

# ELECTRIC CIRCUITS

## ECSE-2010

Lecture 5.1



## LECTURE 5.1 AGENDA

- Wheatstone bridge
- Norton/Thevinin equivalent circuits
- Norton/Thevinin equivalent circuit example

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## BRIDGE CIRCUITS

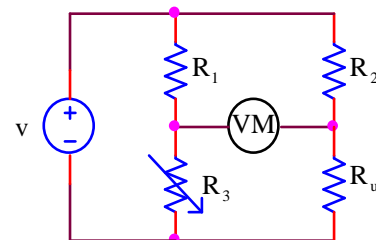
- Bridge circuits are circuits used to accurately measure circuit elements:
  - **Provide measurements with minimal disturbance**
  - A Wheatstone Bridge is used to measure resistance
  - Will Use a Maxwell Bridge later to measure inductance

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## WHEATSTONE BRIDGE



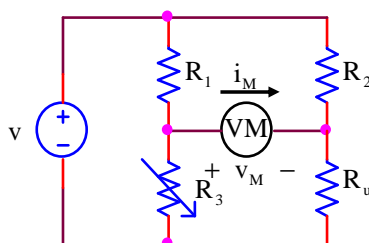
Connect Voltmeter Across "Bridge"  
Adjust  $R_3$  such that VM reads 0 V  
Provides Measurement of  $R_u$

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## WHEATSTONE BRIDGE



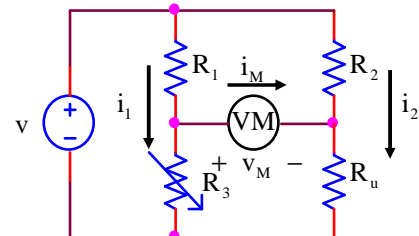
When Bridge is Balanced,  $i_M = 0$ ;  $v_M = 0$   
Meter Draws No Current

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## WHEATSTONE BRIDGE



Bridge Balanced

$$i_1(R_3 + R_1) = i_2(R_u + R_2)$$

$$i_1 R_3 = i_2 R_u; i_1 R_1 = i_2 R_2;$$

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# ELECTRIC CIRCUITS

## ECSE-2010

Lecture 5.2



## LECTURE 5.2 AGENDA

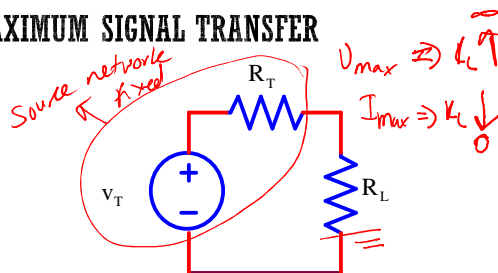
- Maximum signal transfer (Power)
- Interface circuit design

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## MAXIMUM SIGNAL TRANSFER



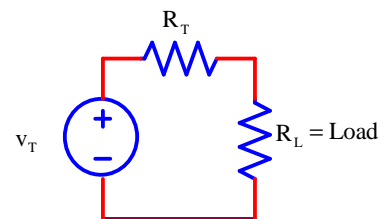
Model for Any Linear Circuit  
Source Network is Often Fixed  
Designer has Control over  $R_L$

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## MAXIMUM SIGNAL TRANSFER



Want to get Maximum Power delivered to Load  
For fixed  $v_T$ ,  $R_T$ ; Want to Choose  $R_L$   
to get Maximum Power to  $R_L$

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## MAXIMUM SIGNAL TRANSFER

$$p_L = v_L \times i$$

$$i = \frac{v_T}{R_T + R_L}$$

$$v_L = \left( \frac{R_L}{R_T + R_L} \right) v_T$$

$$p_L = \frac{R_L v_T^2}{(R_T + R_L)^2}$$

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## MAXIMUM SIGNAL TRANSFER

$$p_L = \frac{R_L v_T^2}{(R_T + R_L)^2}$$

$$\text{For } p_{\max}; \frac{dp_L}{dR_L} = 0$$

$$\text{Solve for } R_L \quad \frac{dp_L}{dR_L} = \frac{R_T - R_L}{(R_T + R_L)^3} v_T^2 = 0$$

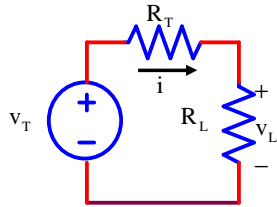
$$\text{For } p_{\max}; \text{Choose } R_L = R_T$$

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## MAXIMUM SIGNAL TRANSFER



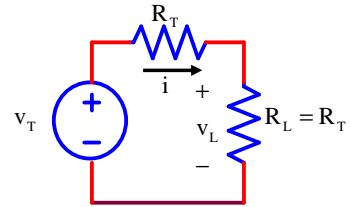
For Maximum Power Transfer; Choose  $R_L = R_T$   
Best You Can Do

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## MAXIMUM SIGNAL TRANSFER



$$p_{\max} = \frac{v_T^2}{4R_T} = \frac{i_N^2 R_T}{4}$$

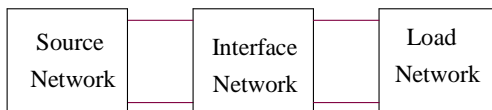
$$\text{Efficiency}_{\max} = \frac{p_{\max}}{p_{\text{source}}} = 50\%$$

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## INTERFACE CIRCUIT DESIGN



Often Need an Interface Network or  
Interface Circuit to Properly "Match"  
a Source Network to a Load Network

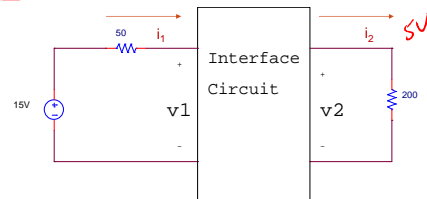
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## DESIGN EXAMPLE

- Design two versions of the interface circuit below that deliver  $v_2 = 5V$  to the  $200\text{-}\Omega$  load. Evaluate the two designs in terms of power loss in the interface circuit.

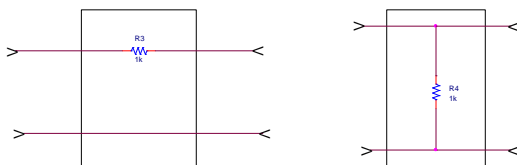


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## DESIGN EXAMPLE

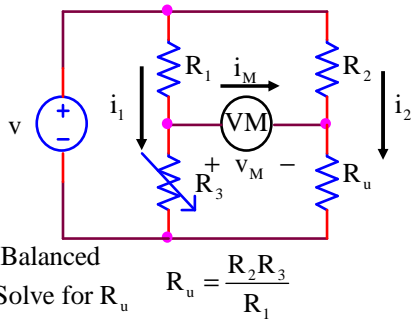


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## WHEATSTONE BRIDGE

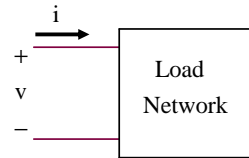


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## LOAD NETWORKS



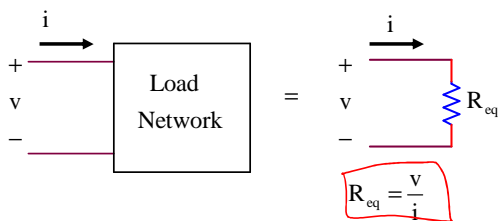
- Resistive (linear) circuit
- No independent sources (Voltage or Current)
- May have dependent sources (VCVS, CCVS, VCCS, CCCs)

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## LOAD NETWORKS



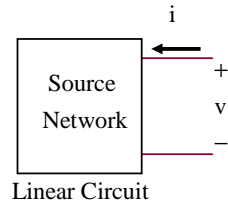
Any Load Network may be Replaced  
by an Equivalent Resistance,  $R_{eq}$

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## SOURCE NETWORKS



At Least One Independent Source  
May have Dependent Sources  
May have Resistors

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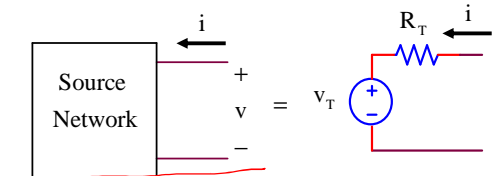
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## THEVENIN'S THEOREM

*Simplify*

*I-V characteristics*



Any Source Network  
May be Replaced with its  
Thevenin Equivalent Circuit

$v_T$  = Thevenin Voltage

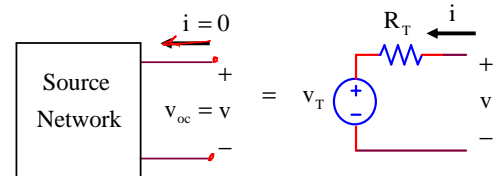
$R_T$  = Thevenin Resistance

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## THEVENIN'S THEOREM



$v_T = v_{oc}$  = Open Circuit Voltage

$v_T = v_{oc} = v$  when  $i = 0$

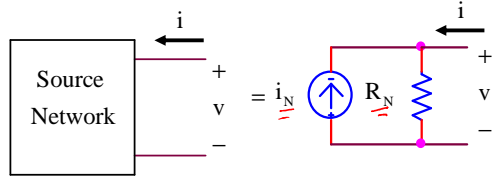
$R_T$  = Thevenin Resistance

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## NORTON'S THEOREM



Any Source Network

$i_N$  = Norton Current

May be Replaced with its  
Norton Equivalent Circuit

$R_N$  = Norton Resistance

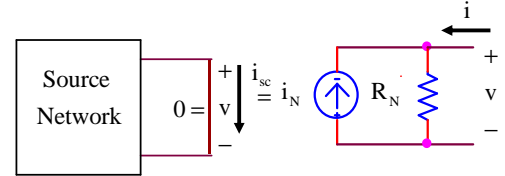
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## NORTON'S THEOREM

$R_N = R_T = \text{Res. Dead Sources}$



$i_N = i_{sc}$  = Short Circuit Current

$i_N = i_{sc}$  = Current Flowing from + to - when  $v = 0$

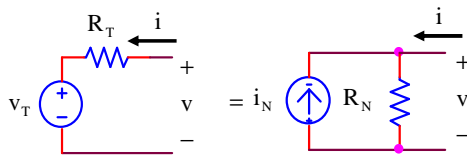
$R_N$  = Norton Resistance

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## EQUIVALENT CIRCUITS



Thevenin Equivalent Circuit    Norton Equivalent Circuit

From Source Conversions:  $i_N = \frac{V_T}{R_T}$  and  $R_N = R_T$

$V_T = V_{oc}$  = Open Circuit Voltage

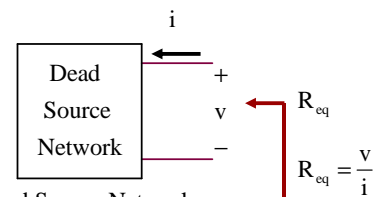
$i_N = i_{sc}$  = Short Circuit Current

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## THEVENIN RESISTANCE



Dead Source Network

Source Network with All Independent Sources Set = 0

Dead Source Network = Load Network

$R_T = R_{eq}$  of Dead Source Network

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## THEVENIN/NORTON SOURCES-SOLVING PROCEDURE

1. **Thevenin - Remove the load**
  1. Find the voltage ( $V_{oc} = V_{TH}$ ) between the two nodes where the load was connected, using any method
2. **Norton - Remove the load and connect a short circuit (wire) between the two nodes where the load was attached**
  1. Find the current ( $I_{sc} = I_N$ ) through that short circuit (wire), using any method
  2. Note: the short circuit may 'combine' nodes. Recognize that you can do KCL at a node to find current through an individual wire connecting components.
3. **Resistance - Remove the load**
  1. Apply a test voltage source,  $V_{test}$ , at the nodes where the load was attached
  2. Short circuit all other independent voltage sources and open circuit all other independent current sources.
  3. Find the current through that source,  $I_{test}$
  4.  $R_{TD} = R_N = R_{TH} = V_{test}/I_{test}$
4. Note: only two of these are needed since  $V_{TH} = (R_{TH})(I_N)$

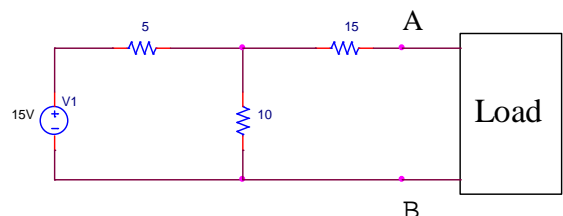
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## EXAMPLE

$V_T$   
 $I_N$   
 $R_T = R_N$   
Applying a Load

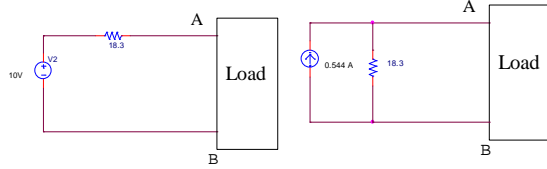


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## EXAMPLE

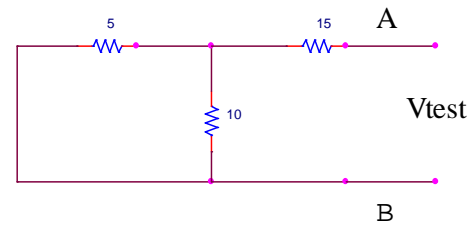


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## EXAMPLE



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