ELECTRIC CIRCUITS
ECSE-2010

Lecture 3.1

LECTURE 3.1 AGENDA

- Introduction to solving tools
- Node voltage analysis basics
- Node voltage analysis examples
- Cramer's Rule

MORE SOLVING TOOLS

- Node Voltage Analysis
  - Systematic method for ***solving for all unknown voltages*** (and hence all unknown currents) in any circuit
- Mesh Current Analysis
  - Systematic method for ***solving for all unknown currents*** (and hence all unknown voltages) in any circuit

NODE VOLTAGE ANALYSIS

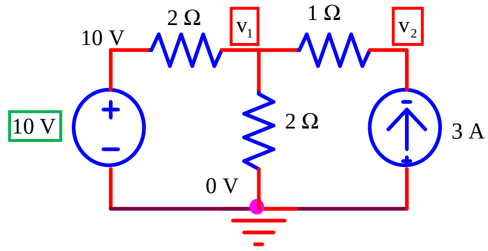
- A systematic technique for ***solving ANY linear circuit:***
  - Will Always Work!
  - Not always the easiest technique
- Will also learn mesh current analysis:
  - Can use either technique; But cannot mix
- These are very powerful techniques!!
  - Will use for rest of course

NODE VOLTAGE ANALYSIS

Procedure:

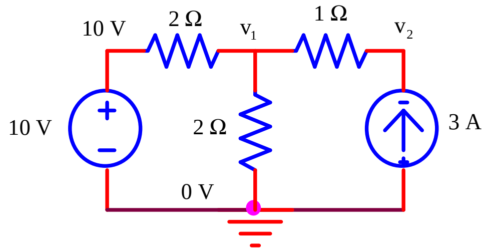
1. Label all node voltages, known and unknown, identifying variables (v_1, v_2, etc.)
   a. # of Unknown Node Voltages = # of Nodes - # of Voltage Sources - 1 (Reference)
   b. Example: 4 Nodes - 1 Voltage Source - 1 = 2 Unknown Node Voltages; v_1, v_2
2. Write a KCL at each unknown node voltage
   a. Best to use: Sum of currents out of node = 0
   b. Makes equations "cleaner"
   c. Express i's in terms of node voltages

EXAMPLE



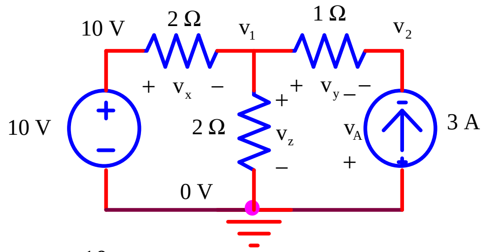
Step 1: Label All Node Voltages
Known and Unknown
2 Unknown Node Voltages, v_1, v_2

EXAMPLE



If Find v_1, v_2, All Voltages and Currents Can Be Determined

EXAMPLE



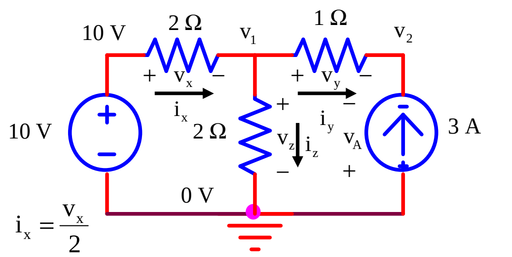
$v_x = 10 - v_1$

$v_y = v_1 - v_2$

$v_z = v_1 - 0 = v_1$

$v_A = 0 - v_2 = -v_2$

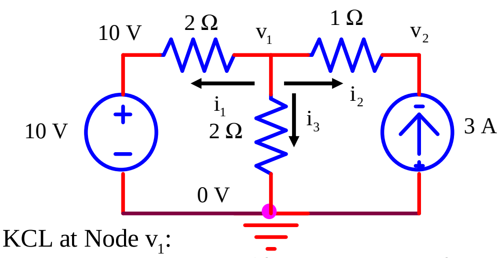
EXAMPLE



$i_x = \frac{v_x}{2}$

$i_y = \frac{v_y}{1} = -3\text{ A} \quad i_z = \frac{v_z}{2} = \frac{v_1}{2} = i_x - i_y = i_x + 3$

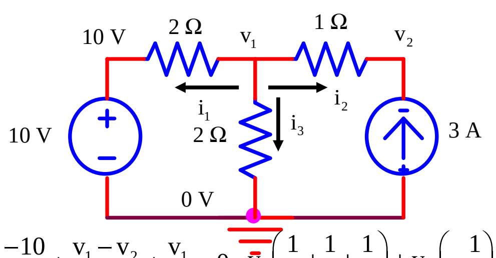
EXAMPLE



KCL at Node v_1:

$i_1 + i_2 + i_3 = 0 \quad \frac{v_1 - 10}{2} + \frac{v_1 - v_2}{1} + \frac{v_1 - 0}{2} = 0$

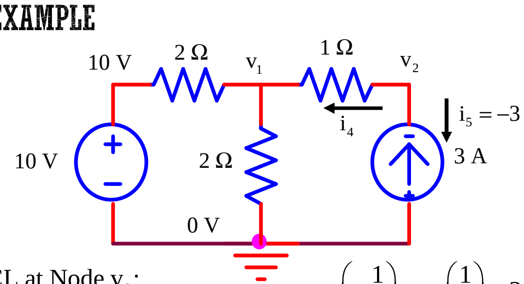
EXAMPLE



$\frac{v_1 - 10}{2} + \frac{v_1 - v_2}{1} + \frac{v_1}{2} = 0 \quad v_1\left(\frac{1}{2} + \frac{1}{1} + \frac{1}{2}\right) + v_2\left(-\frac{1}{1}\right) = 5$

1 Equation, 2 Unknowns $\quad 2v_1 - v_2 = 5$

EXAMPLE



KCL at Node v_2:

$i_4 + i_5 = 0$

$i_4 + (-3) = 0$

$\frac{v_2 - v_1}{1} - 3 = 0$

$v_1\left(-\frac{1}{1}\right) + v_2\left(\frac{1}{1}\right) = 3$

$-v_1 + v_2 = 3$

NODE VOLTAGE ANALYSIS

- **Example:**

$$\text{KCL at } v_1:\ \frac{(v_1-10)}{2}+\frac{(v_1-0)}{2}+\frac{(v_1-v_2)}{1}=0$$

$$\Rightarrow 2v_1 - v_2 = 5$$

$$\text{KCL at } v_2:\ \frac{(v_2-v_1)}{1}+(-3)=0$$

$$\Rightarrow (-v_1+v_2)=3$$

NODE VOLTAGE ANALYSIS

- **Example:**

$$\text{Node } v_1 \Rightarrow \quad 2v_1 - v_2 = 5$$

$$\text{Node } v_2 \Rightarrow \quad -v_1 + v_2 = 3$$

$$\text{Add:} \Rightarrow \quad v_1 = 8\text{ V}$$

$$\Rightarrow \quad v_2 = 11\text{ V}$$

Can now Find All Voltages and All Currents

NODE VOLTAGE ANALYSIS

- Writing a KCL at each unknown node voltage will always provide # of linear equations = # Unknowns:
  - Can always solve for $v_1, v_2, \ldots$.
  - Can solve for all currents after solving for all Unknown Node Voltages
  - Node Voltage Analysis will ALWAYS work
  - If cannot find an easier method => Use Node Voltage Analysis

CRAMER'S RULE

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$$\text{Define } \Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

CRAMER'S RULE

$$\text{Define } \Delta_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix};$$

$$\Delta_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix};\ \text{etc}$$

CRAMER'S RULE

$$x = \frac{\Delta_1}{\Delta}; \quad y = \frac{\Delta_2}{\Delta}; \quad z = \frac{\Delta_3}{\Delta}$$

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Lecture 3.2

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LECTURE 3.2 AGENDA

- Mesh current analysis basics
- Mesh current analysis examples
- Circuit solver

MESH CURRENT ANALYSIS

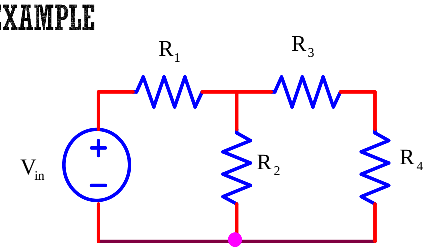
Procedure:

1. Label and define ALL Mesh Currents
   a. Unknown Mesh Currents and currents from current sources
   b. # of Unknown Mesh Currents = # of Meshes - # of Current Sources;
   c. Example: 2 Meshes - 0 Current Sources = 2 Unknown Mesh Currents; i_1 and i_2
2. Write a KVL around each Unknown Mesh Current
   1. Sum of voltages due to all Mesh Currents = 0
   2. Best to go backwards around current arrow
   3. Express v's in terms of Mesh Currents using Ohm's Law

MESH CURRENT ANALYSIS

- ***What is a mesh and what is a mesh current?***
  - Mesh = "Window Pane" in Circuit
  - Mesh Current = A current defined as flowing all the way around a mesh
    - Some Circuit Elements will have more than 1 Mesh Current flowing in them
    - Mesh Currents must satisfy KCL
  - Must define both Unknown Mesh Currents and known currents from current sources
  - May choose any direction for Unknown Mesh Current

EXAMPLE

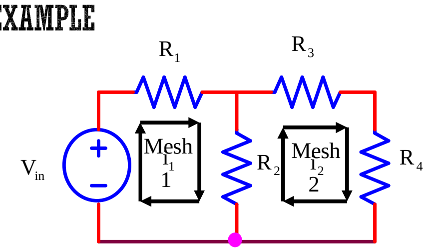


Ladder Circuit:

Could Solve Using Series/Parallel Reduction

Let's Solve Using Mesh Current Analysis

EXAMPLE

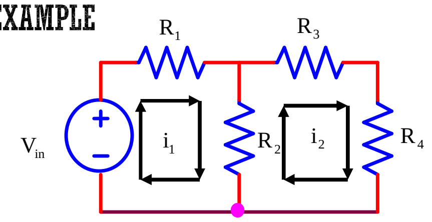


2 "Window Panes" => 2 Meshes

Define Two Mesh Currents, i_1 and i_2

Directions chosen for i_1 and i_2 are arbitrary

EXAMPLE



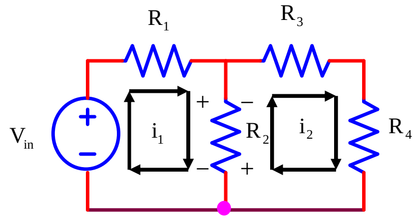
i_1 is the only current in R_1, V_{in}

i_2 is the only current in R_3, R_4

Both i_1 and i_2 flow in R_2 i_1 flows Down; i_2 flows Up

Total Current Down $= i_1 - i_2$

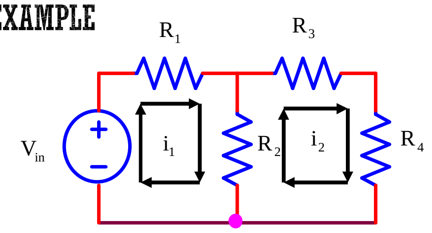
EXAMPLE



KVL Around Mesh i_1: $i_1 R_2 - i_2 R_2 + i_1 R_1 - V_{in} = 0$

KVL Around Mesh i_2: $i_2 R_4 + i_2 R_3 + i_2 R_2 - i_1 R_2 = 0$

EXAMPLE



$i_1(R_1 + R_2) + i_2(-R_2) = v_{in}$ 2 Equations, 2 Unknowns

$i_1(-R_2) + i_2(R_1 + R_2 + R_3) = 0$ Can Solve for i_1 and i_2:

EXAMPLE

In Matrix Form:

$$\begin{bmatrix} R_1 + R_2 & -R_2 \\ -R_2 & R_2 + R_3 + R_4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_{in} \\ 0 \end{bmatrix}$$

$$[R] \qquad [I] \quad = [V_s]$$

ohms   amps   = volts

EXAMPLE

$- \sum_{\text{Common to Mesh 1 and Mesh 2}}$ R's

In Matrix Form:

$$\begin{bmatrix} R_1 + R_2 & -R_2 \\ -R_2 & R_2 + R_3 + R_4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_{in} \\ 0 \end{bmatrix}$$

$\sum_{\text{Around Mesh 1}}$ R's $\sum_{\text{Around Mesh 2}}$ R's

EXAMPLE

In Matrix Form:

$$\begin{bmatrix} R_{11} & -R_{12} \\ -R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_{in} \\ 0 \end{bmatrix}$$

Diagonal Terms are ≥ 0

Off-Diagonal Terms are ≤ 0

Matrix is Symmetric ($R_{12} = R_{21}$, etc.)

Only for $\sum_{\text{Backwards Around Arrow}}$ v's $= 0$

SUMMARY

- **Node Voltage Analysis:**
  - **Label All Node Voltages, Known and Unknown, Identifying Variables (v_1, v_2, etc.)**
  - **# of Unknown Node Voltages = # of Nodes - # of Voltage Sources - 1 (Reference)**
  - **Write a KCL at Each Unknown Node Voltage**
    - **Best to Use: Sum of Currents Out of Node = 0**
    - **Express i's in terms of Node Voltages**
  - **Solve Algebraic Equations for Node Voltages**
    - **Use MAPLE, MATLAB, Cramer's Rule, etc.**
  - **Solve for Currents Using Ohm's Law**

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NODE ANALYSIS -SOLVING PROCEDURE

1. **Identify all nodes**
2. **Choose a reference node (ground)**
3. **Label the unknown nodes**
4. **Locate all voltage sources**
   1. **Determine either absolute or relative voltages based on the voltage sources**
5. **Write a KCL equation for each node**
6. **Use Ohm's Law to rewrite the currents as voltage differences over resistance**
   1. **If a voltage source is on one of the current paths, 'follow' it to the next node to get an expression for current.**
7. **Set up the linear system**
8. **Solve the matrix**

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SUMARRY

- **Mesh Current Analysis:**
  - **Label and Define ALL Mesh Currents**
    - **Unknown Mesh Currents <u>and</u> Currents from Current Sources**
  - **# of Unknown Mesh Currents = # of Meshes - # of Current Sources;**
  - **Write a KVL around Each Unknown Mesh Current**
    - **Sum of Voltages due to All Mesh Currents = 0**
    - **Best to Go Backwards Around Current Arrow**
  - **Solve Algebraic Equations for Mesh Currents (Maple, Cramer's Rule, etc.)**
  - **Solve for Voltages Using Ohms Law**

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MESH ANALYSIS -SOLVING PROCEDURE

1. **Identify all loops**
2. **Locate all current sources**
3. **If possble, simplify the problem by redrawing the circuit with current sources on the 'outside'**
4. **Label the currents in each loop**
5. **Assign the current directly if a current source is on the 'outside'**
6. **Assign a relative current expression if the current source is shared by two loops.**
7. **Write a KVL expression for each loop**
   1. **If a current source is shared by two loops, combine them to form a larger loop.**
8. **Use Ohm's Law to write the KVL in terms of currents**
9. **Set up the linear system**
10. **Solve the matrix**

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17 CIRCUIT SOLVER

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CIRCUIT SOLVER

- An Interactive Learning Module (ILM) developed by Academy for Electronic Media
- 1 of many ILM's developed at Rensselaer
- Link to Modules on Course WebCT Homepage
- Or via: http://www.academy.rpi.edu/projects/ccli
- Click on Circuit Solver
- "2 Mesh" is same as our Example
- We'll do Activity 4-2 using circuit solver (number on ILM is 4-3)

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