ELECTRIC CIRCUITS
ECSE-2010

Lecture 17 Review

Rensselaer

REVIEW: LECTURE 9

- First order RC and RL circuits
- Already a solved problem!
- Get RC and RL into the form of the solved problem
  - Find Thevenin Equivalent circuit
  - Find τ
- Find coefficients
  - Need $t \rightarrow \infty$ and initial condition t=0+

DYNAMIC CIRCUITS

$$y(t) = y_H + y_P$$

Homogeneous Response + Particular Response

$$y(t) = y_N + y_F$$

Natural Response + Forced Response

$$y_N = y_H; \quad y_F = y_P$$

$$\boxed{y(t) = y_{ZI} + y_{ZS}}$$

Zero-Input Response + Zero-State Response

Rensselaer

RC CIRCUITS

Solution to Any Current or Voltage in Any Circuit Containing 1 C plus R's, Independent Sources and Dependent Sources, with a Switched DC Input:

$$\boxed{y(t) = y_{SS} + (y_0 - y_{SS})e^{-(t-t_0)/\tau}} \quad \text{for } t \geq t_0$$

$$\tau = R_{eq}C$$

Can Find y_0, y_{SS}, τ Directly From Circuit

R_{eq} = Equivalent Resistance Seen at Terminals of C

Rensselaer

RL CIRCUITS

Solution to Any Current or Voltage in Any Circuit Containing 1 L plus R's, Independent Sources and Dependent Sources, with a Switched DC Input:

$$\boxed{y(t) = y_{SS} + (y_0 - y_{SS})e^{-(t-t_0)/\tau}} \quad \text{for } t \geq t_0$$

$$\tau = \frac{L}{R_{eq}}$$

Can Find y_0, y_{SS}, τ Directly From Circuit

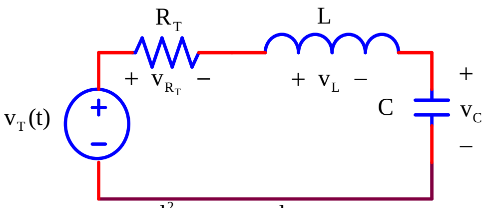
R_{eq} = Equivalent Resistance Seen at Terminals of L

Rensselaer

REVIEW: LECTURE 10

- Second order Series RLC and Parallel RLC
- Already solved problems!
- Get into standard from and find α, ω_0 and β (if needed)
- Compare α, ω_0 to find form of solution
- Find coefficients
  - Need $t \rightarrow \infty$ and initial conditions both Vc(0+) and dVc(0+)/dt for example

SERIES RLC CIRCUITS



$$LC\frac{d^2v_C}{dt^2}+R_TC\frac{dv_C}{dt}+v_C=v_T$$

$$\frac{d^2v_C}{dt^2}+\frac{R_T}{L}\frac{dv_C}{dt}+\frac{1}{LC}v_C=\frac{1}{LC}v_T$$

Rensselaer

SERIES RLC CIRCUITS

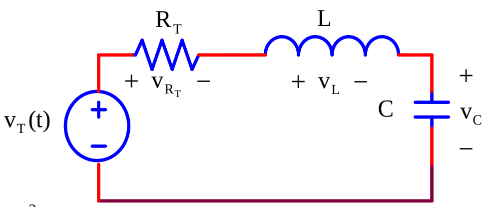
$$\frac{d^2v_C}{dt^2}+\frac{R_T}{L}\frac{dv_C}{dt}+\frac{1}{LC}v_C=\frac{1}{LC}v_T$$

$$\left[\frac{1}{LC}\right]=\frac{1}{(\text{seconds})^2}=\omega_0^2 \qquad \left[\frac{R_T}{L}\right]=\frac{1}{\text{seconds}}=2\alpha$$

$$\frac{d^2v_C}{dt^2}+2\alpha\frac{dv_C}{dt}+\omega_0^2v_C=\omega_0^2v_T$$

Rensselaer

SERIES RLC CIRCUITS



$$\frac{d^2v_C}{dt^2}+2\alpha\frac{dv_C}{dt}+\omega_0^2v_C=\omega_0^2v_T$$ Need Initial Conditions

$v_C(0^+)$ and $\frac{dv_C}{dt}(0^+)$ $\qquad$ $\frac{dv_C}{dt}(0^+)=\frac{1}{C}i_L(0^+)$

Rensselaer

SERIES RLC CIRCUITS

$$\frac{d^2v_{CN}}{dt^2}+2\alpha\frac{dv_{CN}}{dt}+\omega_0^2v_{CN}=0$$

Assume $v_{CN}(t)=Ke^{st}$

$$s^2+2\alpha s+\omega_0^2=0$$

Characteristic Equation

Roots are $s_1,s_2=-\alpha\pm\sqrt{\alpha^2-\omega_0^2}$

$$\boxed{v_{CN}(t)=K_1e^{s_1t}+K_2e^{s_2t}}$$

Rensselaer

SERIES RLC CIRCUITS

$$s^2+2\alpha s+\omega_0^2=0$$

Roots are $s_1,\ s_2=-\alpha\pm\sqrt{\alpha^2-\omega_0^2}$

3 Possible Cases:

Case 1: $\alpha^2>\omega_0^2$: 2 Real, Unequal Roots

Case 2: $\alpha^2=\omega_0^2$: 2 Real, Equal Roots

Case 3: $\alpha^2<\omega_0^2$: 2 Complex Conjugate Roots

Rensselaer

SERIES RLC CIRCUITS

Case 1: $\alpha^2>\omega_0^2$: 2 Real, Unequal Roots

$$s_1=-\alpha+\sqrt{\alpha^2-\omega_0^2} \qquad \alpha=\frac{R_T}{2L}$$

$$s_2=-\alpha-\sqrt{\alpha^2-\omega_0^2} \qquad \omega_0^2=\frac{1}{LC}$$

$$\boxed{v_{CN}=K_1e^{s_1t}+K_2e^{s_2t}}$$

2 Decaying Exponentials

Circuit is Overdamped

Rensselaer

SERIES RLC CIRCUITS

Case 2: $\alpha^2 = \omega_0^2$: 2 Real, Equal Roots

$$s_1 = -\alpha \qquad \alpha = \frac{R_T}{2L} \qquad \omega_0^2 = \frac{1}{LC}$$
$$s_2 = -\alpha$$

$$\boxed{v_{CN} = K_1 e^{-\alpha t} + K_2 t e^{-\alpha t}}$$

Decaying Exponential + Exponentially Damped Ramp

Circuit is Critically Damped

Rensselaer

SERIES RLC CIRCUITS

Case 3: $\alpha^2 < \omega_0^2$: 2 Complex Conjugate Roots

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -\alpha + j\sqrt{\omega_0^2 - \alpha^2} = -\alpha + j\beta$$
$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -\alpha - j\sqrt{\omega_0^2 - \alpha^2} = -\alpha - j\beta$$

$$v_{CN} = K_1 e^{(-\alpha + j\beta)t} + K_2 e^{(-\alpha - j\beta)t}$$
$$v_{CN} = A e^{-\alpha t} \cos(\beta t + \phi)$$

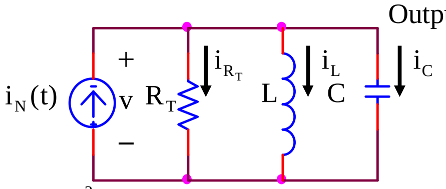
Exponentially Damped Sinusoid

Circuit is Underdamped

Rensselaer

PARALLEL RLC CIRCUITS

Output $= i_L$



$$\frac{d^2 i_L}{dt^2} + 2\alpha \frac{di_L}{dt} + \omega_0^2 i_L = \omega_0^2 i_N \qquad \alpha = \frac{1}{2R_T C}$$

Same Form of Equation as for Series RLC $\qquad \omega_0^2 = \frac{1}{LC}$

Slightly Different α

PARALLEL RLC CIRCUITS

Parallel RLC Circuits

LHS of Differential Equation is Same for Any Output

Natural Response for Any Output

$$\frac{d^2 y_N}{dt^2} + 2\alpha \frac{dy_N}{dt} + \omega_0^2 y_N = 0$$

Same as for Series RLC Circuits

PARALLEL RLC CIRCUITS

Natural Response

$$\frac{d^2 y_N}{dt^2} + 2\alpha \frac{dy_N}{dt} + \omega_0^2 y_N = 0$$

Characteristic Equation

$$s^2 + 2\alpha s + \omega_0^2 = 0$$

Same Roots as for Series RLC

Overdamping, Critical Damping, Underdamping

REVIEW: LECTURE 12 AND 13

- Laplace transforms
- Finding poles and zeros
- Partial Fraction Expansion
  - Simple real poles
  - Complex conjugate poles
  - Double poles
- Relationship to differential equations
- S-domain impedances (zero and non-zero initial conditions)

LAPLACE TRANSFORMS

Signal	f(t)	F(s)
Impulse	$\delta(t)$	1
Step	$u(t)$	$\frac{1}{s}$
Constant	$Au(t)$	$\frac{A}{s}$
Ramp	$tu(t)$	$\frac{1}{s^2}$

LAPLACE TRANSFORMS

Signal	f(t)	F(s)
Exponential	$e^{-\alpha t}u(t)$	$\frac{1}{s+\alpha}$
Damped Ramp	$[te^{-\alpha t}]u(t)$	$\frac{1}{(s+\alpha)^2}$
Cosine Wave	$[\cos\beta t]u(t)$	$\frac{s}{s^2+\beta^2}$
Damped Cosine	$[e^{-\alpha t}\cos\beta t]u(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\beta^2}$

LAPLACE TRANSFORMS

Time Domain	s-Domain
$Af_1(t)+Bf_2(t)$	$AF_1(s)+BF_2(s)$
$\int_0^t f(\tau)\,d\tau$	$\frac{F(s)}{s}$
$\frac{df(t)}{dt}$	$sF(s)-f(0^-)$
$e^{-\alpha t}f(t)$	$F(s+\alpha)$
$t\,f(t)$	$-dF(s)/ds$
$f(t-a)u(t-a)$	$e^{-as}F(s)$

POLES AND ZEROS

$$F(s)=\frac{b_m s^m+\;......\;+b_1 s+b_0}{a_n s^n+......\;+a_1 s+a_0}$$

Factor F(s):

$$F(s)=K\frac{(s-z_1)\,(s-z_2)\,(.....)\,(s-z_m)}{(s-p_1)\,(s-p_2)\,(.....)\,(s-p_n)}$$

$$K=\frac{b_m}{a_n}=\text{Scale Factor}$$

POLES AND ZEROS

$$F(s)=K\frac{(s-z_1)\,(s-z_2)\,(.....)\,(s-z_m)}{(s-p_1)\,(s-p_2)\,(.....)\,(s-p_n)}$$

At $s=z_i$ => $F(s)\to 0$ => Zeros of F(s)

At $s=p_j$ => $F(s)\to\infty$ => Poles of F(s)

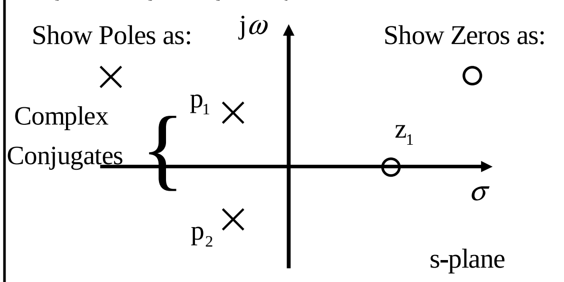
Poles and Zeros are "Critical Frequencies" of F(s)

Useful to Plot "Pole-Zero Diagram" in s-plane

POLE-ZERO DIAGRAMS

Show Poles as: ×

Show Zeros as: ○



PARTIAL FRACTION EXPANSION

There are only 3 Types of Poles:

Simple, Real Poles: $(s-4)$, $\Rightarrow p_1 = 4$

Real, Equal Poles: $(s+3)^2$, $\Rightarrow p_1 = p_2 = -3$

Complex Conjugate Poles: $(s^2+8s+25)$

$\Rightarrow p_1, p_2 = -4 \pm j3$

Rensselaer

PARTIAL FRACTION EXPANSION

For $m < n$:

- Simple Real Poles

In General:

Expand: $F(s) = \dfrac{A_1}{s-p_1} + \dfrac{A_2}{s-p_2} + \dfrac{A_3}{s-p_3} + \ .....$

$A_n = [(s-p_n)F(s)]\big|_{s=p_n}$; Cover-Up Rule

$\Rightarrow\ f(t) =)A_1 e^{p_1 t} + A_2 e^{p_2 t} + A_3 e^{p_3 t} + \ .....) \quad t \geq 0$

Rensselaer

PARTIAL FRACTION EXPANSION

- Complex Conjugate Poles

In General:

Expand $F(s) = \dfrac{A_1}{s-p_1} + \ .... + \dfrac{A}{s+\alpha-j\beta} + \dfrac{A^*}{s+\alpha+j\beta}$

Find A_1 and $A = |A|\underline{/\phi}$ from Cover-Up Rule

$\Rightarrow f(t) = A_1 e^{p_1 t} + \ .... + 2|A|e^{-\alpha t}\cos(\beta t + \phi) \quad t \geq 0$

Simple Poles Complex Poles

Rensselaer

PARTIAL FRACTION EXPANSION

- Real, Equal Poles – Double Pole:

Expand $F(s) = \dfrac{A_1}{s-p_1} + \ .. + [\dfrac{A_{n1}}{s-p_n} + \dfrac{A_{n2}}{(s-p_n)^2}]$

$A_{n2} = \left[(s-p_n)^2 F(s)\right]\big|_{s=p_n}$; Cover-Up Rule

Usually Find A_{n1} from evaluating F(0) or F(1)

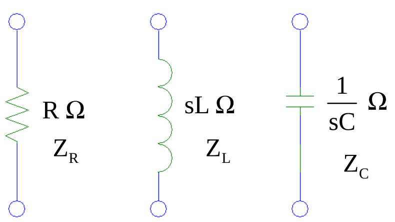
$\Rightarrow f(t) = (A_1 e^{p_1 t} + \ .... + A_{n1} e^{p_n t} + A_{n2} t e^{p_n t}) \quad t \geq 0$

Simple Poles Repeated Poles

Rensselaer

IMPEDANCE

Zero initial conditions

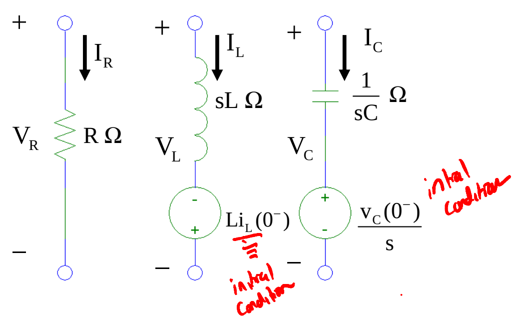


$Z = \text{Impedance} = \dfrac{V(s)}{I(s)}$

sawyes@rpi.edu www.rpi.edu/~sawyes

Rensselaer 29

NON-ZERO INITIAL CONDITIONS

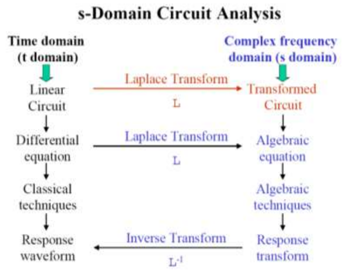


Rensselaer

REVIEW: LECTURE 14 CIRCUIT ANALYSIS

- Unit 1 + Unit 2 in one problem

GENERAL PROCESS



CIRCUITS WITH LAPLACE

1. Find Initial Conditions
2. Determine Laplace Equivalent circuit
3. Use Unit 1 concepts (node/mesh/voltage dividers etc.) to find an expression for the parameter of interest (impedances)
   a. "Clean up" expression to have $\frac{N(s)}{D(s)}$
4. Find poles (zeros, Unit 3)
5. Partial fraction expansion
   a. Cover up rule for coefficients or F(0), F(1)
6. Inverse Laplace gives time domain response

EXAM DETAILS

All questions are similar to problems done in class, homework, or lab!

1) Short answer section
   May include any mini-analysis

2) First order circuit (Diff. Eq)

3) Second order circuit (Diff. Eq.)

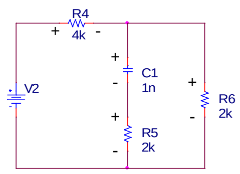
4) Second order circuit (Diff. Eq.)

5) Second order circuit (Laplace)

OTHER SHORT ANSWER QUESTIONS

- Voltage/Current continuity
- Overdamped, underdamped, critically damped analysis
- Laplace transforms
- Equivalent impedance conversion in s-domain
- Partial fraction expansion
- Pole-zero diagram

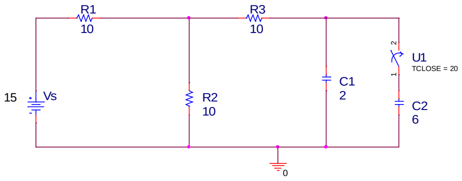
VOLTAGE/CURRENT CONTINUITY



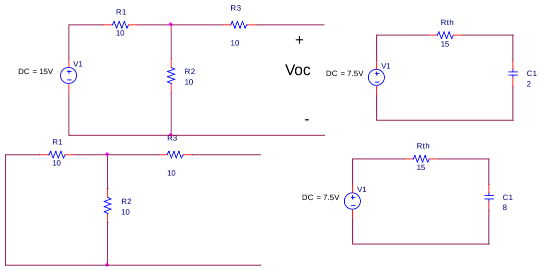
In the above circuit, the voltage is defined as follows:

$V1 = \begin{cases} 5V & t<0 \\ 10V & 0<t \end{cases}$ (the voltage source changes from 5V to 10V at t = 0)

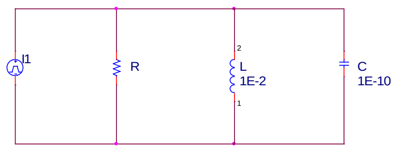
FIRST ORDER DIFFERENTIAL EQUATIONS



In the above circuit, the source turns on at t = 0 with a voltage of 15V, Vs=15u(t)V. Additionally, at t = 20s the switch in series with C2 is <u>closed</u>. You can (should) ignore C2 for part a) of this problem.



SECOND ORDER DIFFERENTIAL EQUATIONS



In the above circuit, the source current is 20mA for t < 0 and 0 for t > 0 (the source turns off at t = 0).

SECOND ORDER LAPLACE WITH INITIAL CONDITIONS