ELECTRIC CIRCUITS
ECSE-2010

Lecture 12.1

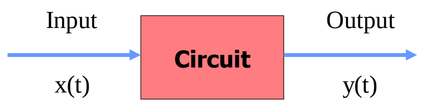
Rensselaer

LECTURE 12.1 AGENDA

- Laplace transforms
- Poles and zeros
- Pole-zero diagram
- Example

DYNAMIC CIRCUITS

Laplace Transforms

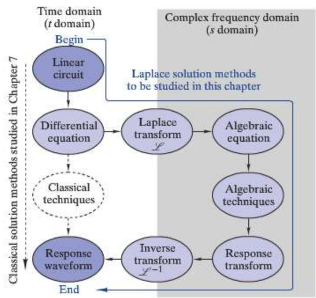


x(t) can be a Current or Voltage

y(t) can be a Current or Voltage

DYNAMIC CIRCUITS

FIGURE 9–1 *Flow diagram of dynamic circuit analysis with Laplace transforms.*



LAPLACE TRANSFORMS

- For most 2nd order circuits and for all higher order circuits, finding the output by solving differential equations is either impossible or, at best, difficult
- Would ***prefer to solve algebraic equations*** rather than differential equations:
- Will use laplace transforms:

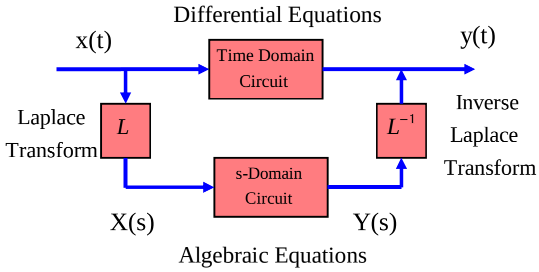
LAPLACE TRANSFORMS

- Laplace transforms:
  - Powerful mathematical tool
  - Will transform differential equations into algebraic equations
  - Can then use all the circuit analysis techniques we developed for resistive circuits!!

LAPLACE TRANSFORMS

- Laplace transforms will allow us to find the complete time response, $y(t) = y_N(t) + y_F(t)$, for <u>Any Circuit</u> with <u>Any Input</u>:
  - Very powerful technique
  - Will use in other courses

LAPLACE TRANSFORMS



Techniques from Unit I

LAPLACE TRANSFORMS

Concept:

- Take Laplace Transform of x(t)

$$L[x(t)] = X(s)$$

- X(s) is Input to s-domain Circuit
- Find Output to s-domain Circuit, Y(s)

  Algebraic Equations

- Take Inverse Laplace Transform of Y(s)

$$L^{-1}[Y(s)] = y(t)$$

LAPLACE TRANSFORMS

$$\text{Define } s \overset{\Delta}{=} \text{Complex Frequency} = \sigma + j\omega$$

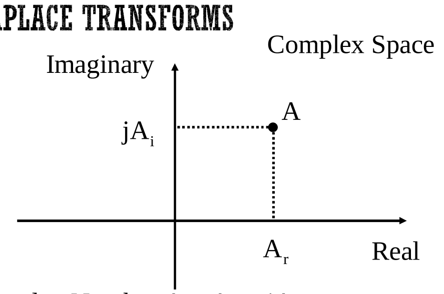
$$\sigma = \text{Real Part; } \omega = \text{Imaginary Part}$$

$$j = \sqrt{-1}$$

When $\sigma = \omega = 0$; => $s = 0$; => DC Steady State

When $\sigma = 0$; $s = j\omega$ => AC Steady State (Unit IV)

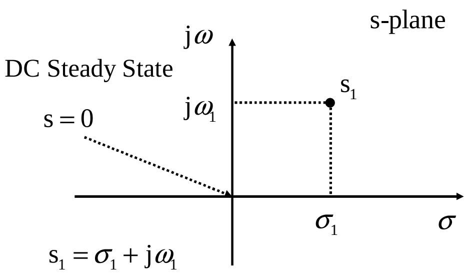
LAPLACE TRANSFORMS



Complex Number $A = A_r + jA_i$

j = Unit Vector in Vertical Direction $= \sqrt{-1}$

LAPLACE TRANSFORMS



LAPLACE TRANSFORMS

Definitions:

Laplace Transform of f(t) = F(s)

$$L\,[f(t)] = F(s) = \int_{0^-}^{\infty} f(t)\, e^{-st}\, dt$$

Will Use Tables to Find F(s) from f(t)

See Table 9.2

LAPLACE TRANSFORMS

Definitions:

Inverse Laplace Transform of F(s) = f(t)

$$L^{-1}\,[F(s)] = f(t) = \frac{1}{2\pi j}\int_{\alpha - j\infty}^{\alpha + j\infty} F(s)\, e^{st}\, ds$$

Will Use Tables or Partial Fraction Expansion to find f(t) from F(s)

See Table 9-2

LAPLACE TRANSFORMS

Signal	f(t)	F(s)
Impulse	$\delta(t)$	1
Step	u(t)	$\frac{1}{s}$
Constant	Au(t)	$\frac{A}{s}$
Ramp	tu(t)	$\frac{1}{s^2}$

LAPLACE TRANSFORMS

Signal	f(t)	F(s)
Exponential	$e^{-\alpha t}u(t)$	$\frac{1}{s+\alpha}$
Damped Ramp	$[te^{-\alpha t}]u(t)$	$\frac{1}{(s+\alpha)^2}$
Cosine Wave	$[\cos\beta t]u(t)$	$\frac{s}{s^2+\beta^2}$
Damped Cosine	$[e^{-\alpha t}\cos\beta t]u(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\beta^2}$

LAPLACE TRANSFORMS

Time Domain	s-Domain
$Af_1(t)+Bf_2(t)$	$AF_1(s)+BF_2(s)$
$\int_0^t f(\tau)\,d\tau$	$\frac{F(s)}{s}$
$\frac{df(t)}{dt}$	$sF(s)-f(0^-)$
$e^{-\alpha t}f(t)$	$F(s+\alpha)$
$t\,f(t)$	$-dF(s)/ds$
$f(t-a)u(t-a)$	$e^{-as}F(s)$

POLES AND ZEROS

Can usually express the Laplace Transform of signals of interest to us as a Ratio of Polynomials:

$$F(s) = \frac{N(s)}{D(s)} = \frac{b_m s^m + \;......\; + b_1 s + b_0}{a_n s^n + ...\; + a_1 s + a_0}$$

For Physically Realizable Circuits (systems), $m \le n$

POLES AND ZEROS

$$F(s) = \frac{b_m s^m + \;\ldots\ldots\; + b_1 s + b_0}{a_n s^n + \ldots\ldots \; + a_1 s + a_0}$$

Factor F(s):

$$F(s) = K\frac{(s - z_1)\,(s - z_2)\,(\ldots\ldots)\,(s - z_m)}{(s - p_1)\,(s - p_2)\,(\ldots\ldots)\,(s - p_n)}$$

$$K = \frac{b_m}{a_n} = \text{Scale Factor}$$

POLES AND ZEROS

$$F(s) = K\frac{(s - z_1)\,(s - z_2)\,(\ldots\ldots)\,(s - z_m)}{(s - p_1)\,(s - p_2)\,(\ldots\ldots)\,(s - p_n)}$$

At $s = z_i$ => $F(s) \to 0$ => Zeros of F(s)

At $s = p_j$ => $F(s) \to \infty$ => Poles of F(s)

Poles and Zeros are "Critical Frequencies" of F(s)

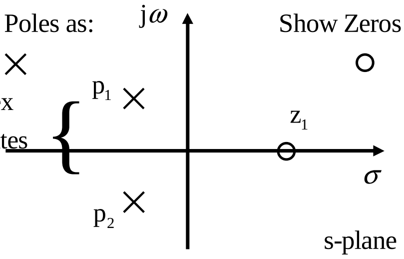
Useful to Plot "Pole-Zero Diagram" in s-plane

POLE-ZERO DIAGRAMS

Show Poles as: ×

Show Zeros as: ○

Complex Conjugates



s-plane

EXAMPLE

$$F(s) = \frac{s^2 - 3s}{s^2 + 8s + 25}$$

Factor F(s):

$$F(s) = \frac{s(s - 3)}{s^2 + 2\alpha s + \omega_0^2}$$

$$2\alpha = 8 => \alpha = 4;\;\; \omega_0{}^2 = 25$$

$\omega_0{}^2 > \alpha^2$ => Complex Conjugate Roots

EXAMPLE

$$F(s) = \frac{s(s - 3)}{s^2 + 2\alpha s + \omega_0^2}$$

$$2\alpha = 8 => \alpha = 4;\;\; \omega_0{}^2 = 25$$

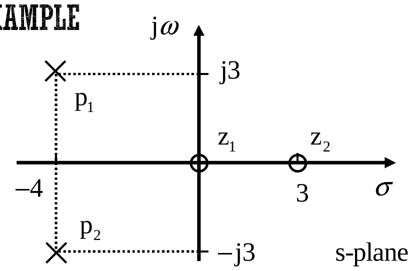
Zeros: $z_1 = 0;\; z_2 = 3$

Poles: $p_1,\; p_2 = -\alpha \pm j\beta;\;\; \beta = \sqrt{\omega_0^2 - \alpha^2}$

$$\beta = \sqrt{25 - 16} = 3$$

Poles: $p_1,\; p_2 = -4 \pm j3$

EXAMPLE



Pole-Zero Diagram

POLE-ZERO DIAGRAMS

Will soon see that we can learn a lot about a circuit's behavior from its Pole-Zero Diagram

Form of Natural Response

Determine the Stability

DC and AC Steady State Responses

Frequency Response