IMG}} \]

\[ \theta = \text{Angle of Impedance} \]
\[ \theta = \tan^{-1} \left( \frac{Z_{\text{IMG}}}{Z_{\text{REAL}}} \right) \]

\( \theta > 0 \) => \( I \) lags \( V \) (Ind.) \( \theta < 0 \) => \( I \) leads \( V \) (Cap.)

Complex Power

1) \( S = V_{RMS} I_{RMS}^* = |V_{RMS}| |I_{RMS}| (\angle V - \angle I) \)
2) \( S = |I_{RMS}|^2 Z = |I_{RMS}|^2 (\angle Z) \) (should be
3) \( S = \frac{|V_{RMS}|^2}{Z^*} = \frac{|V_{RMS}|^2}{|Z|} (\angle Z) \)
## Bode Plots Crib Sheet

### Bode Plots

Decade – a change in frequency by one order of magnitude, for example
- \(100 \text{ rad/s} \rightarrow 1000 \text{ rad/s}\)
- \(10^4 \text{ Hz} \rightarrow 10^5 \text{ Hz}\)

\(\text{dB} – \text{decibel}\)

\(\text{dB} = 20 \log |F(j\omega)|\)

Note the argument of the logarithm is a magnitude expression

A change of 20dB corresponds to a change of \(|F(j\omega)|\) by one order of magnitude

### Bode plot magnitude approximations

\[ H(s) \propto s^n \]

Slope +20dB/decade

\[ H(s) \propto \frac{1}{s^n} \]

Slope -20dB/decade

\[ H(s) \propto K \]

‘Flat’, dB = 20\log|K|

### Sketching Bode plot magnitudes (real poles and zeros)

- **Crossing an n-pole**: Slope changes by \(-20n\) dB/decade
- **Crossing an n-zero**: Slope changes by \(+20n\) dB/decade

‘\(n\)’ indicates the number of poles or zeros

‘Crossing’ rules apply when going from a lower frequency to a higher frequency

### Sketching Bode plot phases (real poles and zeros)

- **Crossing an n-pole**: Phase changes by \(-n\frac{\pi}{2}\)
- **Crossing an n-zero**: Phase changes by \(+n\frac{\pi}{2}\)

Phase changes are ‘spread out’ over two decades, one decade on either side of the pole or zero

### Corrections for Bode plot magnitudes (real poles and zeros)

- At an n-pole: The ‘real’ dB value is -3n dB ‘below’ the asymptote
- At an n-zero: The ‘real’ dB value is +3n dB ‘above’ the asymptote

The asymptote is the straight line approximation of the Bode plots
‘Far away’ from poles and zeros, the asymptotes are an accurate representation of the Bode plot
### Bode Plots Crib Sheet

#### Second Order Circuits

| Damping ratio, $\delta = \frac{\alpha}{\omega_o}$, a metric of the damping | $\delta > 1$, overdamped |
| $\alpha$ is the attenuation constant | $\delta = 1$, critically damped |
| $\omega_o$ is the resonant frequency | $\delta < 1$, underdamped |

**Lowpass/Highpass filters**
- Overdamped and critically damped cases, the Bode plots follow the procedure on the previous page
- Underdamped cases, use the critically damped approximation, add a ‘correction’ of $20\log\frac{1}{2\delta}$ at the resonant frequency, $\omega_o$

**Bandpass filters**
- Overdamped, the Bode plots follow the procedure on the previous page
- Critically damped and underdamped cases
  - At the resonant frequency, the magnitude Bode plot is 0dB
  - The vertex where the stopbands meet is $20\log|2\delta|$

*Note: The above discussion is for second order circuits only. If there is a gain stage, the Bode plot moves ‘up’ or ‘down’ and the dB value of the gain determines the reference for adding corrections/stopband vertices*

#### Cascaded Filters – Magnitude Bode Plots

$H(s) = H_1(s)H_2(s)H_3(s)$ (three stages) $\rightarrow$

$$
\text{dB} = 20\log|H_1(j\omega)H_2(j\omega)H_3(j\omega)| = 20\log|H_1(j\omega)| + 20\log|H_2(j\omega)| + 20\log|H_3(j\omega)|
$$

$$
\text{angle} = \angle[H_1(j\omega)H_2(j\omega)H_3(j\omega)] = \angle H_1(j\omega) + \angle H_2(j\omega) + \angle H_3(j\omega)
$$
Bode Plots Crib Sheet

**First order filters**

\[ \omega_c = \frac{1}{RC} \quad \omega_c = \frac{R}{L} \]

<table>
<thead>
<tr>
<th>Filter name</th>
<th>Schematic(s)</th>
<th>( H(s) )</th>
<th>pole/zero ID</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low pass filter</td>
<td><img src="image" alt="Low pass filter" /></td>
<td>( \frac{\omega_c}{s + \omega_c} )</td>
<td>1 pole</td>
</tr>
<tr>
<td>High pass filter</td>
<td><img src="image" alt="High pass filter" /></td>
<td>( \frac{s}{s + \omega_c} )</td>
<td>1 zero at zero, 1 pole</td>
</tr>
</tbody>
</table>
# Bode Plots Crib Sheet

## Second order filters

<table>
<thead>
<tr>
<th>Filter name</th>
<th>Schematic(s)</th>
<th>( H(s) )</th>
<th>pole/zero ID</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low pass filter</td>
<td></td>
<td>( \frac{\omega_o^2}{s^2 + 2\alpha s + \omega_o^2} )</td>
<td>2 poles</td>
</tr>
<tr>
<td>High pass filter</td>
<td><img src="image" alt="High pass filter schematic" /></td>
<td>( \frac{s^2}{s^2 + 2\alpha s + \omega_o^2} )</td>
<td>2 zeros at zero</td>
</tr>
<tr>
<td>Bandpass filter</td>
<td><img src="image" alt="Bandpass filter schematic" /></td>
<td>( \frac{2\alpha s}{s^2 + 2\alpha s + \omega_o^2} )</td>
<td>1 zero at zero</td>
</tr>
<tr>
<td>Bandstop filter</td>
<td></td>
<td>( \frac{s^2 + \omega_o^2}{s^2 + 2\alpha s + \omega_o^2} )</td>
<td></td>
</tr>
</tbody>
</table>