ELECTRIC CIRCUITS
ECSE-2010

Lecture 8.1

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LECTURE 8.1 AGENDA

- Signals and waveforms
  - DC waveforms
  - Unit step functions
  - Ramp functions
  - Exponential functions
  - Sinusoidal functions

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SIGNALS AND WAVEFORMS

- To date, we have looked at circuits with only DC inputs:
  - DC voltage sources, DC current sources
  - Independent and dependent sources
  - DC steady state (No changes with time)
- Have developed circuit analysis techniques for resistive circuits:
  - DC inputs
  - Circuits containing independent sources, dependent sources and resistors

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SIGNALS AND WAVEFORMS

- Circuit analysis techniques:
  - KCL and KVL
  - Series/parallel resistors
  - Voltage and current dividers
  - Equivalent resistance
  - Source conversion
  - Node voltage analysis
  - Mesh current analysis
  - Thevenin/Norton equivalent circuits
  - Linearity and superposition

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SIGNALS AND WAVEFORMS

- Almost all interesting circuits involve inputs that change with time
- The *rest of the course* is focused on finding the output, ***y(t)***, given the input, ***x(t)***
  - Will also add capacitors and inductors to our circuit elements

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ELECTRIC CIRCUITS

x(t) can be a Current or Voltage

So Far, x(t) = Constant (DC)

y(t) can be a Current or Voltage

=> y(t) = Constant (DC)

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SIGNALS AND WAVEFORMS

- **Signal = Any Input to a Circuit:**
- **Waveform = Equation or Graph that Defines a Signal as a Function of Time:**

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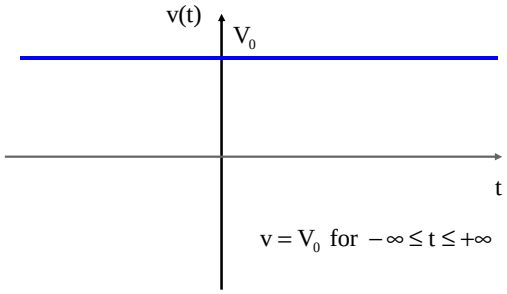
DC WAVEFORM



$v = V_0$ for $-\infty \le t \le +\infty$

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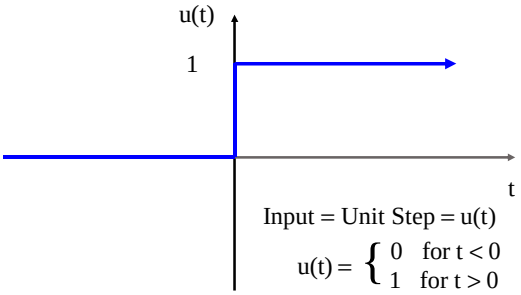
UNIT STEP FUNCTION



Input = Unit Step = u(t)

$$u(t) = \begin{cases} 0 & \text{for } t < 0 \\ 1 & \text{for } t > 0 \end{cases}$$

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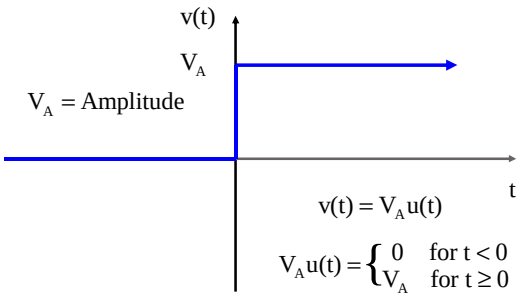
STEP FUNCTION



V_A = Amplitude

$v(t) = V_A u(t)$

$$V_A u(t) = \begin{cases} 0 & \text{for } t < 0 \\ V_A & \text{for } t \ge 0 \end{cases}$$

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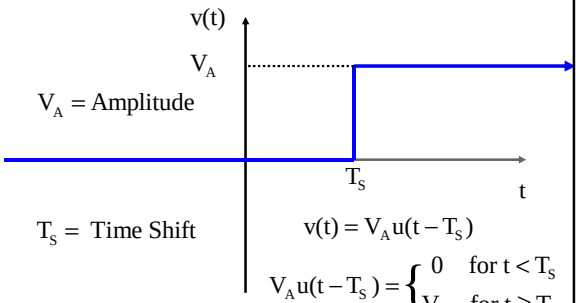
DELAYED STEP FUNCTION



V_A = Amplitude

T_S = Time Shift

$v(t) = V_A u(t - T_S)$

$$V_A u(t - T_S) = \begin{cases} 0 & \text{for } t < T_S \\ V_A & \text{for } t \ge T_S \end{cases}$$

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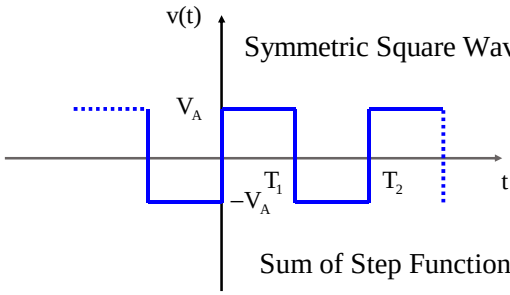
SQUARE WAVE



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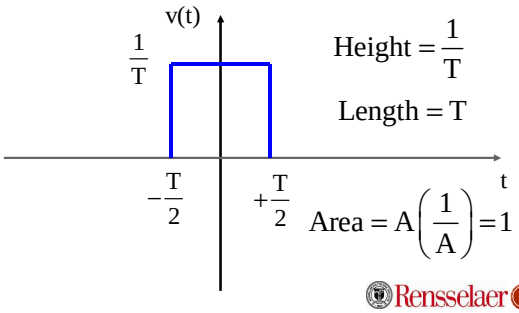
UNIT IMPULSE FUNCTION



$$\text{Height} = \frac{1}{T}$$

$$\text{Length} = T$$

$$\text{Area} = A\left(\frac{1}{A}\right) = 1$$

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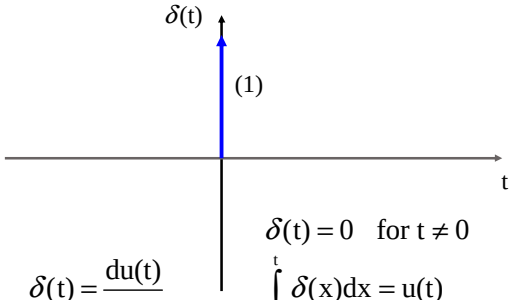
UNIT IMPULSE FUNCTION



$$\delta(t) = 0 \quad \text{for } t \neq 0$$

$$\delta(t) = \frac{du(t)}{dt}$$

$$\int_{-\infty}^{t} \delta(x)dx = u(t)$$

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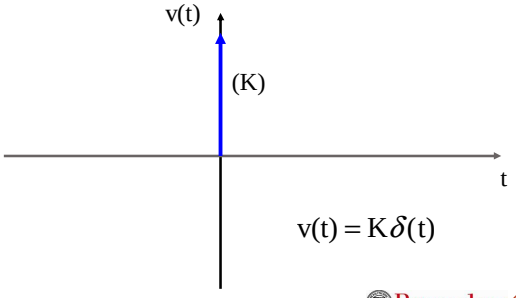
IMPULSE FUNCTION



$$v(t) = K\delta(t)$$

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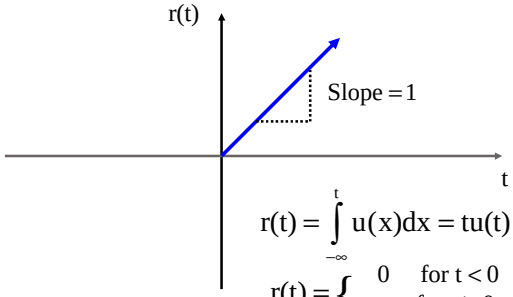
UNIT RAMP FUNCTION



$$r(t) = \int_{-\infty}^{t} u(x)dx = tu(t)$$

$$r(t) = \begin{cases} 0 & \text{for } t < 0 \\ t & \text{for } t \geq 0 \end{cases}$$

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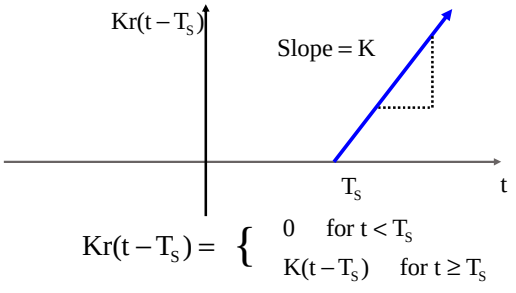
RAMP FUNCTION



$$Kr(t - T_S) = \begin{cases} 0 & \text{for } t < T_S \\ K(t - T_S) & \text{for } t \geq T_S \end{cases}$$

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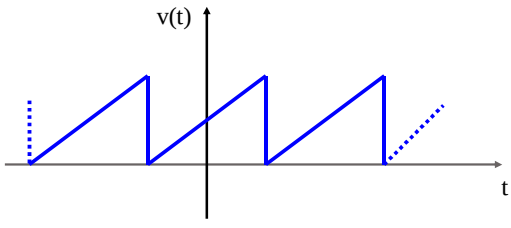
SAWTOOTH WAVEFORM



Sum of Ramp Functions

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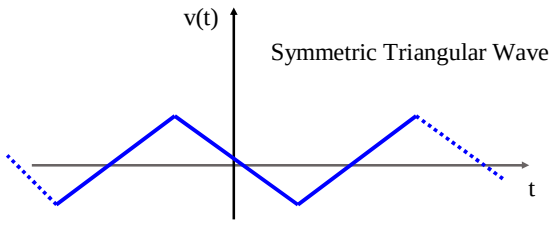
TRIANGULAR WAVE



Sum of Ramp Functions

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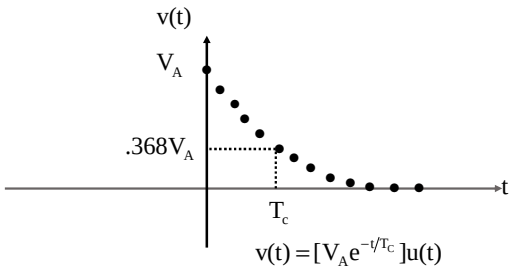
EXPONENTIAL FUNCTION



$$v(t) = [V_A e^{-t/T_C}]u(t)$$

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SINUSOIDAL WAVEFORM

See Pages 219 - 226 Thomas and Rosa

$$v(t) = V_A \cos(\omega t + \phi)$$

https://www.youtube.com/watch?v=QFi16s4RXXY

$$\text{Amplitude} = V_A; \text{ volts}$$

$$\text{Angular Frequency} = \omega = 2\pi f; \text{ radians/sec}$$

$$f = \text{Frequency} = \frac{1}{T_0}; \text{ hertz}$$

http://www.analyzemath.com/unitcircle/unit_circle_applet.html

$$T_0 = \text{Period}; \text{ seconds}$$

$$\phi = \text{Phase Angle}; \text{ degrees}$$

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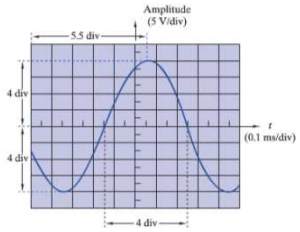
The figure shows an oscilloscope display of a sinusoid. The vertical axis (amplitude) is calibrated at 5V per division, and the horizontal axis (time) is calibrated at 0.1 ms per division. Derive an expression for the sinusoid displayed below.



EXAMPLE 5-7

1. Find V_A
2. Find T_0
3. Find f_0 and ω_0
4. Determine T_s
5. Calculate Φ_0 Fourrier coefficients
6. Write equations

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SINUSOIDAL WAVEFORM

All of Unit IV will be on AC Steady State

All Inputs are Sinusoidal

All Steady State Outputs are Sinsoidal

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ELECTRIC CIRCUITS
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LECTURE 8.2 AGENDA

- R, L, C circuits
- Impedance
- Op Amp Integrator
- Op Amp Differentiator

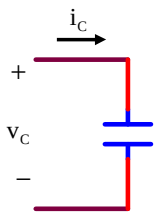
RESISTANCE

- For Resistive Circuits:
  - v = i R; => v(t) = i(t) R
  - Resistor does not affect time behavior
  - Resistors only absorb energy (get hot)
  - Resistors convert electrical energy to thermal energy

CIRCUITS WITH R, L, & C

- R, L, C Circuits:
  - L = Inductor; C = Capacitor
  - v, i are now time dependent
  - v(t) and i(t) may be quite different waveforms
  - L and C can store electrical energy!
  - Makes circuits far more interesting
  - Must find Time Behavior of circuit

CAPACITANCE



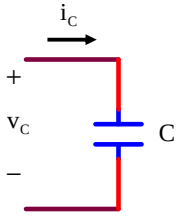
$$i_C = C\frac{dv_C}{dt}$$

$$v_C = v_C(t_0) + \frac{1}{C}\int_{t_0}^{t} i_C\, dt$$

C [farads]

1 farad = 1 amp-sec/volt

CAPACITANCE



$$i_C = C\frac{dv_C}{dt}$$

$$v_C = v_C(t_0) + \frac{1}{C}\int_{t_0}^{t} i_C\, dt$$

In DC Steady State; $\frac{d}{dt} = 0$

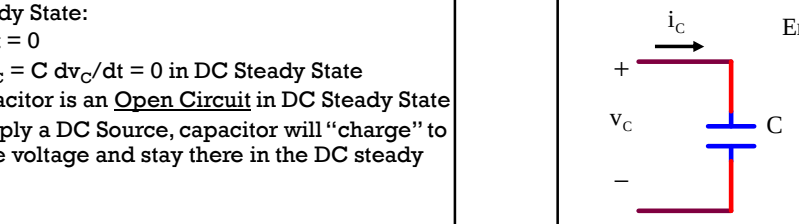
$i_{CSS} = 0$ => Open Circuit

Capacitor is an Open Circuit in DC Steady State

CAPACITANCE

- DC Steady State:
  - d /dt = 0
  - => $i_C = C\, dv_C/dt = 0$ in DC Steady State
  - Capacitor is an Open Circuit in DC Steady State
  - If Apply a DC Source, capacitor will "charge" to some voltage and stay there in the DC steady state

CAPACITANCE



Energy is Stored in Electric Field

$$w_C = \frac{1}{2} C\, v_C^2$$

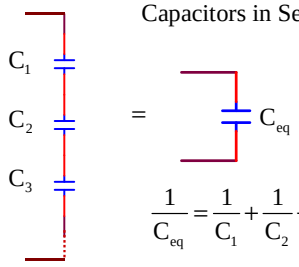
$$v_C(t_0^+) = v_C(t_0^-)$$

Cannot Change Energy Instantaneously

v_C Cannot Change Instantaneously

CAPACITANCE

Capacitors in Series

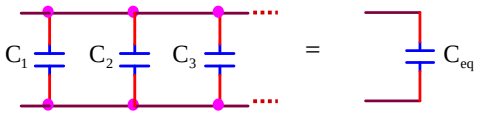


$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ..$$

Similar to Resistors in Parallel

CAPACITANCE

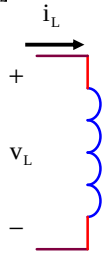
Capacitors in Parallel



$$C_{eq} = C_1 + C_2 + C_3 + ..$$

Similar to Resistors in Series

INDUCTANCE



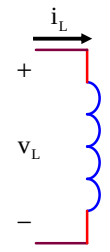
$$v_L = L \frac{di_L}{dt}$$

$$i_L = i_L(t_0) + \frac{1}{L} \int_{t_0}^{t} v_L \, dt$$

L [henries]

1 henry = 1 volt-sec/amp

INDUCTANCE



$$v_L = L \frac{di_L}{dt}$$

$$i_L = i_L(t_0) + \frac{1}{L} \int_{t_0}^{t} v_L \, dt$$

L [henries]

In DC Steady State; $\frac{d}{dt} = 0$

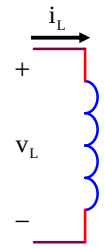
$v_{LSS} = 0$ => Short Circuit

Inductor is a Short Circuit in DC Steady State

INDUCTANCE

- DC Steady State:
  - d /dt = 0
  - => $v_L = L\, di_L/dt = 0$ in DC Steady State
  - Inductor is a short circuit in DC Steady State
  - If apply a DC source, inductor *will have current flowing* in it, but *no voltage across* it in the DC Steady State

INDUCTANCE



L [henries]

Energy is Stored in Magnetic Field

$$w_L = \frac{1}{2} L\, i_L^2$$

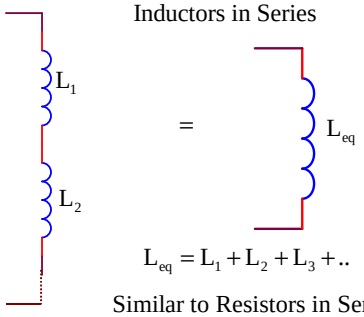
$$i_L(t_0^+) = i_L(t_0^-)$$

Cannot Change Energy Instantaneously

i_L Cannot Change Instantaneously

INDUCTANCE

Inductors in Series

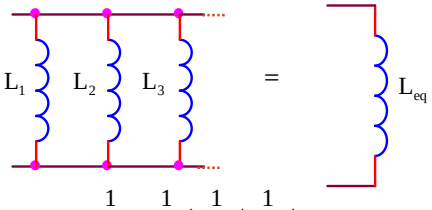


$$L_{eq} = L_1 + L_2 + L_3 + ..$$

Similar to Resistors in Series

INDUCTANCE

Inductors in Parallel

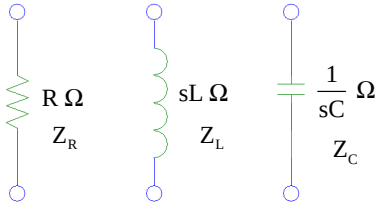


$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + ..$$

Similar to Resistors in Parallel

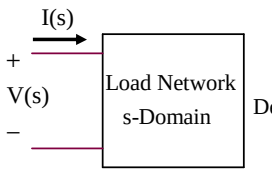
IMPEDANCE

No Initial Stored Energy



$$Z = \text{Impedance} = \frac{V(s)}{I(s)}$$

IMPEDANCE

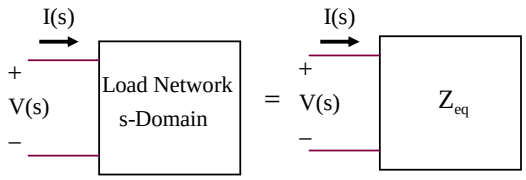


R's, L's, C's
Dependent Sources

$$\text{Impedance} = Z(s) = \frac{V(s)}{I(s)}$$

Measured in Ohms

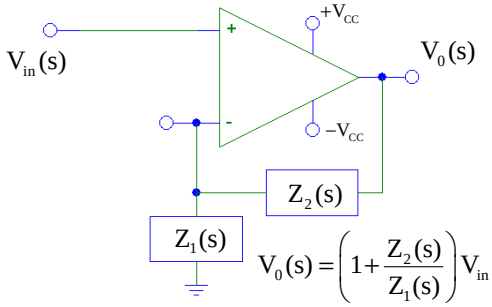
EQUIVALENT IMPEDANCE



Can replace any Load Network with Z_{eq}

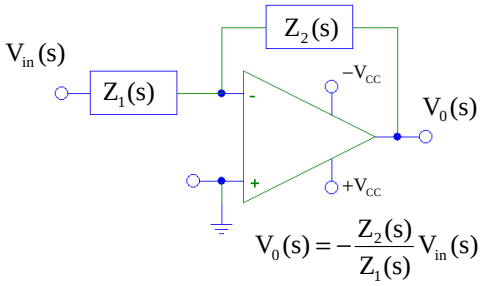
$$Z_{eq} = \frac{V(s)}{I(s)}$$

NON-INVERTING AMPLIFIER



$$V_0(s) = \left(1 + \frac{Z_2(s)}{Z_1(s)}\right) V_{in}(s)$$

INVERTING AMPLIFIER

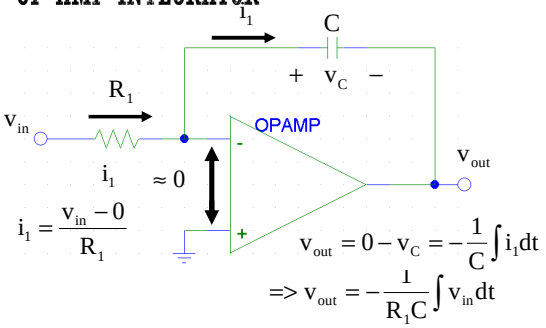


$$V_0(s) = -\frac{Z_2(s)}{Z_1(s)} V_{in}(s)$$

DYNAMIC OP AMP CIRCUITS

- Can make very useful circuits by using capacitors (or inductors) in Op Amp Circuits
- Let's replace R_2 with C in an inverting voltage amplifier:
- Then Replace R_1 with C in an inverting voltage amplifier:

OP AMP INTEGRATOR

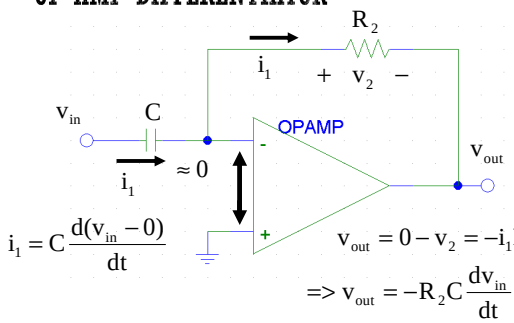


$$i_1 = \frac{v_{in} - 0}{R_1}$$

$$v_{out} = 0 - v_C = -\frac{1}{C}\int i_1 dt$$

$$\Rightarrow v_{out} = -\frac{1}{R_1 C}\int v_{in} dt$$

OP AMP DIFFERENTIATOR



$$i_1 = C\frac{d(v_{in} - 0)}{dt}$$

$$v_{out} = 0 - v_2 = -i_1 R_2$$

$$\Rightarrow v_{out} = -R_2 C\frac{dv_{in}}{dt}$$

Circuit	Block diagram	Gains
Noninverting	$v_1(t) \rightarrow K \rightarrow v_o(t)$	$K = \frac{R_1 + R_2}{R_2}$
Inverting	$v_1(t) \rightarrow K \rightarrow v_o(t)$	$K = -\frac{R_2}{R_1}$
Summer	$v_1(t) \rightarrow K_1$, $v_2(t) \rightarrow K_2$, $+$, $+$ $\rightarrow v_o(t)$	$K_1 = -\frac{R_F}{R_1}$ $K_2 = -\frac{R_F}{R_2}$
Subtractor	$v_1(t) \rightarrow K_1$, $v_2(t) \rightarrow K_2$, $+$, $-$ $\rightarrow v_o(t)$	$K_1 = -\frac{R_2}{R_1}$ $K_2 = \left(\frac{R_1 + R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right)$
Integrator	$v_1(t) \rightarrow K \rightarrow \int \rightarrow v_o(t)$	$K = -\frac{1}{RC}$
Differentiator	$v_1(t) \rightarrow K \rightarrow \frac{d}{dt} \rightarrow v_o(t)$	$K = -RC$

FIGURE 6-17 *Summary of basic OP AMP signal-processing circuits.*