Circuits

Exam 3

Fall 2020

1.	/30
2.	/70
Total	/100

Name ___________________

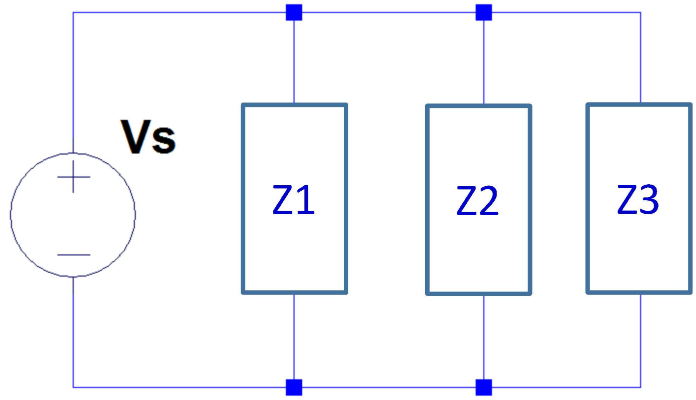
Notes:

1) If you are stuck on one part of the problem, choose 'reasonable' values on the following parts to receive partial credit
2) You don't need to simplify all your numerical calculations. For example, you can leave square root terms in radical form.

Sorry that I forgot to integrity statment. I will use you name placed anywhere on your papers as your signature!

1) Power Circuits (30 points)

In this circuit, the total source power, S, is 50,000 VA with a 60 Hz, 1,000 VRMS voltage. The power factor for the entire parallel load is 0.74 lagging . The loads are described below:



Z1: Capacitive load with value you specifiy ___.____μF

Z2: Inductive load L= 0.2 H with real loss, R=____.____ Ω,

Z3: Unknown load (value(s) depend on your choices above)

1.1: **(28 pts)** Determine the values in the table below. Please show all work for full credit! Every box is worth 2 points. Partial credit is not given for wrong answers in boxes.

	P[W]	Q [VAR]	|S| [VA]	power factor
Load 1				0
Load 2				
Load 3				
Source				0.74 lagging

$S_{Totalabs} := 50000 = 5 \times 10^4$

$pf := 0.74$

$\theta := \text{acos}(pf)$

$\theta = 42.269 \cdot \text{deg}$

$S_{Total} = P_{Total} + j \cdot Q_{Total}$

$P_{Total} := pf \cdot S_{Totalabs}$

$Q_{Total} := S_{Totalabs} \cdot \sin(\theta)$

$P_{Total} = 3.7 \times 10^4$

$Q_{Total} = 3.363 \times 10^4$ VARs

lagging means QTotal is positive

$|S| =$ check $\sqrt{P_{Total}{}^2 + (Q_{Total})^2} = 5 \times 10^4$ VA

Load 1: Capacitive element

$f_1 := 60\text{Hz}$ $\omega_1 := 2\cdot\pi\cdot f_1 = 376.991\frac{1}{s}$

1st way to solve:

Can use source voltage $V_{RMS} := 1000V$ $C_1 := 2.3\mu F$ $Z_{C1} := \frac{-j}{C_1\cdot\omega_1} = -1.153i\times 10^3\Omega$

Another way to solve:

$Z_{C1} = 1.153\cdot 10^3\angle\cdot -90°$ $Z_{C1mag} := 1.153\cdot 10^3$ $\theta_{C1phase} := -90deg$

note: θ is 0 degrees because it is purely resistive. There is not j or imaginary part in for a purely resistive load.

$$S_1 = \frac{(|V_{RMS}|)^2}{Z_{C1mag}}\cdot\cos(\theta_{C1phase}) + j\cdot\frac{(|V_{RMS}|)^2}{Z_{C1mag}}\cdot\sin(\theta_{C1phase})$$

$$S_1 := \left[\frac{(|V_{RMS}|)^2}{Z_{C1mag}}\cdot\cos(\theta_{C1phase}) + \frac{(|V_{RMS}|)^2}{Z_{C1mag}}j\cdot\sin(\theta_{C1phase})\right]$$

$$\cos(\theta_{C1phase}) = 0$$

$$P_1 := \frac{(|V_{RMS}|)^2}{Z_{C1mag}}\cdot 0 = 0$$

$$Q_1 := \frac{(|V_{RMS}|)^2}{Z_{C1mag}}\cdot\sin(\theta_{C1phase}) = -867.303\,V^2$$

$S_1 = 0 - j867$

$P_1 = 0$ W $Q_1 := -867$ VAR $|S_1| = 867VA$ $pf_1 := \frac{P_1}{|S_1|} = 0$

Load 2: Inductive element with real loss, R=5.7Ω, L=0.2H

$Z_2 = Z_{R2} + Z_{L2}$ Now there is a real AND imaginary part to the load.

$Z_{R2} = 5.7\Omega$ $L_2 := 0.2H$ $\omega_1 := 2\cdot\pi\cdot 60 = 376.991$

$Z_{L2} := L_2\cdot\omega_1\cdot j = 75.398iH$ Mathcad turns j into i

$Z_2 = 5.7 + j\cdot 75.4 = 75.43\angle 85.7° = |Z_{2mag}|\cdot\angle\theta_{Z2}$ $\sqrt{5.7^2 + (75.4)^2} = 75.615$

$V_{RMS} = 1000$ Given

$Z_{2mag} := 75.615$ $\theta_{Z2} := 85.689deg$

$\mathrm{atan}\left(\frac{75.62}{5.7}\right) = 85.689 \cdot deg$

$$S_2 = \frac{(|V_{RMS}|)^2}{Z_{2mag}} \cdot \cos\theta_{Z2} + j\frac{(|V_{RMS}|)^2}{Z_{2mag}} \cdot \sin\theta_{Z2}$$

$$\frac{V_{RMS}^2}{Z_{2mag}} \cdot \cos(\theta_{Z2}) = 994.117\,V^2$$

$$\frac{V_{RMS}^2}{Z_{2mag}} \cdot \sin(\theta_{Z2}) = 1.319 \times 10^4\,V^2$$

$S_2 = 994.117 + 1.319 \times 10^4 j$

$P_2 := 994.117$ $Q_2 := 1.319 \times 10^4$ $S_{2abs} := \sqrt{P_2^2 + Q_2^2} = 1.323 \times 10^4$

$$pf_{Load2} := \frac{P_2}{S_{2abs}} = 0.075$$

To get the unknown load we need the total power, then we can subtract S1 and S2 from it.

Load 3:

$S_3 = S_{tot} - S_1 - S_2$ $pf_{Tot} := 0.74$

$$P_3 := P_{Total} - P_1 - P_2 = 3.601 \times 10^4$$

$$Q_3 := Q_{Total} - Q_1 - Q_2 = 2.131 \times 10^4$$

check

$$Q_3 + Q_1 + Q_2 = 3.363 \times 10^4$$

$$Q_{Total} = 3.363 \times 10^4$$

$$S_{3abs} := \sqrt{P_3^2 + Q_3^2} = 4.184 \times 10^4$$

$$pf_3 := \frac{P_3}{S_{3abs}} = 0.861$$

1.2: **(2 pts)** What are the component values for your unknown load? Enter the values that correspond to your circuit. You may leave the component value that doesn't exist in your circuit blank.

$\operatorname{acos}(pf_3) = 30.616 \cdot deg$

$$\frac{(|V_{RMS}|)^2}{Z_{L3mag}} \cdot pf_3 = P_3$$

$$Z_{L3mag} := \frac{(|V_{RMS}|)^2}{P_3} \cdot pf_3 = 23.902\,V^2$$

postive because it is leading pf

$R_3 := 23.902 \cdot \cos(30.616deg) = 20.57$

$X_3 := 23.902 \cdot \sin(30.616deg) = 12.173$

$X_3 = j \cdot L_3 \cdot \omega_1$

$$L_3 := \frac{X_3}{\omega_1} = 0.032$$

My check

$Z_{RL3} := 20.57 + 12.173j$

$X_3 = 12.173$

$L_3 := 0.032$

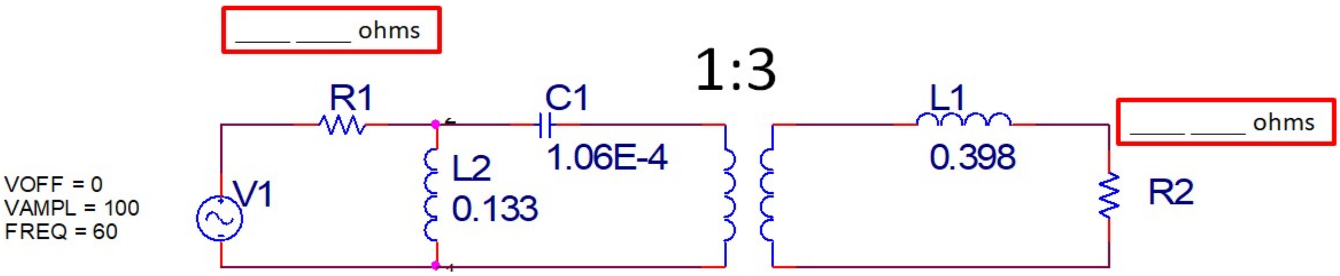
$R_3 := 20.57$

$Z_3 := L_3 \cdot \omega_1 \cdot j = 12.064i$

$\sqrt{R_3^{\,2} + (X_3)^2} = 23.902$

$\operatorname{atan}\left(\frac{X_3}{R_3}\right) = 30.616 \cdot deg$

Problem 2) Ideal Transformer Circuit (70 pts)



2.1: ***30 pts)*** Choose the values of R1 and R2. (i.e 15 ohms and 78 ohms. respectively...) ***You may use either referral method though one referral method is one less step. (30 pts)***

The above circuit has a 100 V, 60 Hz source.

$V_1 := 100V$ $\quad R_1 := 33\Omega$ $\quad C_1 := 1.06\cdot 10^{-4}F$ $\quad R_2 := 55\Omega$

$f := 60Hz$ $\quad L_2 := 0.133H$ $\quad L_1 := 0.398H$

$\omega := 2\cdot\pi\cdot f$ $\quad N_1 := 3$ $\quad Z_{R2} := R_2$

$\omega = 376.991\frac{1}{s}$

$L_2\cdot\omega = 50.14\Omega$ $\quad \frac{-1}{C_1\cdot\omega} = -25.024\Omega$ $\quad L_1\cdot\omega = 150.042\Omega$

$Z_{L2} := 50j\Omega$ $\quad Z_{C1} := -25j\Omega$ $\quad Z_{L1} := 150j\Omega$

Convert to thevenin using a voltage divider (for voltage across L2...)

$$V_{Th} := V_1\cdot\frac{Z_{L2}}{R_1 + Z_{L2}}$$

$$V_{Th} = (69.657 + 45.974i)\,V$$

$$\sqrt{70^2 + 46^2} = 83.762$$

$$\operatorname{atan}\left(\frac{46}{70}\right) = 33.311\cdot deg$$

$$83.8 < 33deg$$

Combine secondary impedances

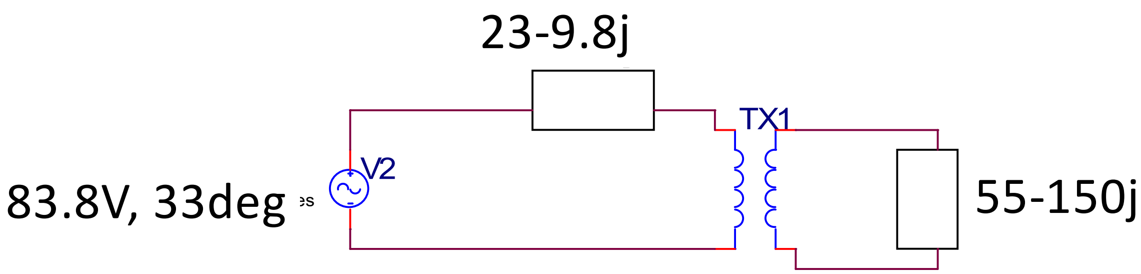
$$Z_{Load} := Z_{L1} + R_2$$

$$Z_{Load} = (55 + 150i)\,\Omega$$

(recognition and correct use of thevenin 7 pts)
They can leave any value in polar or rectangular form.

Find thevenin impedance by shorting source then combining like resistors

$$Z_{Th} := \frac{R_1 \cdot Z_{L2}}{R_1 + Z_{L2}} + Z_{C1} \qquad Z_{Th} = (22.987 - 9.829i)\,\Omega$$



Equivalent circuit:

1st equivalent circuit: Refer to secondary

$$Z_{sps} := N_1^{\,2} \cdot Z_{Th} \qquad Z_{LEQps} := Z_{Load}$$

$$Z_{LEQps} = (55 + 150i)\,\Omega$$

$$Z_{sps} = (206.882 - 88.458i)\cdot\Omega$$

$$V_{sps} := N_1 \cdot V_{Th}$$

$$V_{sps} = (208.972 + 137.921i)\,V$$

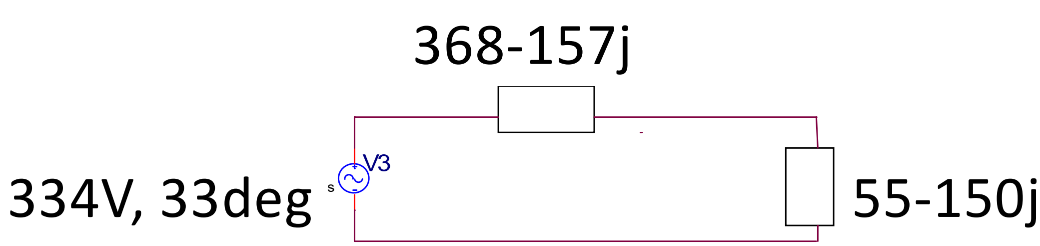
This one should be preferred since you are looking for the voltage across R2.

$$\sqrt{279^2 + 184^2} = 334.211$$

$$\operatorname{atan}\left(\frac{184}{279}\right) = 33.405\cdot deg$$

$$334 < 33deg$$

Correct referral process (either one) 7 pts
Correct values 1 pt



Use refer to secondary. You can use voltage divider to find it.

$$V_{LEQ} := V_{sps} \cdot \left[\frac{Z_{LEQps}}{(Z_{sps}) + (Z_{LEQps})} \right]$$

$$\frac{Z_{LEQps}}{(Z_{sps}) + (Z_{LEQps})} = 0.327 + 0.496i$$

$$V_{LEQ} = (-0.166 + 148.699i)\ V$$

$$\sqrt{(0.124)^2 + 0.357^2} = 0.378$$

$$\operatorname{atan}\left(\frac{0.357}{0.124}\right) = 70.846 \cdot deg$$

$$V_{R2} := V_{LEQ} \cdot \frac{Z_{R2}}{Z_{LEQps}} = (48.042 + 17.676i)\ V$$

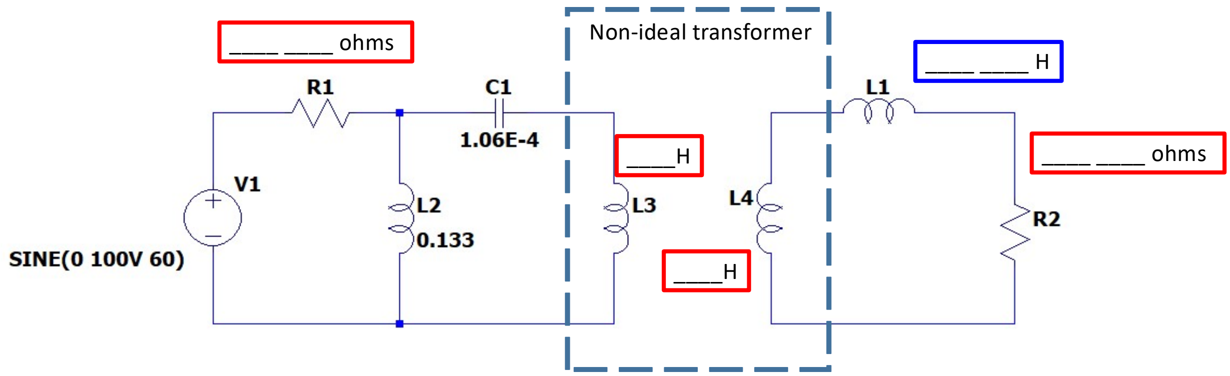
$$\sqrt{35.825^2 + (24.538)^2} = 43.423$$

$$\operatorname{atan}\left(\frac{24.538}{35.825}\right) = 34.409 \cdot deg$$

$43.423 < 34.409deg$

R2	

2.2: ***(35 pts)*** The ideal transformer if 2.1 is now non-ideal. The values of R1 and R2 in 2.1 should be used again. The coupling coefficient is 0.45. Choose values of the transformer self-inductances L3 and L4 between 1 and 5 H. Design your circuit such that your answer for the voltage across R2 in 2.1 matches with this non-ideal transformer. (The easiest way is to adjust the value of L1 at the end if needed which may mean adding resistance or making it a capacitor. If your answer is ridiculous but procedure is right...no problem..you willl still get full points!)



$\omega := 2\cdot\pi\cdot f$

$\omega = 376.991\,\frac{1}{s}$

$V_1 := 100V$ $\quad C_1 := 1.06\cdot 10^{-4}F$ $\quad L_2 := 0.133H$

$R_1 := 33\Omega$ $\quad R_2 := 55\Omega$

$L_2\cdot\omega = 50.14\,\Omega$ $\quad \frac{-1}{C_1\cdot\omega} = -25.024\,\Omega$ $\quad Z_{R1} := R_1 = 33\,\Omega$

$Z_{L2} := 50j\Omega$

$L_3 := 2H$ $\quad L_4 := 5H$

$Z_{C1} := -25j\Omega$ $\quad Z_{R2} := R_2 = 55\,\Omega$

$k := 0.45$

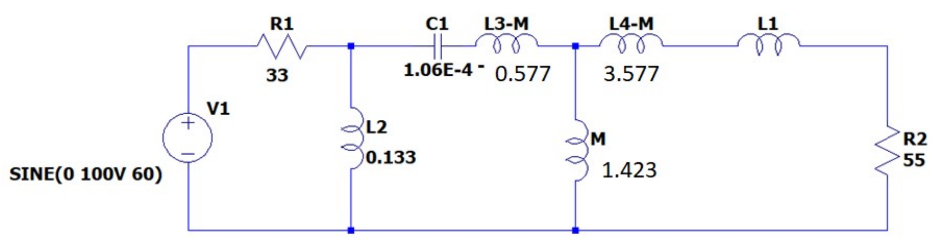
$M := k\cdot\sqrt{L_3\cdot L_4}$ $\quad k = \frac{M}{\sqrt{L_3\cdot L_4}}$

$M = 1.423\,H$

$L_3 - M = 0.577\,H$ $\quad Z_{L3M} := (L_3 - M)j\cdot\omega = 217.514i\,\Omega$

$L_4 - M = 3.577\,H$ $\quad Z_{L4M} := (L_4 - M)\cdot j\cdot\omega = 1.348i\times 10^3\,\Omega$

$Z_M := M\cdot j\cdot\omega = 536.468i\,\Omega$



To find VR2...can use circuit reduction and then voltage divider...make equal to find L1

$L_1 := 0.398H$

$L_1 \cdot \omega = 150.042\,\Omega$

using thevenin again before L1...then will make it equal

$Z_{L1} := 150j\Omega$

$$Z_M + Z_{L3M} + Z_{C1} = 728.982i\,\Omega$$

$$Z_{L2C1L3MM} := \frac{(Z_M + Z_{L3M} + Z_{C1}) \cdot Z_{L2}}{Z_{L2} + Z_M + Z_{L3M} + Z_{C1}} = 46.791i\,\Omega$$

$$V_{ZL2C1L3MM} := V_1 \cdot \frac{Z_{L2C1L3MM}}{Z_{R1} + Z_{L2C1L3MM}} = (66.782 + 47.099i)\,V$$

$$V_M := V_{ZL2C1L3MM} \cdot \frac{Z_M}{Z_M + Z_{L3M} + Z_{C1}} = (49.146 + 34.661i)\,V$$

$$V_{Th2} := V_M$$

$$\sqrt{42.206^2 + 13.53^2} = 44.322$$

$$Z_{TH} := \frac{\left(\frac{Z_{R1} \cdot Z_{L2}}{Z_{R1} + Z_{L2}} + Z_{C1} + Z_{L3M}\right) \cdot Z_M}{\left(\frac{Z_{R1} \cdot Z_{L2}}{Z_{R1} + Z_{L2}} + Z_{C1} + Z_{L3M}\right) + Z_M} + Z_{L4M} = \left(11.935 + 1.499i \times 10^3\right)\Omega$$

$$V_{R2} = (48.042 + 17.676i)\,V$$

$$V_{R2} = V_{Th2} \cdot \frac{Z_{R2}}{Z_{R2} + Z_{L1} + Z_{Th}}$$

$$V_{L1nonideal} := \frac{V_{Th2} \cdot Z_{R2}}{V_{R2}} - Z_{TH} - Z_{R2} = \left(-4.52 - 1.482i \times 10^3\right)\Omega$$

Added resistor

$$R_{ZL1} := -4.52\Omega$$

$$C_{ZL1} := \frac{-j\Omega}{-1.482 \cdot 10^3 \Omega j \cdot \omega \cdot 1} = 1.79 \times 10^{-6}\,s$$

This is my check of your answers to see if it matches your VR2

$$V_{R2} = (48.042 + 17.676i)\,V$$

$$Z_{LZL1} := \frac{-j\Omega}{C_{ZL1} \cdot \omega} = -1.482i \times 10^{3}\,\Omega$$

$$Z_{ZL1} := R_{ZL1} + Z_{R2} = 50.48\,\Omega$$

These resistances add together...

a couple voltage dividers are done...

$$Z_{L4ML1R2} := Z_{ZL1} + Z_{LZL1} + Z_{L4M} = (50.48 - 133.512i)\,\Omega$$

$$Z_{ML4ML1R2} := \frac{Z_{L4ML1R2} \cdot Z_{M}}{Z_{L4ML1R2} + Z_{M}} = (88.091 - 166.714i)\,\Omega$$

$$Z_{C1L3MML4ML1R2} := Z_{ML4ML1R2} + Z_{C1} + Z_{L3M} = (88.091 + 25.801i)\,\Omega$$

$$Z_{L2C1L3MML4ML1R2} := \frac{Z_{C1L3MML4ML1R2} \cdot Z_{L2}}{Z_{C1L3MML4ML1R2} + Z_{L2}} = (16.306 + 35.969i)\,\Omega$$

$$V_{A} := V_{1} \cdot \frac{Z_{L2C1L3MML4ML1R2}}{Z_{L2C1L3MML4ML1R2} + Z_{R1}} = (56.318 + 31.866i)\,V$$

$$V_{B} := \frac{V_{A} \cdot Z_{ML4ML1R2}}{Z_{ML4ML1R2} + Z_{C1} + Z_{L3M}} = (87.256 - 100.273i)\,V$$

$$V_{R2check} := \frac{V_{B} \cdot Z_{R2}}{Z_{L4ML1R2}} = (48.031 + 17.785i)\,V$$

if VR2check doesn't match VR2 something is wrong...

$$V_{R2} = (48.042 + 17.676i)\,V$$