## Circuits

## Exam 1

Fall 2020

| 1. | $/ 25$ |
| :---: | :---: |
| 2. | $/ 25$ |
| 3. | $/ 30$ |
| 4. | $/ 20$ |
| Total | $/ 100$ |

## Name

Notes:

1) Your crib sheet is provided in the Exams Team Space.
2) You may use your calculator (only).
3) You cannot use your computer or cell phone during any part of the exam. Doing so results in an automatic 0 for this exam.

Please sign below:
I have not consulted any person or collaborated with anyone to complete this exam. I did not post and will not post any part of this exam to Chegg.com or any other equivalent websites. I understand that if my exam is found online, I will be given an $F$ for the semester and the academic dishonesty process will be initiated. I did not look for answers on any website to this exam. If any signification portion of this exam is found to match with any other student, I will be given an automatic 0 for the entire exam. Further actions due to academic dishonesty may be warranted after discussion with all parties.

Signature: $\qquad$

## Problem 1) The Laws: KCL, KVL, Ohm's Law (25 pts)

25 pts Using KCL and KVL and Ohm's law only, write every step to find the voltages across each resistor. The answer to the final answer to this problem is not necessary and not graded. You have three components R3, V1, and I1. There are three spots for these components to go. Place them in one of the three spots in any orientation you'd like. You must use all three components in the circuit. There are two spaces for you to fill in to give the values for these components. You MUST choose an integer 1-9, for every space you see. For example, for R3 if I choose 1 for the first space and 2 for the second space, the value of R2 is 1.2 K ohms. Note: 11 is in mA...

1.1: (1 pt) Step 1: Redraw your circuit below. Include your reference marks across the resistor for your passive sign convention "scaffolding". Ground must be where it is originally placed. You can turn your components and give them any polairty you'd like. (Note: Reference marks are absolutely necessary to draw on your circuit. Without them, your 1.2 step will be marked incorrect with no partial credit!)

## My circuit

Other circuit 1:
Other circuit 2:


Note: There are many other potential circuits but these are a few examples.
1.2: (10 pts) Step 2: Write the linear independent equations necessary to solve your problem.

My solution


Other solution 1:


Other solution 2:


$$
\mathrm{R}_{1}:=3 \cdot 10^{3}
$$

$$
\mathrm{R}_{2}:=1 \cdot 10^{3}
$$

$$
\mathrm{R}_{3}:=2.1 \times 10^{3}
$$

$$
\mathrm{R}_{4}:=1 \times 10^{3}
$$

$$
\mathrm{V}_{1}:=3.7
$$

$$
\mathrm{I}_{1}:=4.5 \cdot 10^{-3}
$$

My solution:

KVL at the top:

$$
\begin{aligned}
& \quad \mathrm{V}_{\mathrm{R} 2}+\mathrm{V}_{1}-\mathrm{V}_{\mathrm{R} 3}=0 \\
& \text { (1) } \quad \mathrm{V}_{\mathrm{R} 2}-\mathrm{V}_{\mathrm{R} 3}=-\mathrm{V}_{1}
\end{aligned}
$$

## KVL at the bottom:

$$
-\mathrm{V}_{\mathrm{R} 1}+\mathrm{V}_{\mathrm{R} 3}-\mathrm{V}_{1}+\mathrm{V}_{\mathrm{R} 4}=0
$$

(2)

$$
-\mathrm{V}_{\mathrm{R} 1}+\mathrm{V}_{\mathrm{R} 3}+\mathrm{V}_{\mathrm{R} 4}=\mathrm{V}_{1}
$$

KCL at node Vn1:

$$
\mathrm{I}_{\mathrm{R} 1}+\mathrm{I}_{\mathrm{R} 3}+\mathrm{I}_{\mathrm{R} 2}=0
$$

(3) $\frac{1}{\mathrm{R}_{1}} \cdot \mathrm{~V}_{\mathrm{R} 1}+\frac{1}{\mathrm{R}_{2}} \cdot \mathrm{~V}_{\mathrm{R} 2}+\frac{1}{\mathrm{R}_{3}} \cdot \mathrm{~V}_{\mathrm{R} 3}=0$

KCL at node Vn2:

$$
-\mathrm{I}_{\mathrm{R} 1}+\mathrm{I}_{1}-\mathrm{I}_{\mathrm{R} 4}=0
$$

(4)

$$
-\frac{1}{\mathrm{R}_{1}} \cdot \mathrm{~V}_{\mathrm{R} 1}-\frac{1}{\mathrm{R}_{4}} \cdot \mathrm{~V}_{\mathrm{R} 4}=-\mathrm{I}_{1}
$$

Other solution 1:

KVL at the top:

$$
\mathrm{V}_{\mathrm{R} 2}-\mathrm{V}_{1}-\mathrm{V}_{\mathrm{R} 3}=0
$$

(1) $\mathrm{V}_{\mathrm{R} 2}-\mathrm{V}_{\mathrm{R} 3}=\mathrm{V}_{1}$

KVL outisde:

$$
-\mathrm{V}_{\mathrm{R} 1}+\mathrm{V}_{\mathrm{R} 2}+\mathrm{V}_{\mathrm{R} 4}=0
$$

$$
\begin{equation*}
-\mathrm{V}_{\mathrm{R} 1}+\mathrm{V}_{\mathrm{R} 2}+\mathrm{V}_{\mathrm{R} 4}=0 \tag{2}
\end{equation*}
$$

Other solution 2:
KVL bottom right:

$$
\mathrm{V}_{\mathrm{R} 4}+\mathrm{V}_{1}-\mathrm{V}_{\mathrm{R} 3}=0
$$

(1) $-\mathrm{V}_{\mathrm{R} 3}+\mathrm{V}_{\mathrm{R} 4}=-\mathrm{V}_{1}$

KVL outside:

$$
-\mathrm{V}_{\mathrm{R} 1}+\mathrm{V}_{\mathrm{R} 2}+\mathrm{V}_{\mathrm{R} 4}=0
$$

(2)

$$
-\mathrm{V}_{\mathrm{R} 1}+\mathrm{V}_{\mathrm{R} 2}+\mathrm{V}_{\mathrm{R} 4}=0
$$

KCL at node Vn1:

$$
\mathrm{I}_{\mathrm{R} 4}-\mathrm{I}_{\mathrm{R} 3}-\mathrm{I}_{\mathrm{R} 2}=0
$$

(3) $-\frac{1}{\mathrm{R}_{2}} \cdot \mathrm{~V}_{\mathrm{R} 2}-\frac{1}{\mathrm{R}_{3}} \cdot \mathrm{~V}_{\mathrm{R} 3}+\frac{1}{\mathrm{R}_{4}} \cdot \mathrm{~V}_{\mathrm{R} 4}=0$

## KCL at node Vn2:

$$
-\mathrm{I}_{\mathrm{R} 1}+\mathrm{I}_{1}-\mathrm{I}_{\mathrm{R} 4}=0
$$

(4) $-\frac{1}{\mathrm{R}_{1}} \cdot \mathrm{~V}_{\mathrm{R} 1}-\frac{1}{\mathrm{R}_{4}} \cdot \mathrm{~V}_{\mathrm{R} 4}=-\mathrm{I}_{1}$
1.3: ( 5 pts) Step 3: Write your matrices with component values. Also include the matrix equation to find the resitance across every resistor. You do not have to provide your final answer values. If you do, they are not graded.

My solution:

$$
\mathrm{M}_{1}:=\left(\begin{array}{cccc}
0 & 1 & -1 & 0 \\
-1 & 0 & 1 & 1 \\
\frac{1}{\mathrm{R}_{1}} & \frac{1}{\mathrm{R}_{2}} & \frac{1}{\mathrm{R}_{3}} & 0
\end{array}\right) \quad \mathrm{C}_{1}:=\left(\begin{array}{c}
-\mathrm{V}_{1} \\
\mathrm{~V}_{1} \\
0 \\
-\mathrm{I}_{1}
\end{array}\right)
$$

Students should have some simple equation to calculate the voltages across every resistor. It is also fine if they calculated current and then used ohm's law at the end.

The final answer here is not graded and doesn't need to be marekd.

$$
\mathrm{M}_{1}^{-1} \cdot \mathrm{C}_{1}=\left(\begin{array}{c}
2.121 \\
-1.672 \\
2.028 \\
3.793
\end{array}\right)
$$

Other solution 1:

$$
\begin{gathered}
\mathrm{M}_{1 \mathrm{ol} 1}:=\left(\begin{array}{cccc}
0 & 1 & -1 & 0 \\
-1 & 1 & 0 & 1 \\
0 & -\frac{1}{\mathrm{R}_{2}} & -\frac{1}{\mathrm{R}_{3}} & \frac{1}{\mathrm{R}_{4}} \\
-\frac{1}{\mathrm{R}_{1}} & 0 & 0 & -\frac{1}{\mathrm{R}_{4}}
\end{array}\right) \\
\mathrm{M}_{1 \mathrm{ol}}{ }^{-1} \cdot \mathrm{C}_{1 \mathrm{ol}}=\left(\begin{array}{c}
\mathrm{V}_{1} \\
2.607 \\
-0.976 \\
2.631
\end{array}\right)
\end{gathered}
$$

Other soltuion 2:

$$
\begin{aligned}
& \mathrm{M}_{1 \mathrm{o} 2}:=\left(\begin{array}{cccc}
0 & 0 & -1 & 1 \\
-1 & 1 & 0 & 1 \\
\frac{1}{\mathrm{R}_{1}} & \frac{1}{\mathrm{R}_{2}} & 0 & 0 \\
-\frac{1}{\mathrm{R}_{1}} & 0 & -\frac{1}{\mathrm{R}_{3}} & -\frac{1}{\mathrm{R}_{4}}
\end{array}\right) \\
& \mathrm{M}_{1 \mathrm{o} 2}^{-1} \cdot \mathrm{C}_{1 \mathrm{o} 2}=\left(\begin{array}{c}
-3.652 \\
-3.283 \\
3.331 \\
-0.369
\end{array}\right)
\end{aligned}
$$

More solutions can be calculated. Please simulate and check if your circuit isn't represented above.
(2 pts) The term "scaffolding" was used in regard to reference marks on your circuit in Step 1.1: What does this refer to when it comes to solving a circuit using the passive sign convention?

It means that the reference marks are there to give a preliminary current or voltage reference to provide a structure for consistent mathematical analysis without need of intuition regarding which way current is flowing for example. These marks are not permanent. The actual direction is determined by polarity.
(2 pts) When you need to FIND and ADD a component like a voltage source in LTSpice, what does the symbol look like?

Please make a note if you use another SPICE program like PSpice.


This one
(2 pts) When you BEGIN to simulate your circuit, what symbol do you press to start to edit your simulation command. (After you set up your simulation, you also push this icon to start your simulation.)

Draw or describe this symbol (from your memory, it is not in the question above).

## Running man

(2 pts) How do you rotate a component in LTSpice? If you use PSpice make a note and answer this question.

## Ctrl-R

(2 pts) What is the purpose of ground in a circuit?

Ground is a common reference point for your entire circuit. Key word is reference or any synonym to this.

## Problem 2) Nose/Mesh Analysis (30 points)


2.1: 5 pts Rearrange your circuit to create a SUPERNODE problem. You may accomplish this by moving ground or moving your components. Describe why your rearrangement resulted in a supernode. (If your circuit already had a supernode in Problem 1, find a way to create a different SUPERNODE circuit.)

## My new circuit diagram:



Explanantion of change and why it has a supernode:

I already had a supernode in Problem 1. The only place I could go is to the left since putting the voltage source where ground is wouldn't make it a supernode.

I could also move ground but decided not to for this one. (see other circuit 2)

Students don't have to change their values yet.


.tran 3ms
2.2: ( 5 pts) Rearrange your circuit to create a SUPERMESH problem. You may accomplish this by moving your components. Describe why you rearrangement resulted in a SUPERMESH. (Your circuit likely already had a supermesh in Problem 1. Find a way to create a different SUPERMESH circuit.)

## New Circuit Diagram:



## Explanantion of change and why it has a supermesh:

I already had a supermesh at the bottom. I would move it to the right making it between the top and bottom right loops. However, placing a current source is any of the given positions would make it a supermesh. I solved it with the current source in the bottom below.

Students don't have to change their values yet.
2.3: (15 pts) Using NODAL analysis for your SUPERNODE circuit in 2.1: OR MESH analysis for your SUPERMESH circuit in 2.2:, find the voltage across R2 (VR2). Write all relevant equations and matrix (if used) and label your diagram above with nodes or mesh loops for reference. The final answer value is not graded. FOR THIS PROBLEM YOU MUST PICK DIFFERENT COMPONENT VALUES THAN WHAT YOU CHOSE IN PROBLEM 1!
$\mathrm{R}_{1}=3 \times 10^{3}$
$\mathrm{R}_{2}=1 \times 10^{3}$
$\mathrm{R}_{3 \mathrm{p} 2}:=5.6 \cdot 10^{3}$
$\mathrm{R}_{4}=1 \times 10^{3}$
$\mathrm{I}_{1 \mathrm{p} 2}:=3.9 \cdot 10^{-3}$

$$
\mathrm{V}_{1 \mathrm{p} 2}:=2.8
$$

1. Nodes labeled
2. Ground Vn4=0
3. KCL Equation (student don't need to write this one)


$$
4-1-1=2
$$

## 4. Supernode definition equation

(1) $-\mathrm{V}_{\mathrm{n} 1}+\mathrm{V}_{\mathrm{n} 2}=2.8$
5. KCL equations...(going out)

## KCL supernode

$$
\frac{\mathrm{V}_{\mathrm{n} 1}-\mathrm{V}_{\mathrm{n} 3}}{\mathrm{R}_{2}}+\frac{\mathrm{V}_{\mathrm{n} 1}}{\mathrm{R}_{1}}+\frac{\mathrm{V}_{\mathrm{n} 2}-\mathrm{V}_{\mathrm{n} 3}}{\mathrm{R}_{3 \mathrm{p} 2}}-\mathrm{I}_{1 \mathrm{p} 2}=0
$$

$$
\text { (2) }\left(\frac{1}{\mathrm{R}_{2}}+\frac{1}{\mathrm{R}_{1}}\right) \cdot \mathrm{V}_{\mathrm{n} 1}+\frac{1}{\mathrm{R}_{3 \mathrm{p} 2}} \cdot \mathrm{~V}_{\mathrm{n} 2}-\left(\frac{1}{\mathrm{R}_{2}}+\frac{1}{\mathrm{R}_{3 \mathrm{p} 2}}\right) \cdot \mathrm{V}_{\mathrm{n} 3}=\mathrm{I}_{1 \mathrm{p} 2}
$$

KCL at node Vn3

$$
\frac{\mathrm{V}_{\mathrm{n} 3}-\mathrm{V}_{\mathrm{n} 1}}{\mathrm{R}_{2}}+\frac{\mathrm{V}_{\mathrm{n} 3}}{\mathrm{R}_{4}}+\frac{\mathrm{V}_{\mathrm{n} 3}-\mathrm{V}_{\mathrm{n} 2}}{\mathrm{R}_{3}}=0
$$

(3)

$$
\begin{align*}
& \frac{-1}{\mathrm{R}_{2}} \cdot \mathrm{~V}_{\mathrm{n} 1}-\frac{1}{\mathrm{R}_{3 \mathrm{p} 2}} \cdot \mathrm{~V}_{\mathrm{n} 2}+\left(\frac{1}{\mathrm{R}_{2}}+\frac{1}{\mathrm{R}_{4}}+\frac{1}{\mathrm{R}_{3 \mathrm{p} 2}}\right) \cdot \mathrm{V}_{\mathrm{n} 3}=0  \tag{6}\\
& \mathrm{M}_{2}:=\left[\begin{array}{cc}
-1 & 1 \\
\frac{1}{\mathrm{R}_{2}}+\frac{1}{\mathrm{R}_{1}} & \frac{1}{\mathrm{R}_{3 \mathrm{p} 2}} \\
\begin{array}{c}
\frac{-1}{\mathrm{R}_{2}} \\
-\frac{1}{\mathrm{R}_{3 \mathrm{p} 2}}
\end{array}\left(\begin{array}{c}
\left(\frac{1}{\mathrm{R}_{2}}+\frac{1}{\mathrm{R}_{2}}+\frac{1}{\mathrm{R}_{4}}+\frac{1}{\mathrm{R}_{3 \mathrm{p} 2}}\right)
\end{array}\right] \\
\mathrm{M}_{2}^{-1} \cdot \mathrm{C}_{2}=\left(\begin{array}{c}
4.198 \\
6.998 \\
2.501
\end{array}\right) \quad \mathrm{V}_{\mathrm{n} 1}:=4.198 \\
0
\end{array}\right) \quad \mathrm{C}_{2}:=\left(\begin{array}{c}
2.8 \\
3.9 \cdot 10^{-3} \\
0
\end{array}\right) \\
& \mathrm{V}_{\mathrm{n} 3}:=2.501
\end{align*}
$$

$$
\mathrm{V}_{\mathrm{R} 2}:=\mathrm{V}_{\mathrm{n} 1}-\mathrm{V}_{\mathrm{n} 3}=1.697 \mathrm{~V}
$$

Need this relationship between labeled nodes. My reference marks are plus on left and minus on right. If they get opposite polarity than expected, they should have opposite reference marks.


## 1. labeled mesh loops

2. KVL equations needed

$$
\begin{aligned}
& 3-1=2 \\
& \text { Fori1 } \\
& \mathrm{i}_{1} \cdot \mathrm{R}_{2}+\mathrm{i}_{1} \cdot \mathrm{R}_{3 \mathrm{p} 2}-\mathrm{i}_{3} \cdot \mathrm{R}_{3 \mathrm{p} 2}+\mathrm{V}_{1 \mathrm{p} 2}=0 \\
& \text { (1) }\left(\mathrm{R}_{2}+\mathrm{R}_{3 \mathrm{p} 2}\right) \cdot \mathrm{i}_{1}-\mathrm{R}_{3 \mathrm{p} 2} \cdot \mathrm{i}_{3}=-\mathrm{V}_{1 \mathrm{p} 2}
\end{aligned}
$$

For supermesh

$$
\mathrm{i}_{2} \cdot \mathrm{R}_{1}-\mathrm{V}_{1 \mathrm{p} 2}+\mathrm{i}_{3} \cdot \mathrm{R}_{3 \mathrm{p} 2}-\mathrm{i}_{1} \cdot \mathrm{R}_{3 \mathrm{p} 2}+\mathrm{i}_{3} \cdot \mathrm{R}_{4}=0
$$

(2) $\mathrm{R}_{1} \cdot \mathrm{i}_{2}+\left(\mathrm{R}_{3 \mathrm{p} 2}+\mathrm{R}_{4}\right) \cdot \mathrm{i}_{3}=\mathrm{V}_{1 \mathrm{p} 2}$
3. Need supermesh definition equation

$$
\begin{aligned}
\mathrm{i}_{3}-\mathrm{i}_{2}=\mathrm{I}_{1 \mathrm{p} 2} \\
\text { (3) } \begin{array}{ll}
-\mathrm{i}_{2}+\mathrm{i}_{3}=\mathrm{I}_{1 \mathrm{p} 2} & \mathrm{M}_{3}:=\left(\begin{array}{ccc}
\mathrm{R}_{2}+\mathrm{R}_{3 \mathrm{p} 2} & 0 & -\mathrm{R}_{3 \mathrm{p} 2} \\
-\mathrm{R}_{3 \mathrm{p} 2} & \mathrm{R}_{1} & \mathrm{R}_{3 \mathrm{p} 2}+\mathrm{R}_{4} \\
0 & -1 & 1
\end{array}\right) \\
\mathrm{C}_{3}:=\left(\begin{array}{c}
-\mathrm{V}_{1 \mathrm{p} 2} \\
\mathrm{~V}_{1 \mathrm{p} 2} \\
\mathrm{I}_{1 \mathrm{p} 2}
\end{array}\right) & \mathrm{M}_{3}{ }^{-1} \cdot \mathrm{C}_{3}=\left(\begin{array}{c}
1.698 \times 10^{-3} \\
-1.399 \times 10^{-3} \\
2.501 \times 10^{-3}
\end{array}\right) \quad \mathrm{i}_{1}:=1.698 \times 10^{-3}
\end{array}
\end{aligned}
$$

$$
\mathrm{V}_{\mathrm{R} 2 \mathrm{mesh}}:=\mathrm{i}_{1} \cdot \mathrm{R}_{2}=1.698
$$

Other circuit 1 with new component values:


1. Nodes labeled
2. Ground Vn4=0
3. KCL Equation (student don't need to write this one )

$$
4-1-1=2
$$

## 4. Supernode definition equation

$$
\begin{equation*}
-\mathrm{V}_{\mathrm{n} 2}+\mathrm{V}_{\mathrm{n} 3}=\mathrm{V}_{1 \mathrm{p} 2} \tag{1}
\end{equation*}
$$

5. KCL equations...(going out)

## KCL supernode

$$
\frac{\mathrm{V}_{\mathrm{n} 2}-\mathrm{V}_{\mathrm{n} 1}}{\mathrm{R}_{3 \mathrm{p} 2}}+\frac{\mathrm{V}_{\mathrm{n} 3}}{\mathrm{R}_{4}}+\frac{\mathrm{V}_{\mathrm{n} 3}-\mathrm{V}_{\mathrm{n} 1}}{\mathrm{R}_{2}}-\mathrm{I}_{1 \mathrm{p} 2}=0
$$

(2) $-\left(\frac{1}{\mathrm{R}_{3 \mathrm{p} 2}}+\frac{1}{\mathrm{R}_{2}}\right) \cdot \mathrm{V}_{\mathrm{n} 1}+\frac{1}{\mathrm{R}_{3 \mathrm{p} 2}} \cdot \mathrm{~V}_{\mathrm{n} 2}+\left(\frac{1}{\mathrm{R}_{4}}+\frac{1}{\mathrm{R}_{2}}\right) \cdot \mathrm{V}_{\mathrm{n} 3}=\mathrm{I}_{1 \mathrm{p} 2}$

KCL at node Vn1
$\frac{\mathrm{V}_{\mathrm{n} 1}-\mathrm{V}_{\mathrm{n} 3}}{\mathrm{R}_{2}}+\frac{\mathrm{V}_{\mathrm{n} 1}}{\mathrm{R}_{1}}+\frac{\mathrm{V}_{\mathrm{n} 1}-\mathrm{V}_{\mathrm{n} 2}}{\mathrm{R}_{3 \mathrm{p} 2}}=0$
(3) $\left(\frac{1}{\mathrm{R}_{2}}+\frac{1}{\mathrm{R}_{1}}+\frac{1}{\mathrm{R}_{3 \mathrm{p} 2}}\right) \cdot \mathrm{V}_{\mathrm{n} 1}-\frac{1}{\mathrm{R}_{3 \mathrm{p} 2}} \cdot \mathrm{~V}_{\mathrm{n} 2}-\left(\frac{1}{\mathrm{R}_{2}}\right) \cdot \mathrm{V}_{\mathrm{n} 3}=0$

$$
\mathrm{M}_{2}:=\left[\begin{array}{ccc}
0 & -1 & 1 \\
-\left(\frac{1}{\mathrm{R}_{3 \mathrm{p} 2}}+\frac{1}{\mathrm{R}_{2}}\right) & \frac{1}{\mathrm{R}_{3 \mathrm{p} 2}} & \left(\frac{1}{\mathrm{R}_{4}}+\frac{1}{\mathrm{R}_{2}}\right) \\
\left(\frac{1}{+}+\frac{1}{+}+1\right. & -1 & -1
\end{array}\right] \quad \mathrm{C}_{2}:=\left(\begin{array}{c}
\mathrm{V}_{1 \mathrm{p} 2} \\
\mathrm{I}_{\mathrm{p} 2} \\
0
\end{array}\right)
$$

$$
\left[\left(\frac{1}{\mathrm{R}_{2}}+\frac{1}{\mathrm{R}_{1}}+\frac{1}{\mathrm{R}_{3 \mathrm{p} 2}}\right)-\frac{1}{\mathrm{R}_{3 \mathrm{p} 2}}-\left(\frac{1}{\mathrm{R}_{2}}\right)\right]
$$

Need this relationship between labeled nodes. My reference marks are plus on left and minus

$$
\mathrm{M}_{2}^{-1} \cdot \mathrm{C}_{2}=\left(\begin{array}{l}
2.151 \\
0.383 \\
3.183
\end{array}\right) \quad \begin{aligned}
& \text { Vinds }:=2.151 \\
& \underset{\text { vindan }}{\mathrm{V}^{2}}:=3.183
\end{aligned}
$$

on right. If they get opposite polarity than expected, they should have opposite reference marks.

$$
V_{R 2 n}:=V_{n 1}-V_{n 3}=-1.032 \mathrm{~V}
$$



## 1. labeled mesh loops

2. KVL equations needed

$$
3-1=2
$$

For i3

$$
\mathrm{i}_{3} \cdot \mathrm{R}_{2}+\mathrm{i}_{3} \cdot \mathrm{R}_{3 \mathrm{p} 2}-\mathrm{i}_{1} \cdot \mathrm{R}_{3 \mathrm{p} 2}+\mathrm{V}_{1 \mathrm{p} 2}=0
$$

(1) $-\mathrm{R}_{3 \mathrm{p} 2 \cdot \mathrm{i}_{1}}+\left(\mathrm{R}_{3 \mathrm{p} 2}+\mathrm{R}_{2}\right) \cdot \mathrm{i}_{3}=-\mathrm{V}_{1 \mathrm{p} 2}$

For supermesh

$$
\mathrm{i}_{1} \cdot \mathrm{R}_{1}-\mathrm{V}_{1 \mathrm{p} 2}+\mathrm{i}_{1} \cdot \mathrm{R}_{3 \mathrm{p} 2}-\mathrm{i}_{3} \cdot \mathrm{R}_{3 \mathrm{p} 2}+\mathrm{i}_{2} \cdot \mathrm{R}_{4}=0
$$

(2) $\quad\left(\mathrm{R}_{1}+\mathrm{R}_{3 \mathrm{p} 2}\right) \cdot \mathrm{i}_{1}+\mathrm{R}_{4} \cdot \mathrm{i}_{2}-\mathrm{R}_{3 \mathrm{p} 2 \cdot \mathrm{i}_{3}}=\mathrm{V}_{1 \mathrm{p} 2}$
3. Need supermesh definition equation

$$
\begin{array}{ll}
\mathrm{i}_{2}-\mathrm{i}_{1}=\mathrm{I}_{1 \mathrm{p} 2} \\
\text { (3) } \quad{ }^{-\mathrm{i}_{1}+\mathrm{i}_{2}=\mathrm{I}_{1 \mathrm{p} 2}} & \mathrm{M}_{3}:=\left(\begin{array}{ccc}
-\mathrm{R}_{3 \mathrm{p} 2} & 0 & \mathrm{R}_{3 \mathrm{p} 2}+\mathrm{R}_{2} \\
\mathrm{R}_{1}+\mathrm{R}_{3 \mathrm{p} 2} & \mathrm{R}_{4} & -\mathrm{R}_{3 \mathrm{p} 2} \\
-1 & 1 & 0
\end{array}\right) \\
\mathrm{C}_{3}:=\left(\begin{array}{c}
-\mathrm{V}_{1 \mathrm{p} 2} \\
\mathrm{~V}_{1 \mathrm{p} 2} \\
\mathrm{I}_{1 \mathrm{p} 2}
\end{array}\right) \quad & \mathrm{M}_{3}{ }^{-1} \cdot \mathrm{C}_{3}=\left(\begin{array}{c}
-7.169 \times 10^{-4} \\
3.183 \times 10^{-3} \\
-1.032 \times 10^{-3}
\end{array}\right) \\
\mathrm{i}_{3}:=-1.032 \times 10^{-3}
\end{array}
$$

$$
\mathrm{V}_{\text {RNsmashv }}:=\mathrm{i}_{3} \cdot \mathrm{R}_{2}=-1.032 \mathrm{~V}
$$



1. Nodes labeled
2. Ground Vn3=0
3. KCL Equation (student don't need to write this one)

$$
4-1-1=2
$$

## 4. Supernode definition equation

$$
\begin{equation*}
-\mathrm{V}_{\mathrm{n} 4}+\mathrm{V}_{\mathrm{n} 2}=\mathrm{V}_{1 \mathrm{p} 2} \tag{1}
\end{equation*}
$$

5. KCL equations...(going out)

## KCL supernode

.tran 3ms

KCL at node Vn1
$\frac{\mathrm{V}_{\mathrm{n} 1}}{\mathrm{R}_{2}}+\mathrm{I}_{1 \mathrm{p} 2}+\frac{\mathrm{V}_{\mathrm{n} 1}-\mathrm{V}_{\mathrm{n} 4}}{\mathrm{R}_{1}}=0$
(3)

$$
\begin{gathered}
\left(\frac{1}{\mathrm{R}_{2}}+\frac{1}{\mathrm{R}_{1}}\right) \cdot \mathrm{V}_{\mathrm{n} 1}-\left(\frac{1}{\mathrm{R}_{1}}\right) \cdot \mathrm{V}_{\mathrm{n} 4}=-\mathrm{I}_{1 \mathrm{p} 2} \\
\mathrm{M}_{2}:=\left[\begin{array}{ccc}
0 & 1 & -1 \\
\frac{-1}{\mathrm{R}_{1}} & \frac{1}{\mathrm{R}_{3 \mathrm{p} 2}}\left(\frac{1}{\mathrm{R}_{4}}+\frac{1}{\mathrm{R}_{1}}\right) \\
\left(\frac{1}{\mathrm{R}_{2}}+\frac{1}{\mathrm{R}_{1}}\right) & 0 & -\left(\frac{1}{\mathrm{R}_{1}}\right)
\end{array}\right]
\end{gathered}
$$

$$
\mathrm{M}_{2}^{-1} \cdot \mathrm{C}_{2}=\left(\begin{array}{c}
-2.501 \\
4.497 \\
1.697
\end{array}\right) \quad \stackrel{\mathrm{V}_{n 1}}{ }:=-2.501
$$

Need this relationship between labeled nodes. My reference marks are plus on left and minus on right. If they get opposite polarity than expected, they should have opposite reference marks.

$$
V_{R_{2} x_{n}}:=V_{\mathrm{n} 1}=-2.501 \mathrm{~V}
$$



## 1. labeled mesh loops

2. KVL equations needed

$$
3-1=2
$$

For i2

$$
\mathrm{i}_{2} \cdot \mathrm{R}_{4}+\mathrm{i}_{2} \cdot \mathrm{R}_{3 \mathrm{p} 2}-\mathrm{i}_{3} \cdot \mathrm{R}_{3 \mathrm{p} 2}-\mathrm{V}_{1 \mathrm{p} 2}=0
$$

(1) $\left(\mathrm{R}_{4}+\mathrm{R}_{3 \mathrm{p} 2}\right) \cdot \mathrm{i}_{2}+-\left(\mathrm{R}_{3 \mathrm{p} 2}\right) \cdot \mathrm{i}_{3}=\mathrm{V}_{1 \mathrm{p} 2}$

For supermesh

$$
\mathrm{i}_{1} \cdot \mathrm{R}_{1}+\mathrm{V}_{1 \mathrm{p} 2}+\mathrm{i}_{3} \cdot \mathrm{R}_{2}+\mathrm{i}_{3} \cdot \mathrm{R}_{3 \mathrm{p} 2}-\mathrm{i}_{2} \cdot \mathrm{R}_{3 \mathrm{p} 2}=0
$$

.tran 3ms
(2) $\quad\left(\mathrm{R}_{1}\right) \cdot \mathrm{i}_{1}-\mathrm{R}_{3 \mathrm{p} 2} \cdot \mathrm{i}_{2}+\left(\mathrm{R}_{3 \mathrm{p} 2}+\mathrm{R}_{2}\right) \cdot \mathrm{i}_{3}=-\mathrm{V}_{1 \mathrm{p} 2}$
3. Need supermesh definition equation
$\mathrm{i}_{1}-\mathrm{i}_{3}=\mathrm{I}_{1 \mathrm{p} 2}$
(3)

$$
\mathrm{i}_{1}-\mathrm{i}_{3}=\mathrm{I}_{1 \mathrm{p} 2}
$$

$$
M_{3}:=\left(\begin{array}{ccc}
0 & \mathrm{R}_{4}+\mathrm{R}_{3 \mathrm{p} 2} & -\mathrm{R}_{3 \mathrm{p} 2} \\
\mathrm{R}_{1} & -\mathrm{R}_{3 \mathrm{p} 2} & \mathrm{R}_{3 \mathrm{p} 2}+\mathrm{R}_{2} \\
1 & 0 & -1
\end{array}\right)
$$

$\mathrm{C}_{3}:=\left(\begin{array}{c}\mathrm{V}_{1 \mathrm{p} 2} \\ -\mathrm{V}_{1 \mathrm{p} 2} \\ \mathrm{I}_{1 \mathrm{p} 2}\end{array}\right)$

$$
\mathrm{M}_{3}^{-1} \cdot \mathrm{C}_{3}=\left(\begin{array}{c}
1.399 \times 10^{-3} \\
-1.698 \times 10^{-3} \\
-2.501 \times 10^{-3}
\end{array}\right) \quad \text { id } \quad \text { i.v }:=-2.501 \times 10^{-3}
$$

$$
\mathrm{V}_{\mathrm{R} 2 \mathrm{manesh}}:=\mathrm{i}_{3} \cdot \mathrm{R}_{2}=-2.501 \mathrm{~V}
$$

## Problem 3) Thevenin/Norton Dependent Circuits (30 pts)

For this problem you my rerrange you component parts using the dependent source with a given value beow. The components can be put in any orientation you'd like. You may use other values that are consistent with other parts of the exam or choose completely new ones. You final answer values ARE graded in this problem.

3.1: $\mathbf{1 0}$ pts $\mathbf{R 2}$ is the load. Find $V_{T H}$ using the Open Circuit method.

My strategy, with the loadoff, only the bottom loops remain. Therefore, I'd make a supermesh problem with the current source at the bottom so I have one supermesh loop, a supermesh defintion, and a dependent source definition problem only.


Supermesh KVL

$$
\mathrm{i}_{1} \cdot \mathrm{R}_{1}-4 \mathrm{Ix}+\mathrm{i}_{2} \cdot \mathrm{R}_{3 \mathrm{p} 2}+\mathrm{i}_{2} \cdot \mathrm{R}_{4}=0
$$

(1) $\mathrm{R}_{1} \cdot \mathrm{i}_{1}+\left(\mathrm{R}_{3 \mathrm{p} 2}+\mathrm{R}_{4}\right) \cdot \mathrm{i}_{2}-4 \mathrm{I}_{\mathrm{x}}=0$
supermesh defintion

$$
\mathrm{i}_{2}-\mathrm{i}_{1}=\mathrm{I}_{1 \mathrm{p} 2}
$$

(2) $-\mathrm{i}_{1}+\mathrm{i}_{2}=\mathrm{I}_{1 \mathrm{p} 2}$
dependent source defintion
(3) $\mathrm{i}_{2}+\mathrm{I}_{\mathrm{x}}=0$

$$
\begin{gathered}
\mathrm{M}_{5}:=\left(\begin{array}{ccc}
\mathrm{R}_{1} & \mathrm{R}_{3 \mathrm{p} 2}+\mathrm{R}_{4} & -4 \\
-1 & 1 & 0 \\
0 & 1 & 1
\end{array}\right) \quad \mathrm{V}_{\mathrm{Th}} \\
\mathrm{C}_{5}:=\left(\begin{array}{c}
0 \\
\mathrm{I}_{1 \mathrm{p} 2} \\
0
\end{array}\right) \\
\end{gathered}
$$

## Watch polarity here!!!

3.2: $\mathbf{1 0}$ pts Find INorton uisng short circuit current (lsc).

Supermesh KVL


$$
\begin{aligned}
& i_{1} \cdot R_{1}-4 V_{x}+i_{2} \cdot R_{3 p 2}-i_{3} \cdot R_{3 p 2}+i_{2} \cdot R_{4}=0 \\
& \text { (1) } R_{1} \cdot i_{1}+\left(R_{3 p 2}+R_{4}\right) \cdot i_{2}-i_{3} \cdot R_{3 p 2}-4 I_{x}=0
\end{aligned}
$$

supermesh defintion

$$
\begin{aligned}
\mathrm{i}_{2}-\mathrm{i}_{1} & =\mathrm{I}_{1 \mathrm{p} 2} \\
\text { (2) }-\mathrm{i}_{1}+\mathrm{i}_{2} & =\mathrm{I}_{1 \mathrm{p} 2}
\end{aligned}
$$

dependent source defintion
(3) $\mathrm{i}_{2}+\mathrm{I}_{\mathrm{X}}=0$

$$
\mathrm{M}_{5}:=\left(\begin{array}{cccc}
\mathrm{R}_{1} & \mathrm{R}_{3 \mathrm{p} 2}+\mathrm{R}_{4} & -\mathrm{R}_{3 \mathrm{p} 2} & -4 \\
-1 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 \\
0 & -\mathrm{R}_{3 \mathrm{p} 2} & \mathrm{R}_{3 \mathrm{p} 2} & 4
\end{array}\right)
$$

(4) $-\mathrm{R}_{3 \mathrm{p} 2} \cdot \mathrm{i}_{2}+\mathrm{i}_{3} \cdot \mathrm{R}_{3 \mathrm{p} 2}+4 \mathrm{I}_{\mathrm{x}}=0$

$$
\mathrm{C}_{5}:=\left(\begin{array}{c}
0 \\
\mathrm{I}_{1 \mathrm{p} 2} \\
0 \\
0
\end{array}\right) \quad \mathrm{M}_{5}^{-1} \cdot \mathrm{C}_{5}=\left(\begin{array}{c}
-9.75 \times 10^{-4} \\
2.925 \times 10^{-3} \\
2.927 \times 10^{-3} \\
-2.925 \times 10^{-3}
\end{array}\right) \quad \mathrm{i}_{\mathrm{sc}}:=2.927 \times 10^{-3}
$$

3.3. $\mathbf{8} \boldsymbol{p t s}$ Find Rth using the Test Method.
loop i2


$$
1+\mathrm{i}_{2} \cdot \mathrm{R}_{3 \mathrm{p} 2}-\mathrm{i}_{1} \cdot \mathrm{R}_{3 \mathrm{p} 2}+4 \mathrm{I}_{\mathrm{x}}=0
$$

(1) $-\mathrm{R}_{3 \mathrm{p} 2} \cdot \mathrm{i}_{1}+\mathrm{R}_{3 \mathrm{p} 2} \cdot \mathrm{i}_{2}+4 \mathrm{I}_{\mathrm{x}}=-1$
loop i1

$$
\mathrm{i}_{1} \cdot \mathrm{R}_{1}-4 \cdot \mathrm{Ix}+\mathrm{i}_{1} \cdot \mathrm{R}_{3 \mathrm{p} 2}-\mathrm{i}_{2} \cdot \mathrm{R}_{3 \mathrm{p} 2}+\mathrm{i}_{1} \cdot \mathrm{R}_{4}=0
$$

$$
\begin{equation*}
\left(R_{1}+R_{3 p 2}+R_{4}\right) \cdot i_{1}-R_{3 p 2} \cdot i_{2}-4 I_{x}=0 \tag{2}
\end{equation*}
$$

$$
\mathrm{M}_{5}:=\left[\begin{array}{ccc}
-\mathrm{R}_{3 \mathrm{p} 2} & \mathrm{R}_{3 \mathrm{p} 2} & 4 \\
\left(\mathrm{R}_{1}+\mathrm{R}_{3 \mathrm{p} 2}+\mathrm{R}_{4}\right) & -\mathrm{R}_{3 \mathrm{p} 2} & -4 \\
1 & 0 & 1
\end{array}\right]
$$

$$
\text { (3) } \mathrm{i}_{1}+\mathrm{I}_{\mathrm{x}}=0
$$

$$
\begin{aligned}
& \mathrm{i}_{2}:=-4.288 \times 10^{-4} \\
& \mathrm{C}_{5}:=\left(\begin{array}{c}
-1 \\
0 \\
0
\end{array}\right) \quad \mathrm{M}_{5}{ }^{-1} \cdot \mathrm{C}_{5}=\left(\begin{array}{c}
-2.5 \times 10^{-4} \\
-4.288 \times 10^{-4} \\
2.5 \times 10^{-4}
\end{array}\right) \\
& \frac{1}{\mathrm{i}_{\text {Test }}}:=-\mathrm{i}_{2} \\
& \frac{1}{{ }^{\mathrm{i}} \text { Test }}=2.332 \times 10^{3} \mathrm{R}_{\text {Th }} \\
&(\Omega) \\
& \hline
\end{aligned}
$$

3.4: $\mathbf{2}$ pts Verify your answers from 3.1-3:3. (Verify they are correct).


$$
\frac{\mathrm{V}_{\mathrm{TH}}}{\mathrm{I}_{\mathrm{N}}}=\mathrm{R}_{\mathrm{TH}}
$$

## 4) Cascading Multi-stage Op AmpCircuits andDesign Concepts (20 pts)

4.1: (10 pts) You have 2 inputs, V1 and V2. Design and draw a cascade of op amps that result in the equation $-2 \mathrm{~V} 1+3 \mathrm{~V} 2$ when V 1 and V 2 are both positive values. You are limited to 3 op amps. Make sure to add in your resistor values.

inverting summer but V2 must have a inverting amp before it. need correct ratio of resistors 2 in first op amp, 3 is second. Don't have to draw power sources in for this one.
4.2: (4 pts) Name two voltage divider applications and briefly describe the circuit around it. It cannot be just to "divide the voltage" or to reduce voltage from high voltage to a lower voltage. These answers result in zero points.
examples: bridge circuit, resistive sensor, double voltage divider filter, needs AC input, capacitor or inductor
4.3: (2 pts) What is the difference between closed loop gain and open loop gain? Describe how they functions to define op amp characteristics?

Open loop gain is infinity and intrinsic to op amp characteristics.. used as ideal characteristic Closed loop gain is finite and is derived from op amp configuration including the feed back loop, used to help define how the op amp configuration changes an input
4.4: Determine whether the statements about comparators are true or false. You need to write a brief explanation for full credi.

Comparators are decision circuits that provide a binary output

Explanation: Key words or phrases "saturation.... set high and low voltage... depending on input...power source."

F Comparators use a feedback loop.
Explanation: Key words of phrases, "linear region or saturation region"
Comparators operate in the linear region

Explanation: Key words of phrases "saturation region or high gain"
Comparators must always output both polarities of a given DC value. (i.e. +5 and -5 V )
Explanation: Key words or phrases "power source voltage"

