

Circuits

Exam 2

Fall 2020

1.	/52.5
2.	/47.5
Total	/100

Name _____

Notes:

- 1) Your crib sheet is provided in the Exams Team Space.
- 2) You may use your calculator (only).
- 3) You cannot use your computer or cell phone during any part of the exam. Doing so results in an automatic 0 for this exam.

Please sign below:

I have not consulted any person or collaborated with anyone to complete this exam. I did not post and will not post any part of this exam to Chegg.com or any other equivalent websites. I understand that if my exam is found online, I will be given an F for the semester and the academic dishonesty process will be initiated. I did not look for answers on any website to this exam. If any signification portion of this exam is found to match with any other student, I will be given an automatic 0 for the entire exam. Further actions due to academic dishonesty may be warranted after discussion with all parties.

Signature: _____

Problem 1) Differential Equations (50 pts)

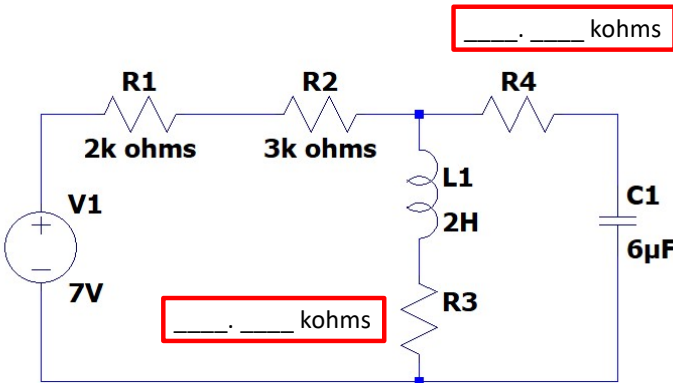
Please solve all sections of Problem 1 using Differential Equations

(time-domain analysis). If you solve any problem in Problem 1 using Laplace (s-domain analysis), you will receive 0 pts for that problem. Please refer to the circuit(s) in the problem sub-section.

1.1: (10 pts) Finding Initial Conditions Step: Find initial conditions for one of the circuits below.

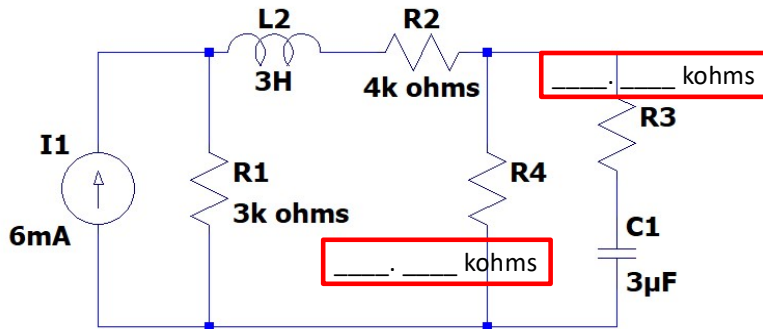
Please choose your circuit (circle it). Enter in your integer values for R3 and R4 (1-9). Do not include zeros. Find the initial conditions for the capacitor and inductor. $V_c(0^+)$ and $i_L(0^+)$

Note: These circuits need switches to provide a change for dynamic components. But can still be solved for initial conditions....they just match the final conditions.

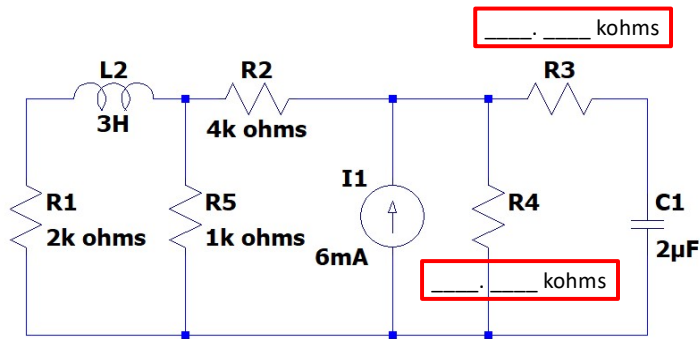


Please circle below

Choice #1



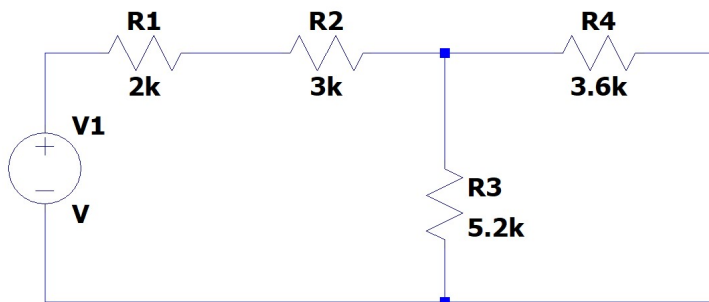
Choice #2



Choice #3

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Choice 1



$$V_{11c1} := 7V \quad R_{11c1} := 2000\Omega \quad R_{21c1} := 3000\Omega$$

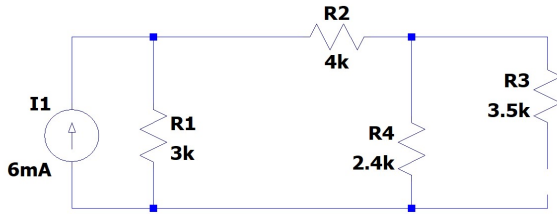
$$R_{31c1} := 5200\Omega$$

$$R_{41c1} := 3600\Omega$$

$$I_{L1c1} := \frac{V_{11c1}}{R_{11c1} + R_{21c1} + R_{31c1}} = 0.686 \cdot \text{mA}$$

$$V_{C1c1} := V_{11c1} \cdot \frac{R_{31c1}}{(R_{11c1} + R_{21c1} + R_{31c1})} = 3.569 \text{ V}$$

Choice 2



$$I_{11c2} := 5\text{mA} \quad R_{11c2} := 3000\Omega \quad R_{21c2} := 4000\Omega$$

$$R_{31c2} := 3500\Omega$$

$$R_{41c2} := 2400\Omega$$

$$I_{L1c2} := I_{11c2} \cdot \frac{R_{11c2}}{R_{11c2} + R_{21c2} + R_{41c2}} = 1.596 \cdot \text{mA}$$

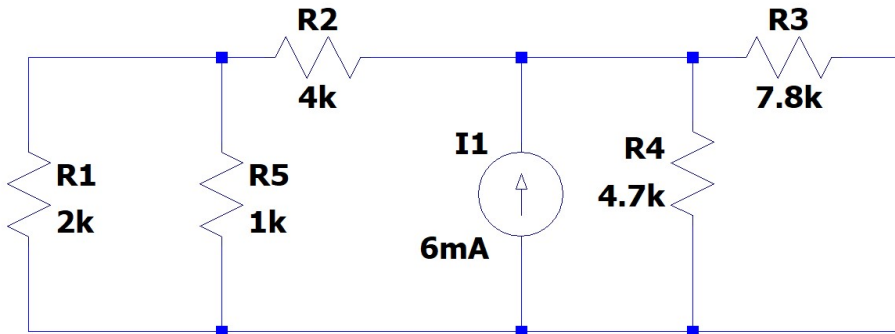
Current divider or equivalent

Same as VR4

$$V_{C1c2} := I_{L1c2} \cdot R_{41c2} = 3.83 \text{ V}$$

Choice # _____	$V_c(0+) =$
	$I_L(0+) =$

Choice 3



Double current divider to find IR1 which is the same as IL2

$$I_{11c3} := 6\text{mA} \quad R_{11c3} := 2000\Omega \quad R_{21c3} := 4000\Omega$$

$$R_{31c3} := 7800\Omega$$

$$R_{41c3} := 4700\Omega$$

$$R_{51c3} := 1000\Omega$$

$$R_{151c3} := \frac{R_{11c3} \cdot R_{51c3}}{R_{11c3} + R_{51c3}} = 666.667 \Omega$$

$$R_{1512c3} := R_{151c3} + R_{21c3} = 4.667 \times 10^3 \Omega$$

$$I_{R1512c3} := I_{11c3} \cdot \frac{R_{41c3}}{R_{41c3} + R_{1512c3}} = 3.011 \cdot \text{mA}$$

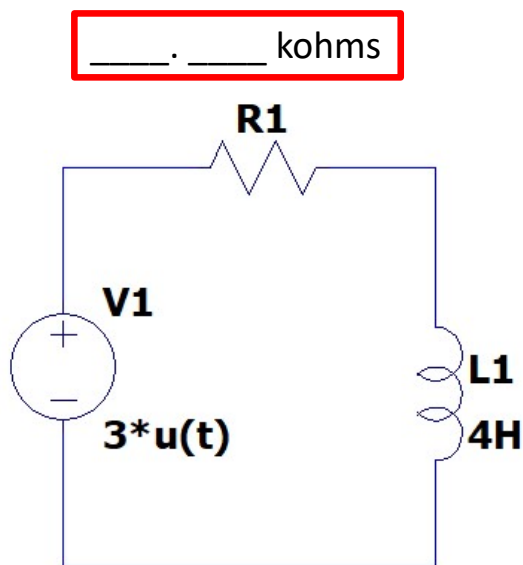
$$I_{R1c3} := I_{R1512c3} \cdot \frac{R_{51c3}}{R_{11c3} + R_{51c3}} = 1.004 \cdot \text{mA}$$

$$I_{R4} := I_{11c3} \cdot \frac{R_{1512c3}}{R_{41c3} + R_{1512c3}} = 2.989 \cdot \text{mA}$$

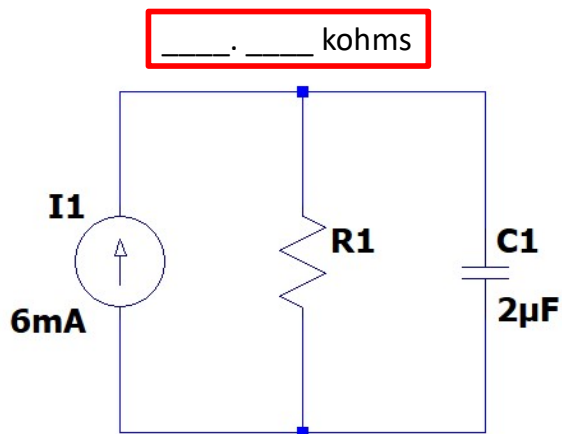
$$V_{R4} := I_{R4} \cdot R_{41c3} = 14.05 \text{ V}$$

1.2: (10 pts) Deriving the Differential Equation Step: Derive the differential equation for one of the small circuits below.

Please choose your circuit (circle it). Enter in your integer values for R1 (1-9). Do not include zeros.



Choice #1
Find $v_L(t)$



Choice #2
Find $i_C(t)$

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Choice #1 $V_{R1} + V_L = V_1$
 $i_L \cdot R_1 + V_L = V_1$

$$V_L = \frac{L di_L}{dt}$$

$$I_L = \frac{1}{L} \left(\int V_L dt \right)$$

$$\frac{1}{L} \left(\int V_L dt \right) \cdot R_1 + V_L = V_1$$

$R_{1p12} := 1200$
 $L_{1p12} := 4H$

$$\frac{R_{1p12}}{L_{1p12}} = 300 \frac{1}{H}$$

$$\frac{dV_L}{dt} + \frac{R_1}{L} \cdot V_L = \frac{dV_1}{dt}$$

$$\frac{dV_L}{dt} + 300 \cdot V_L = \frac{d(V_1)}{dt}$$

Choice #2

$$-I_1 + I_{R1} + I_C = 0$$

$$I_{R1} + I_C = I_1$$

$$\frac{V_C}{R_1} + I_C = I_1$$

$$\frac{1}{R_1 \cdot C} \int I_C dt + I_C = I_1$$

$$\frac{1}{R_1 \cdot C} \cdot I_C + \frac{dI_C}{dt} = \frac{dI_1}{dt}$$

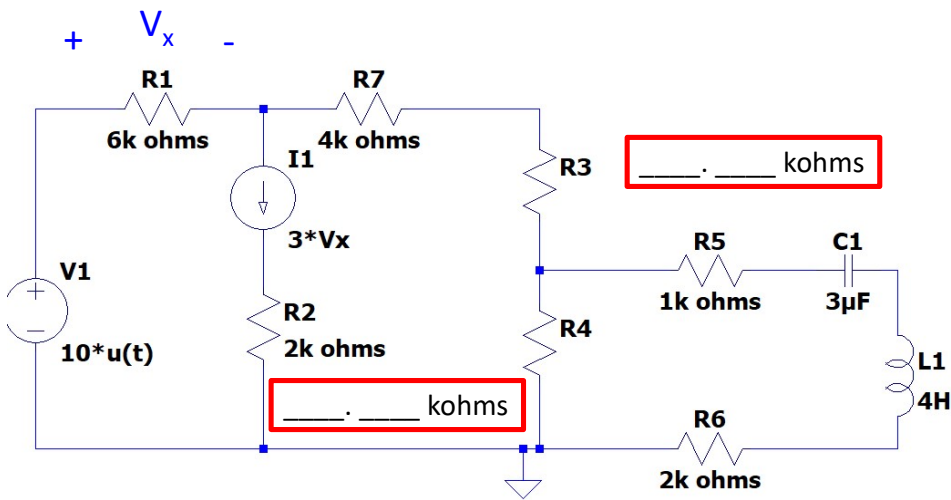
$C_1 := 2 \cdot 10^{-6} F$
 $I_C = \frac{CdV_c}{dt}$
 $V_C = \frac{1}{C} \int I_C dt$

$$\frac{dI_C}{dt} + \frac{1}{R_1 \cdot C} \cdot I_C = \frac{dI_1}{dt}$$

$$\frac{1}{C_1 \cdot R_{1p12}} = 416.667 \frac{1}{F}$$

$$\frac{dI_C}{dt} + 417 \cdot I_C = \frac{dI_1}{dt}$$

1.3: Solving the Differential Equation (20 pts): Find the solution for $V_{C1}(t)$ below using an important Unit 1 concept (that you knew was coming again on Exam 2). You **DO NOT** have to find the coefficients. You should find α and ω_0 , (and β if necessary). Write the solution without finding the coefficients.



Make C1 and L1 the load. Find VR4 at Voc.

$$R_{13} := 6000$$

$$V_{13} := 10$$

$$I_1 = i_1 - i_2$$

$$R_{23} := 2000$$

$$R_{33} := 4500$$

$$(1) \quad i_1 - i_2 - 3V_x = 0$$

$$R_{43} := 3200$$

$$R_{73} := 1000$$

$$-10 + i_1 \cdot R_1 + i_2 \cdot R_7 + i_2 \cdot R_3 + i_2 \cdot R_4 = 0$$

$$R_{53} := 1000$$

$$R_{63} := 2000$$

$$V_{\text{Test}} := 1$$

$$(2) \quad R_{13} i_1 + (R_{73} + R_{33} + R_{43}) i_2 = 10$$

$$V_x = i_1 \cdot R_{13}$$

$$(3) \quad i_1 \cdot R_{13} - V_x = 0$$

$$M := \begin{pmatrix} 1 & -1 & -3 \\ R_{13} & R_{73} + R_{33} + R_{43} & 0 \\ R_{13} & 0 & -1 \end{pmatrix}$$

$$C_{m1} := \begin{pmatrix} 0 \\ 10 \\ 0 \end{pmatrix}$$

$$M^{-1} \cdot C_{m1} = \begin{pmatrix} -6.386 \times 10^{-8} \\ 1.149 \times 10^{-3} \\ -3.832 \times 10^{-4} \end{pmatrix}$$

$$i_2 := 1.149 \times 10^{-3}$$

$$V_{\text{oc}} := R_{43} \cdot i_2 = 3.677$$

R_{TH}

$$I_1 = i_1 - i_2$$

$$(1) \quad i_1 - i_2 - 3V_x = 0$$

$$i_1 \cdot R_1 + i_2 \cdot R_7 + i_2 \cdot R_3 + i_2 \cdot R_4 - i_3 \cdot R_4 = 0$$

$$(2) \quad R_{13} i_1 + (R_{73} + R_{33} + R_{43}) i_2 - R_{43} i_3 = 0$$

$$V_x = i_1 \cdot R_{13}$$

$$(3) \quad i_1 \cdot R_{13} - V_x = 0$$

$$i_3 \cdot R_4 - i_2 \cdot R_4 + i_3 \cdot R_5 + i_3 \cdot R_6 + V_{\text{Test}} = 0$$

$$(4) \quad -R_{43} i_2 + (R_{43} + R_{53} + R_{63}) i_3 = -V_{\text{Test}}$$

$$M_1 := \begin{pmatrix} 1 & -1 & 0 & -3 \\ R_{13} & R_{73} + R_{33} + R_{43} & -R_{43} & 0 \\ R_{13} & 0 & 0 & -1 \\ 0 & -R_{43} & R_{43} + R_{53} + R_{63} & 0 \end{pmatrix}$$

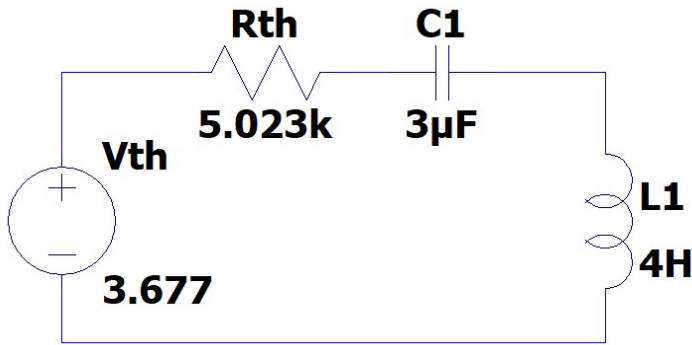
$$C_{m2} := \begin{pmatrix} 0 \\ 0 \\ 0 \\ -V_{\text{Test}} \end{pmatrix}$$

$$M_1^{-1} \cdot C_{m2} = \begin{pmatrix} 4.069 \times 10^{-9} \\ -7.323 \times 10^{-5} \\ -1.991 \times 10^{-4} \\ 2.441 \times 10^{-5} \end{pmatrix}$$

$$i_3 := -1.991 \times 10^{-4}$$

$$i_{\text{Test}} := -i_3 = 1.991 \times 10^{-4}$$

$$R_{\text{TH}} := \frac{V_{\text{Test}}}{i_{\text{Test}}} = 5.023 \times 10^3$$



$$L_{1p1} := 4$$

$$C_{1p1} := 3 \cdot 10^{-6}$$

$$\alpha := \frac{R_{TH}}{2 \cdot L_{1p1}} = 627.825$$

$$\omega_o := \sqrt{\frac{1}{L_{1p1} \cdot C_{1p1}}} = 288.675$$

$\alpha > \omega_o$ overdamped

$$s_1 := -\alpha + \sqrt{\alpha^2 - \omega_o^2} = -70.303$$

$$s_2 := -\alpha - \sqrt{\alpha^2 - \omega_o^2} = -1.185 \times 10^3$$

$$V_C(t) = A_1 \cdot e^{-70.3t} + A_2 \cdot e^{-1.185 \cdot 10^3 t} + A_3$$

$V_{C1}(t)$	
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Check if values chosen result in an underdamped or critically damped circuit

$$R_{TH2} := 2 \cdot L_{1p1} \cdot \sqrt{\frac{1}{L_{1p1} \cdot C_{1p1}}} = 2.309 \times 10^3$$

For my resistor choices:

Overdamped $R_{TH} > 2.309k\Omega$

Critically damped $R_{TH} = 2.309k\Omega$

Underdamped $R_{TH} < 2.309k\Omega$

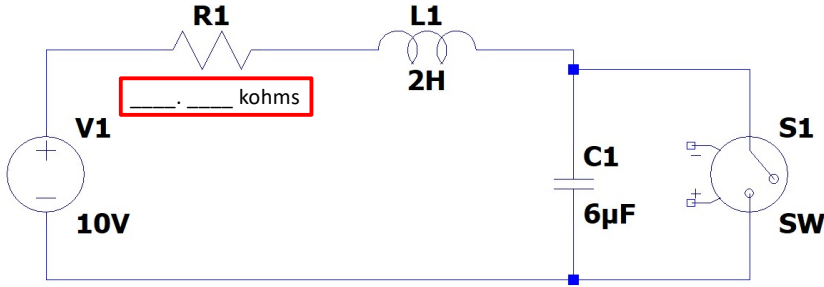
$$V_{C1}(t) = A_1 e^{-288.63t} + A_2 t e^{-288.63t} + A_3 \quad \frac{2.309 \cdot 10^3}{2 \cdot L_{1p1}} = 288.625$$

$$V_{C1}(t) = e^{-125t} \cdot [A_1 \cdot \cos(260.2t) + A_2 \cdot \sin(260.2t)] + A_3$$

$$\beta := \sqrt{\omega_o^2 - 125^2} = 260.208 \quad \frac{1 \cdot 10^3}{2 \cdot L_{1p1}} = 125$$

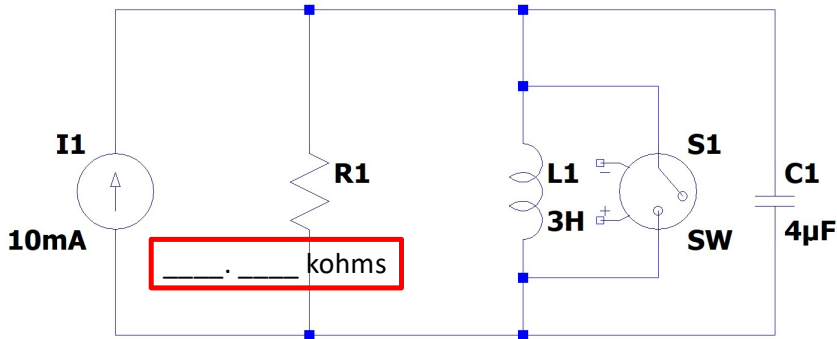
1.4: Using Initial Conditions to find Coefficients (10 pts): Find the solution and coefficients for one of the circuits below. Your solution will depend on your choice of R1. You do not have to derive the differential

equations for the RLC circuits below you can simply use the differential equation known for RLC series and parallel circuits.



Choice #1
Find $i_c(t)$

The switch S1 opens at $t=0$.
(for both circuits)



Choice #2
Find $v_L(t)$

Choice	
# _____	

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Choice 1:

$$V_{14c1} := 10 \quad R_{14c1} := 2400 \quad L_{14c1} := 2 \quad C_{14c1} := 6 \cdot 10^{-6}$$

RLC parallel circuit so using $v_c(0) = 0$ $i_{L0} := \frac{V_{14c1}}{R_{14c1}} = 4.167 \times 10^{-3}$

$$\alpha_{c1} := \frac{R_{14c1}}{2 \cdot L_{14c1}} = 600 \quad \omega_{oc1} := \sqrt{\frac{1}{L_{14c1} \cdot C_{14c1}}} = 288.675 \quad \alpha > \omega_o$$

Critically damped borderline for grading $R_{1test} := 2 \cdot L_{14c1} \sqrt{\frac{1}{L_{14c1} \cdot C_{14c1}}} = 1.155 \times 10^3$

overdamped $R_1 > 1.155 \times 10^3$

critically damped $R_1 = 1.155 \times 10^3$

underdamped $R_1 < 1.155 \times 10^3$

Finding $v_c(t)$ $s_{1c1} := -\alpha_{c1} + \sqrt{\alpha_{c1}^2 - \omega_{oc1}^2} = -74.009$

$$s_{2c1} := -\alpha_{c1} - \sqrt{\alpha_{c1}^2 - \omega_{oc1}^2} = -1.126 \times 10^3 \quad \frac{dI_L}{dt} = \frac{v_{c0}}{L_{14c1}}$$

$$V_c(t) = A_1 e^{-74t} + A_2 \cdot e^{-1.126 \times 10^3 \cdot t} + A_3 \quad \text{final conditions} \quad A_3 := 10$$

Initial conditions find coefficients using $v_c(0)$ and $dvc(0)/dt = i_L(0)/C$
take derivative and multiply by C

$$0 = A_1 + A_2 + 10$$

$$A_1 + A_2 = -10$$

$$-74A_1 - 1.126 \cdot 10^3 A_2 = 694.4$$

$$\frac{i_{L0}}{C_{14c1}} = 694.444$$

$$M_1 := \begin{bmatrix} 1 & 1 \\ -74 & -(1.126 \cdot 10^3) \end{bmatrix}$$

$$C_{m3} := \begin{pmatrix} -10 \\ 694.4 \end{pmatrix}$$

$$M_1^{-1} \cdot C_{m3} = \begin{pmatrix} -10.043 \\ 0.043 \end{pmatrix}$$

$$V_c(t) = -10.043 \cdot e^{-74t} + 0.043 \cdot e^{-1.126 \cdot 10^3 t} + 10$$

$$C_{14c1} \cdot -74 \cdot -10.043 = 4.459 \times 10^{-3}$$

$$i_c(t) = C \cdot \frac{dV_c}{dt}$$

$$C_{14c1} \cdot 0.043 \cdot -1.126 \cdot 10^3 = -2.905 \times 10^{-4}$$

$$i_c(t) = 4.459 \times 10^{-3} \cdot e^{-74t} - 2.905 \times 10^{-4} \cdot e^{-1.126 \cdot 10^3 t}$$

Of course, the type of circuit and answer depends on the value of R1.

$$R_1 = 1.155 \times 10^3 \text{ critically damped}$$

$$R_1 < 1.155 \times 10^3 \text{ underdamped}$$

Choice 2:

$$I_{14c1} := 10\text{mA} \quad R_{14c1} := 400 \quad L_{14c1} := 3 \quad C_{14c1} := 4 \cdot 10^{-6}$$

RLC parallel circuit so using

$$v_c(0) = 0 \quad i_{L0} := I_{14c1} = 0.01 \text{ A}$$

$$\alpha_{oc1} := \frac{1}{2 \cdot R_{14c1} \cdot C_{14c1}} = 312.5 \quad \omega_{oc1} := \sqrt{\frac{1}{L_{14c1} \cdot C_{14c1}}} = 288.675 \quad \alpha > \omega_o$$

Critically damped borderline for grading

$$R_{1test} := \left(2 \cdot C_{14c1} \sqrt{\frac{1}{L_{14c1} \cdot C_{14c1}}} \right)^{-1} = 433.013$$

$$\text{overdamped} \quad R_1 < 433$$

$$\text{critically damped} \quad R_1 = 433$$

$$\text{underdamped} \quad R_1 > 433$$

Finding $i_L(t)$

$$s_{1oc1} := -\alpha_{oc1} + \sqrt{\alpha_{oc1}^2 - \omega_{oc1}^2} = -192.822$$

$$s_{2oc1} := -\alpha_{oc1} - \sqrt{\alpha_{oc1}^2 - \omega_{oc1}^2} = -432.178$$

$$\frac{dI_L}{dt} = \frac{v_{c0}}{L_{14c1}}$$

final conditions

$$i_L(t) = A_1 e^{-192.822t} + A_2 \cdot e^{-432.178 \cdot t} + A_3$$

$$A_3 := 10\text{mA}$$

Initial conditions

find coefficients using $i_L(0)$ and $dI_L(0)/dt = v_c(0)/L$
take derivative and multiply by L

$$0 = A_1 + A_2 + 10\text{mA}$$

$$A_1 + A_2 = -10\text{mA}$$

$$-192.822A_1 - 432.178A_2 = 0$$

$$\frac{v_c(0)}{L_{14c1}} = 0$$

$$M_1 := \begin{bmatrix} 1 & 1 \\ -192.822 & -(432.178) \end{bmatrix} \quad C_{m3} := \begin{pmatrix} -0.01 \\ 0 \end{pmatrix} \quad M_1^{-1} \cdot C_{m3} = \begin{pmatrix} -0.018 \\ 8.056 \times 10^{-3} \end{pmatrix}$$

$$V_c(t) = -0.018 \cdot e^{-192.822t} + 8.056 \times 10^{-3} \cdot e^{-432.178 \cdot t} + 0.01$$

$$v_L(t) = L \cdot \frac{di_L}{dt}$$

$$L_{14c1} \cdot -192.822 \cdot -0.018 = 10.412$$

$$L_{14c1} \cdot (-432.178) \cdot (8.056 \times 10^{-3}) = -10.445$$

$$i_c(t) = 10.412 \cdot e^{-192.822t} - 10.445 \cdot e^{-432.178 \cdot t}$$

Of course, the type of circuit and answer depends on the value of R_1 .

$$R_1 = 433 \quad \text{critically damped}$$

$$R_1 > 433 \quad \text{underdamped}$$

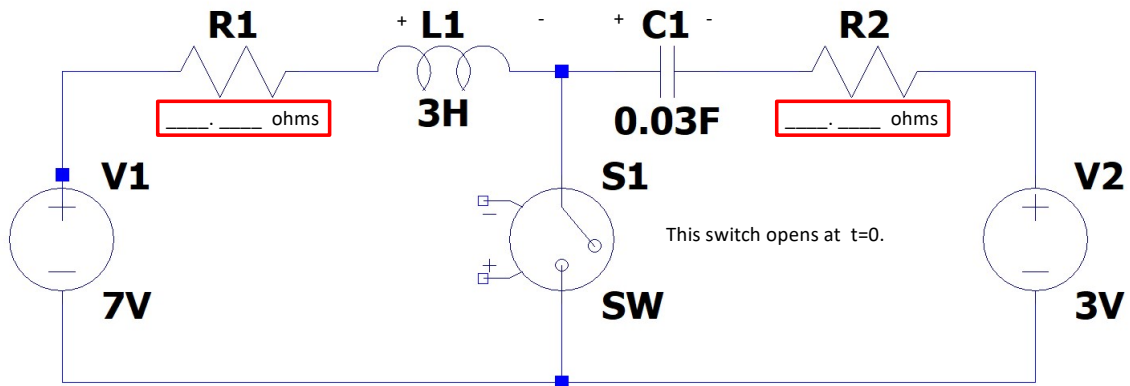
1.5 Conceptual Question (2.5 pts): When solving problem 1.4 can you find $V_L(t)$ or $I_c(t)$ **directly** for the series RLC circuit or parallel RLC circuit respectively using differential equations? Why or why not? Highlight any part of the process that best describes your answer.

No $\frac{di_c}{dt}$ and $\frac{dv_L}{dt}$ don't have values

Problem 2) Laplace Transforms (50 pts)

Please solve all sections of Problem 2 using Laplace transforms (s-domain analysis). Please refer to the circuit(s) in the problem sub-section.

2.1: (15 pts) Convert to S-domain Circuit: Convert the circuit in the time domain to its s-domain equivalent circuit. You will need to find initial conditions. You can choose either voltage sources or current sources to represent the s-domain circuit. You must show calculations and drawings for full credit.



In the above circuit, switch S1 is closed (shorted) for $t < 0$ and opens at $t = 0$. Be careful with polarities which are indicated for L and C. Write your values for R1 and R2 using integers (1-9). Do not use 0.

$$R_{1p21} := 1200 \quad R_{2p21} := 4500 \quad L_{1p21} := 3 \quad C_{1p21} := 0.03 \quad V_{1p21} := 7$$

$$V_{2p21} := 3$$

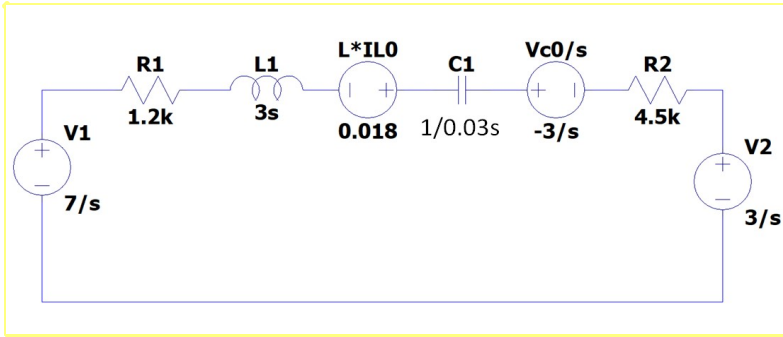
draw $t=0^-$...solve left and right circuits for IL and VC respectively...

$$I_{L10} := \frac{V_{1p21}}{R_{1p21}} = 5.833 \times 10^{-3} \quad L_{1p21} \cdot I_{L10} = 0.018$$

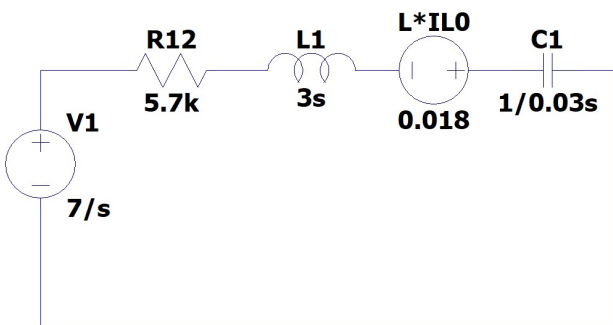
$$V_{C10} := -V_{2p21} = -3$$

$$V_{1p21} - V_{2p21} - V_{C10} = 7$$

S-domain circuit



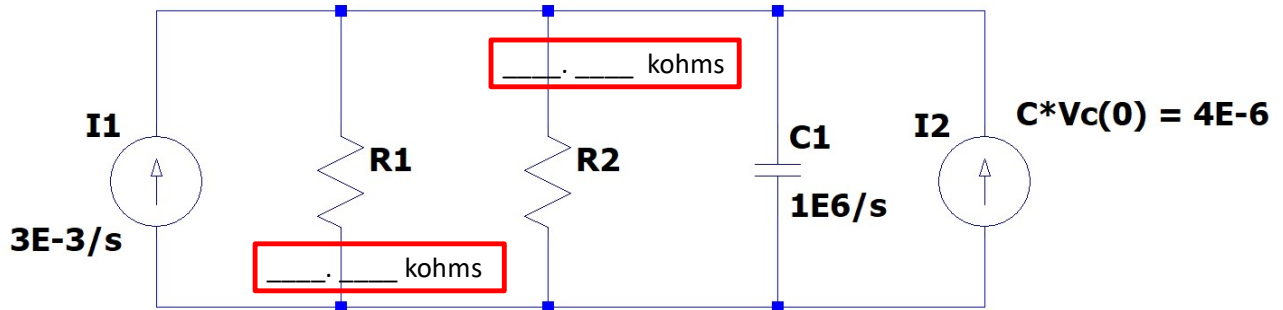
Simplified



Students don't need simplified version...

2.2: (15 pts) S-domain circuit to s-ratio: Use any method from Unit 1 to convert the s-domain circuit below to a simplified s-domain. (i.e. a ratio that is ready for partial fraction expansion). You do not have to use PFE for this portion of the problem.

Write your values for R1 and R2 using integers (1-9). Do not use 0.



For this solution I found IR2. The output was not specified but if you chose IC1 or VC1, the solving process will be evaluated according to what your output was. Please send a regrade if you are confident your solving process is correct.

One way is to use current divider with the norton circuit

$$R_1 := 20 \times 10^3 \quad R_2 := 10 \cdot 10^3 \quad 1 \cdot 10^6 \cdot 20 \cdot 10^3 = 2 \times 10^{10}$$

$$R1C1 = \frac{\frac{1 \cdot 10^6}{s} \cdot 20 \cdot 10^3}{\left(\frac{1 \cdot 10^6}{s}\right) + 20 \cdot 10^3} \quad \frac{1 \cdot 10^6 \cdot 20 \cdot 10^3}{1 \cdot 10^6 + 20 \cdot 10^3 s} \quad R1C1 = \frac{2 \times 10^{10}}{s + 50} = \frac{2 \times 10^{10}}{s + 50}$$

Use current divider with norton circuit to find R1C1

$$\left(\frac{3 \cdot 10^{-3}}{s} + 4 \cdot 10^{-6}\right) \cdot \left(\frac{\frac{2 \times 10^{10}}{s+50}}{10 \cdot 10^3 + \frac{2 \times 10^{10}}{s+50}}\right) = \frac{3 \cdot 10^{-3}}{s} \cdot \frac{\frac{2 \times 10^{10}}{s+50}}{10 \cdot 10^3 + \frac{2 \times 10^{10}}{s+50}} + 4 \cdot 10^{-6} \cdot \frac{\frac{2 \times 10^{10}}{s+50}}{10 \cdot 10^3 + \frac{2 \times 10^{10}}{s+50}}$$

$$\frac{2 \times 10^{10}}{10 \cdot 10^3 \cdot (s + 50) + 2 \times 10^{10}} \cdot \frac{3 \cdot 10^{-3}}{s} + 4 \cdot 10^{-6} \cdot \frac{2 \times 10^{10}}{10 \cdot 10^3 \cdot (s + 50) + 2 \times 10^{10}}$$

$$\frac{2 \times 10^{10}}{10 \cdot 10^3 s + 5 \cdot 10^5 + 2 \times 10^{10}} \cdot \frac{3 \cdot 10^{-3}}{s} + 4 \cdot 10^{-6} \cdot \frac{2 \times 10^{10}}{10 \cdot 10^3 s + 5 \cdot 10^5 + 2 \times 10^{10}}$$

$$\frac{6 \cdot 10^3}{s \cdot (s + 2 \cdot 10^6)} + \frac{8}{(s + 2 \cdot 10^6)}$$

2.3: S-domain ratio to time domain using PFE (15 pts): Choose one s-domain ratio below. Use partial fraction expansion to convert it back to the time domain.

$$F(s) = \frac{\frac{s}{1 \cdot 10^{-6}}}{s^2 + 5 \cdot 10^9 s + 10^{18}} \cdot \left(\frac{10}{s} + 0 - \frac{5}{s}\right)$$

Choice #1

$$F(s) = \frac{\frac{s}{1 \cdot 10^{-6}}}{s^2 + 5 \cdot 10^9 s + 10^{18}} \cdot \left(\frac{10}{s} + 0 - \frac{5}{s} \right)$$

$$s^2 + 5 \cdot 10^9 s + 10^{18} = 0$$

$$\frac{5 \cdot 10^6}{s^2 + 5 \cdot 10^9 s + 10^{18}} \quad \left(\begin{array}{l} 500000000 \cdot \sqrt{21} - 2500000000 \\ -500000000 \cdot \sqrt{21} - 2500000000 \end{array} \right)$$

PFE

$$500000000 \cdot \sqrt{21} - 2500000000 = -2.087 \times 10^8$$

$$-500000000 \cdot \sqrt{21} - 2500000000 = -4.791 \times 10^9$$

$$\frac{5 \cdot 10^6}{(s + 2.09 \cdot 10^8)(s + 4.8 \cdot 10^9)} = \frac{A_1}{(s + 2.09 \cdot 10^8)} + \frac{A_2}{(s + 4.8 \cdot 10^9)}$$

For A1

$$\frac{5 \cdot 10^6}{-2.09 \cdot 10^8 + 4.8 \cdot 10^9} = 1.089 \times 10^{-3}$$

For A2

$$\frac{5 \cdot 10^6}{-4.8 \cdot 10^9 + 2.09 \cdot 10^8} = -1.089 \times 10^{-3}$$

$$F(t) = 1.1 \cdot 10^{-3} \cdot e^{-2.09 \cdot 10^8 t} - 1.1 \cdot 10^{-3} \cdot e^{-4.8 \cdot 10^9 t}$$

$$F(s) = \frac{-0.02s}{s^2 + 2000s + 1 \cdot 10^8}$$

$$s^2 + 2000s + 1 \cdot 10^8 = 0$$

$$\left(\begin{array}{l} -1000 + 3000i \cdot \sqrt{11} \\ -1000 - 3000i \cdot \sqrt{11} \end{array} \right) \quad 3000 \cdot \sqrt{11} = 9.95 \times 10^3$$

complex conjugate poles $-1000 + 9.95 \cdot 10^3 j$

$-1000 - 9.95 \cdot 10^3 j$

Choice #2

$$\frac{-0.02s}{(s + 1000 - 9.995j \cdot 10^3) \cdot (s + 1000 + 9.995j \cdot 10^3)} = \frac{A_1}{(s + 1000 - 9.995j \cdot 10^3)} + \frac{A_2}{(s + 1000 + 9.995j \cdot 10^3)}$$

For A1 $\frac{-0.02s \cdot (s + 1000 + 9.995j \cdot 10^3)}{(s + 1000 - 9.995j \cdot 10^3) \cdot (s + 1000 + 9.995j \cdot 10^3)}$ for s = -1000 + 9.995 * 10³j

$$\frac{-0.02(-1000 + 9.995j \cdot 10^3)}{(-1000 + 9.995j \cdot 10^3 + 1000 + 9.995j \cdot 10^3)} = -0.01 - 1.001i \times 10^{-3}$$

$$A_1 = -0.01 - 1.001j \cdot 10^{-3}$$

For A2 $A_2 := -0.01 + 1.001j \cdot 10^{-3}$

$$F(t) = (-0.01 - 1.001j \cdot 10^{-3}) \cdot e^{(-1000 + 9.95 \cdot 10^3 j)t} + (-0.01 + 1.001j \cdot 10^{-3}) \cdot e^{(-1000 - 9.95 \cdot 10^3 j)t}$$

$$F(s) = \frac{2 \cdot s}{s(s^2 + 6 \cdot 10^3 s + 9 \cdot 10^6)}$$

Choice #3

This problem didn't have s next to 6*10³...

$$\frac{2}{(s + 3000)^2} \quad s^2 + 6 \cdot 10^3 s + 9 \cdot 10^6 = 0$$

$$\frac{A_1}{s + 3000} + \frac{A_2}{(s + 3000)^2} \quad \begin{pmatrix} -3000 \\ -3000 \end{pmatrix}$$

...actually can just take the inverse laplace since it is already in form!

for A2

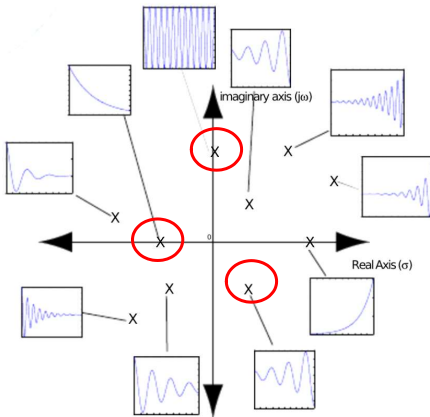
$$2t \cdot e^{-3000t}$$

$$A_2 = \frac{2 \cdot (s + 3000)^2}{(s + 3000)^2} = 2$$

for A1 $\frac{2}{(0 + 3000)^2} = \frac{A_1}{0 + 3000} + \frac{2}{(0 + 3000)^2}$

0

2.4 Conceptual Question (2.5 pts): Explain any details about the three circled poles in the pole-zero diagram. Details may include the type of circuit it is, the circuit stability, and comments about its characteristics relative to the poles around it. Please grade with some combination of points that adds up to 2.5...



1. All poles on right are unstable because they are increasing exponentially...the instability should be mentioned for all poles chosen on the right.

2. All poles on left are stable because they are decreasing exponentially...the stability should be mentioned for all poles chosen on the left.

3. All oscillating poles that aren't on the y axis are one of the complex conjugate pairs. This means the circuit is underdamped. Should be mentioned for all poles not on the x axis.

4. The further away from 0 on the x axis, for underdamped circuits, the faster the sinusoid dampens.

5. The further away from 0 on the y axis, for an underdamped circuit the more it oscillates.

One of these should be mentioned for all poles not on the x axis except the one that is on the y axis...this one doesn't have a comparison point.

6. If the pole on the y axis is chosen it doesn't have a real component so it oscillates forever. Should be mentioned for the only one on the y axis.

7. Exponential decay or rising always on the x axis. No imaginary component. Should be mentioned for all poles on x-axis.