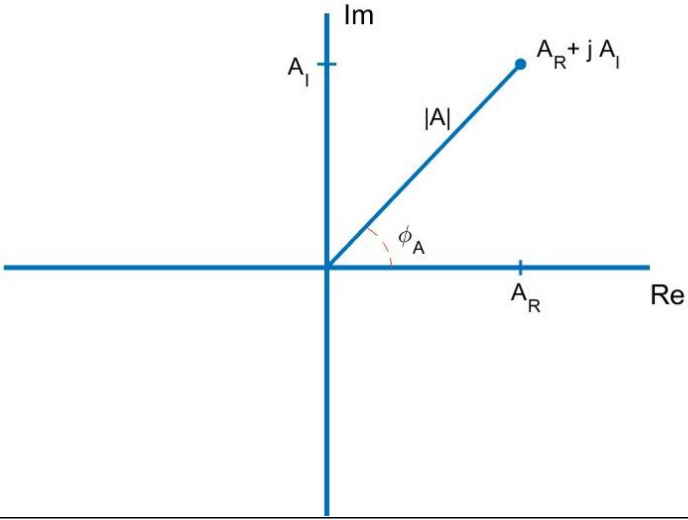
Exam 3 Crib Sheet

Complex Numbers



Rectangular form:

$A = A_R + jA_I$

Polar form:

$|A|\angle\varphi_A$

Rectangular to polar

$|A| = \sqrt{(A_R)^2 + (A_I)^2}$

$\varphi_A = tan^{-1}\left(\frac{A_I}{A_R}\right)$

Polar to rectangular

$A_R = |A|\,cos(\varphi_A)$

$A_I = |A|\,sin(\varphi_A)$

Euler's Law: $e^{j\theta} = \boldsymbol{cos}(\theta) + j\,\boldsymbol{sin}(\theta)$

Mathematics with complex number

Addition/Subtraction – Rectangular form

$A + B = (A_R + B_R) + j(A_I + B_I)$

$A - B = (A_R - B_R) + j(A_I - B_I)$

Complex conjugate

$A = A_R + jA_I$ $\quad A^* = A_R - jA_I$

Multiplication/Dvision – Rectangular form

$AB = |A||B|\angle(\varphi_A + \varphi_B)$

$\frac{A}{B} = \frac{|A|}{|B|}\angle(\varphi_A - \varphi_B)$

Complex conjugate

$A = |A|\angle\varphi_A$ $\quad A^* = |A|\angle - \varphi_A$

AC Steady State signals

Time domain signals

$\mathrm{F(t)} = \mathrm{A}_{\boldsymbol{o}}\mathbf{cos}\,(\boldsymbol{\omega}\mathrm{t} + \boldsymbol{\theta})$

A_o – amplitude

ω – radial frequency, 2πf

Θ – phase

Phasor signals

$\tilde{F} = A_o\angle\theta$

A_o – amplitude

Θ – phase

(Rectangular form) $F(t) = A_o\,\mathbf{cos}(\omega t + \theta) \leftrightarrow A_o\mathbb{R}\{e^{j(\omega t+\theta)}\} \leftrightarrow A_o e^{j\theta} \leftrightarrow A_o\angle\theta$ **(Phasor form)**

Impedances – Laplace domain (zero initial conditions)

$Z_R = R$	$Z_L = sL$	$Z_C = \frac{1}{sC}$

Impedances – AC steady state

$Z_R = R$ $Z_R = R\angle 0°$	$Z_L = j\omega L$ $Z_L = \omega L\angle 90°$	$Z_C = \frac{1}{j\omega C}$ $Z_C = \frac{1}{\omega C}\angle - 90°$

Impedance, Z [Ω], properties have the same characteristics as resistance

In series add, $Z_{EQ} = Z_1 + Z_2$ $\quad$ ***In parallel, inverse relationship,*** $Z_{EQ} = \left(\frac{1}{Z_1} + \frac{1}{Z_2}\right)^{-1} = \frac{Z_1 Z_2}{Z_1 + Z_2}$

Admittance, Y [mho], properties have characteristics that are the 'inverse' of impedance

In parallel, add, $Y_{EQ} = Y_1 + Y_2$ $\quad$ ***In series, inverse relationship,*** $Y_{EQ} = \left(\frac{1}{Y_1} + \frac{1}{Y_2}\right)^{-1} = \frac{Y_1 Y_2}{Y_1 + Y_2}$

AC Steady State Power

$S = P + jQ$

S – Complex power

Using Ohm's Law relationships for impedances (Z)

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P – Real power, [W]
Q – Reactive power, [VAR]

|S| –Apparent Power, [VA]

$P = I_{RMS}^2 |Z| \cos\theta$
$= I_{RMS}^2 \ R(\omega)$
$= V_{RMS} I_{RMS} \cos\theta$

Equivalent ways of expressing Real Power [Watts]

$Q = I_{RMS}^2 |Z| \sin\theta$
$= I_{RMS}^2 \ X(\omega)$
$= V_{RMS} I_{RMS} \sin\theta$

Equivalent ways of expressing Reactive Power [VAR's]

If using V_{RMS}^2 version of equations also divide by |Z| (phasor form) *cos or sin θ OR must use complex conjugate of Z (rectangular form)

Capacitive reactance is negative (Q < 0)

Inductive reactance is positive (Q > 0)

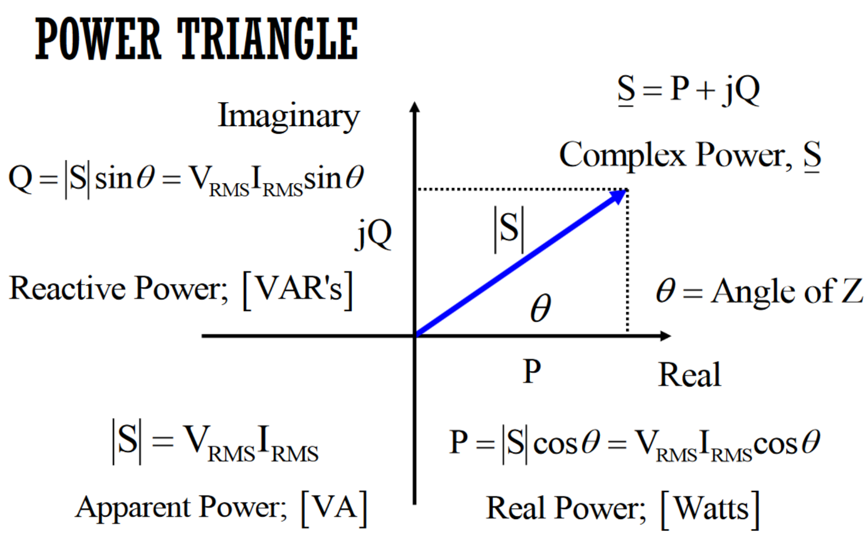
Power produced by the source(s) is equal to the sum of the power produced/stored for each impedance in the circuit

Power factor – a metric over how efficient power consumption/production appears to be

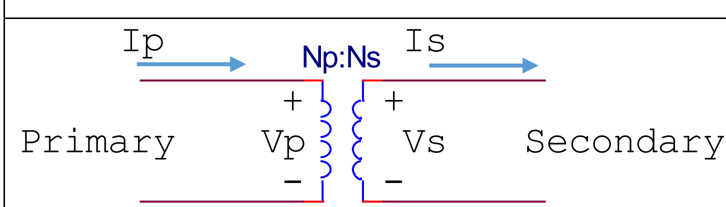
$0 < \text{power factor} < 1$

$\text{Power factor} = \frac{P}{|S|} = \cos(\varphi_S)$

Power Triangle



Ideal Transformers



Np : number of windings on the primary
Ns : number of windings on the secondary

Primary: source side of the transformer
Secondary: load side of the transformer
The winding ratio, $N = \frac{Ns}{Np}$
Voltage relationship, $Vs = NVp$
Current relationship, $Is = \frac{Ip}{N}$

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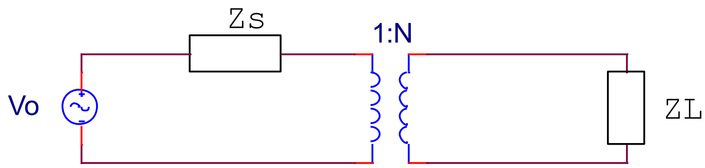


Refer to secondary (voltage source):

$$Vo_{eq} = NVo \qquad Zs_{eq} = N^2 Zs$$

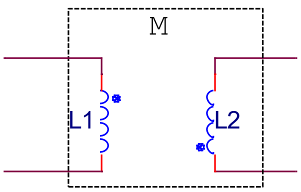
Refer to secondary (current source):

$$Io_{eq} = \frac{Io}{N} \qquad Zs_{eq} = N^2 Zs_{eq}$$

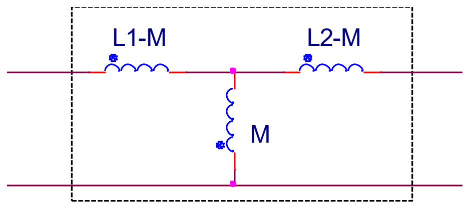
Refer to primary:

$$ZL_{eq} = \frac{ZL}{N^2}$$

Mutual Inductance



Coupled Inductors



Tee Model

The Tee model for coupled inductors represents an equivalent circuit.

M is the mutual inductance, the coupling between the two inductors.

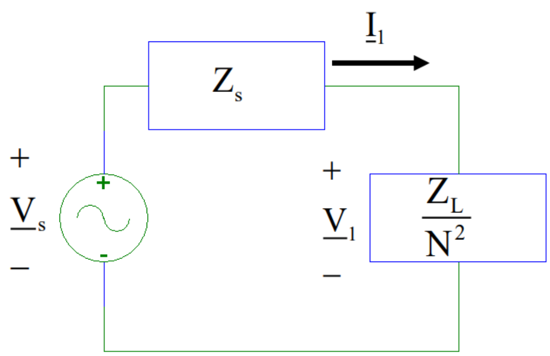
$$M = k\sqrt{L_1 L_2}$$

where k is the coupling coefficient

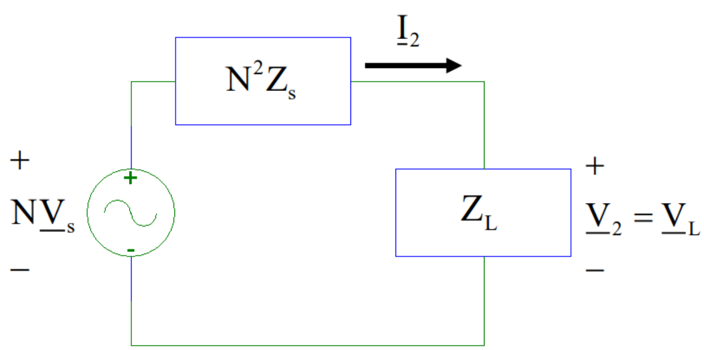
$0 < k < 1$

Student Requested Add-ons

REFERRAL TO PRIMARY



REFERRAL TO SECONDARY



Complex Power	
$P = I^2_{RMS}\|Z\|\cos\theta$	$Q = I^2_{RMS}\|Z\|\sin\theta$
$P = I^2_{RMS}R(\omega)$	$Q = I^2_{RMS}X(\omega)$
$P = V_{RMS}I_{RMS}\cos\theta$	$Q = V_{RMS}I_{RMS}\sin\theta$
Notes	
$R(\omega) = Z_{REAL}$	$X(\omega) = Z_{IMG}$
θ = *Angle of Impedance*	$\theta = \tan^{-1}(\frac{Z_{IMG}}{Z_{REAL}})$
$\theta > 0$ => *I lags V (Ind.)*	$\theta < 0$ => *I leads V (Cap.)*