ELECTRIC CIRCUITS ECSE-2010

Lecture 18.1: Introduction

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LECTURE 18.1 AGENDA

- Transfer Functions
- Phasors
- Phasor Math

TRANSFER FUNCTIONS

$$\text{H(s)} = \text{One of the most important things we can find for a circuit}$$

H(s) depends only the circuit

H(s) does NOT depend on the Input

Find H(s) by ASSUMING there is no Initial Stored Energy

and then finding the Ratio of $\dfrac{\text{Output(s)}}{\text{Input(s)}}$

Will use H(s) to find the Frequency Response of a Circuit

TRANSFER FUNCTIONS

FREQUENCY RESPONSE

For No Initial Stored Energy

$$Y(s) = H(s)\,X(s)$$

For AC Steady State: $s = j\omega \quad (\sigma = 0)$

$$Y_{SS}(j\omega) = H(j\omega)\,X(j\omega)$$

$Y_{SS}(j\omega)$, $H(j\omega)$, $X(j\omega)$ are all Complex Numbers

Each has an Amplitude and a Phase

FREQUENCY RESPONSE

$$\left|Y_{SS}\right| = \left(\left|H\right|\right)\left(\left|X\right|\right)$$

$$\text{Output Amplitude} = \left|H\right| \text{ x Input Amplitude}$$

$$\phi_Y = \theta + \phi_X$$

$$\text{Output Phase} = \theta \ + \ \text{Input Phase}$$

If we know Input and Network Function

=> Can Find Amplitude and Phase of Output

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FREQUENCY RESPONSE

$$\left|H\right| = \left|H(j\omega)\right| \qquad \theta = \theta(j\omega)$$

Amplitude and Phase of Network Function
are Functions of Frequency

Plots of $\left|H\right|$ and θ vs. ω => Frequency Response

Describe how a circuit "behaves" as a function
of frequency in the AC Steady State

FREQUENCY RESPONSE

$$|H| = \frac{|Y_{SS}|}{|X|} = \text{"Gain" of Circuit}$$

$$\theta = \phi_Y - \phi_X = \text{"Phase Shift" of Circuit}$$

Frequency Response is described by plots of
the Gain and Phase Shift of the Network Function
as a function of frequency

BODE PLOT: DECIBELS AND LOG

$\lvert\mathbf{H}\rvert$	$\lvert\mathbf{H}\rvert_{dB}$
1	$20\log(1) = 0 \text{ dB}$
$\sqrt{2}$	$20\log(\sqrt{2}) = 10\log 2 = 3 \text{ dB}$
2	$20\log(2) = 6 \text{ dB}$
4	$20\log(4) = 12 \text{ dB}$
5	$20\log(5) = 14 \text{ dB}$
10	$20\log(10) = 20 \text{ dB}$

http://my.ece.ucsb.edu/York/Bobsclass/2B/Frequency%20Response.pdf. Bob York, 2009

BODE PLOT: SIMPLE POLE

$$\mathrm{H(s)} = \frac{1}{(s+1)} \quad \Rightarrow \quad \mathrm{H(j\omega)} = \frac{1}{(j\omega+1)}$$

Type equation here.

The magnitude of the transfer function is then given by

$$|H(j\omega)| \frac{1}{\sqrt{\omega^2+1}} = (\omega^2 + 1)^{-1/2}$$

The magnitude of the transfer function is given by

$$< H(j\omega) = -\tan^{-1}\frac{\omega}{1}$$

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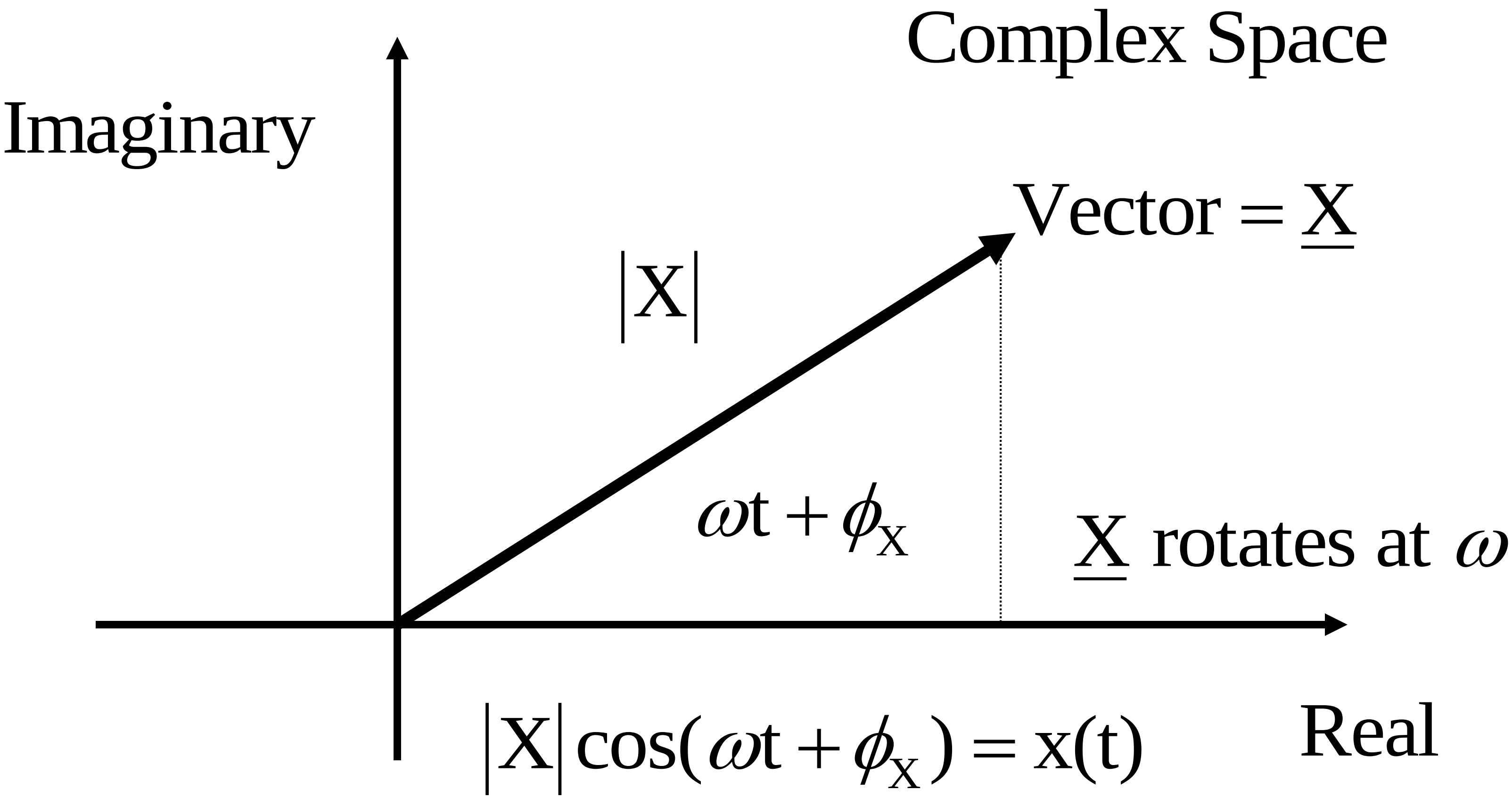
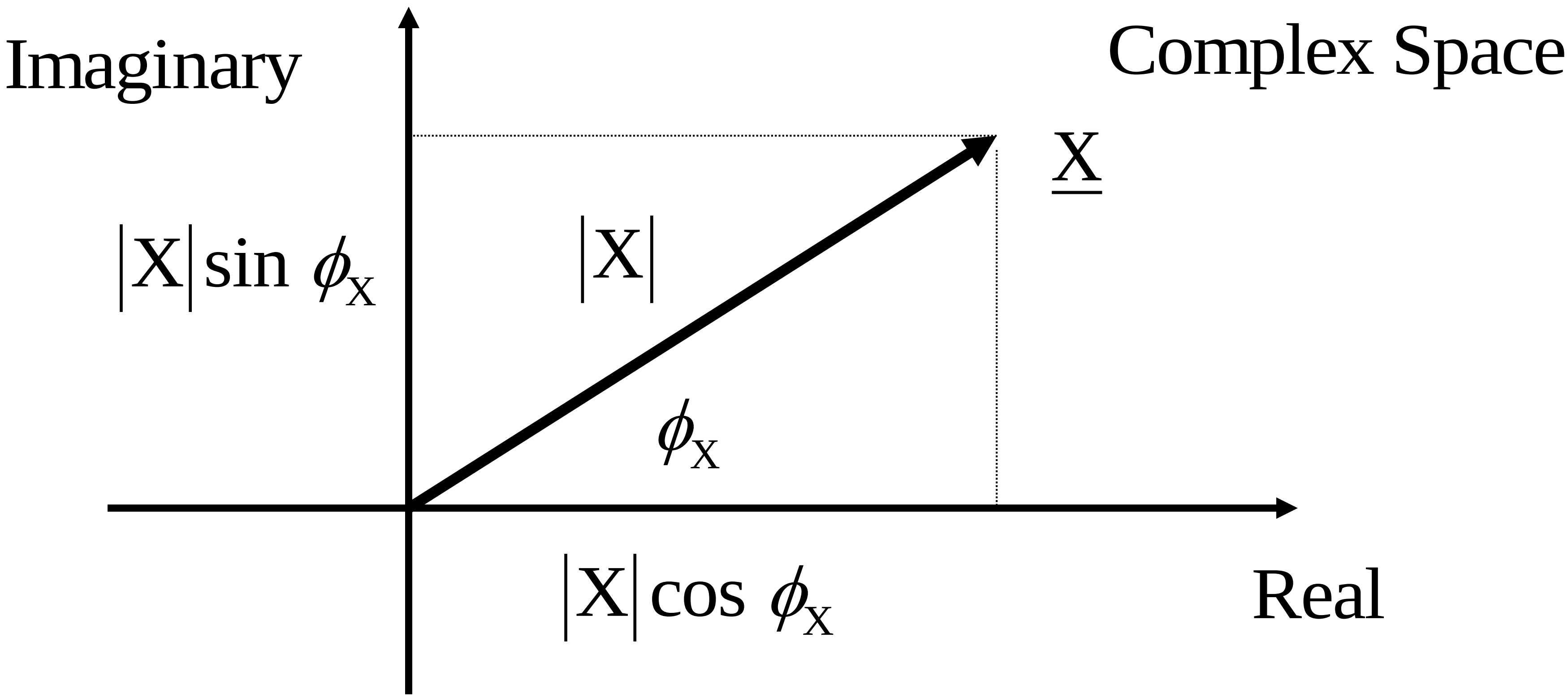
PHASORS



Input = Projection of $\underline{X}$ on Real Axis

PHASORS

Will Show $\underline{X}$ as a Snapshot at $t = 0$:



$$\underline{X} = |X| \underline{/ \phi_X} = \text{Polar Form}$$

PHASORS

Phasor = Rotating Vector in Complex Space

Phasor $\underline{X}$ Rotates at Angular Frequency ω

Amplitude (or Magnitude) of $\underline{X} = |X|$

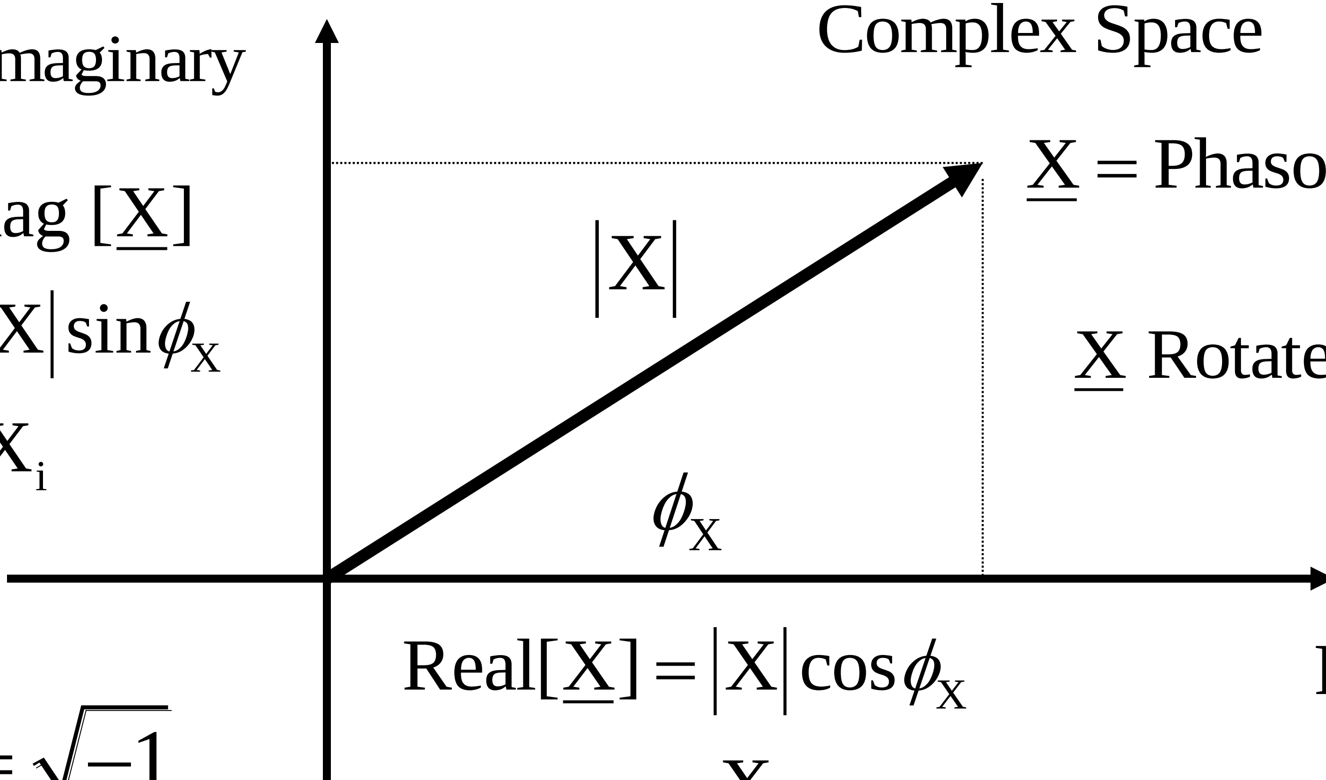
Angle (or Phase) of $\underline{X} = \underline{/X} = \phi_X$

$=>$ Can Express as $\underline{X} = |X|\underline{/\phi_X}$ $=>$ "$|X|$ at an angle of ϕ_X"

$\underline{X} = |X|\underline{/\phi_X}$ = "Polar Form for $\underline{X}$"

PHASORS

Complex Space



$\underline{X}$ = Phasor

$\underline{X}$ Rotates at ω

$$\text{Imag}[\underline{X}] = |X|\sin\phi_X = X_i$$

$$\text{Real}[\underline{X}] = |X|\cos\phi_X = X_r$$

$$j = \sqrt{-1}$$

$$\underline{X} = X_r + jX_i = \text{Rectangular Form}$$

PHASORS

$$\underline{X} = X_r + jX_i \Rightarrow \text{Rectangular Form}$$

$$\underline{X} = |X| \underline{/\phi_X} \Rightarrow \text{Polar Form}$$

$$X_r = |X|\cos\phi_X \qquad X_i = |X|\sin\phi_X$$

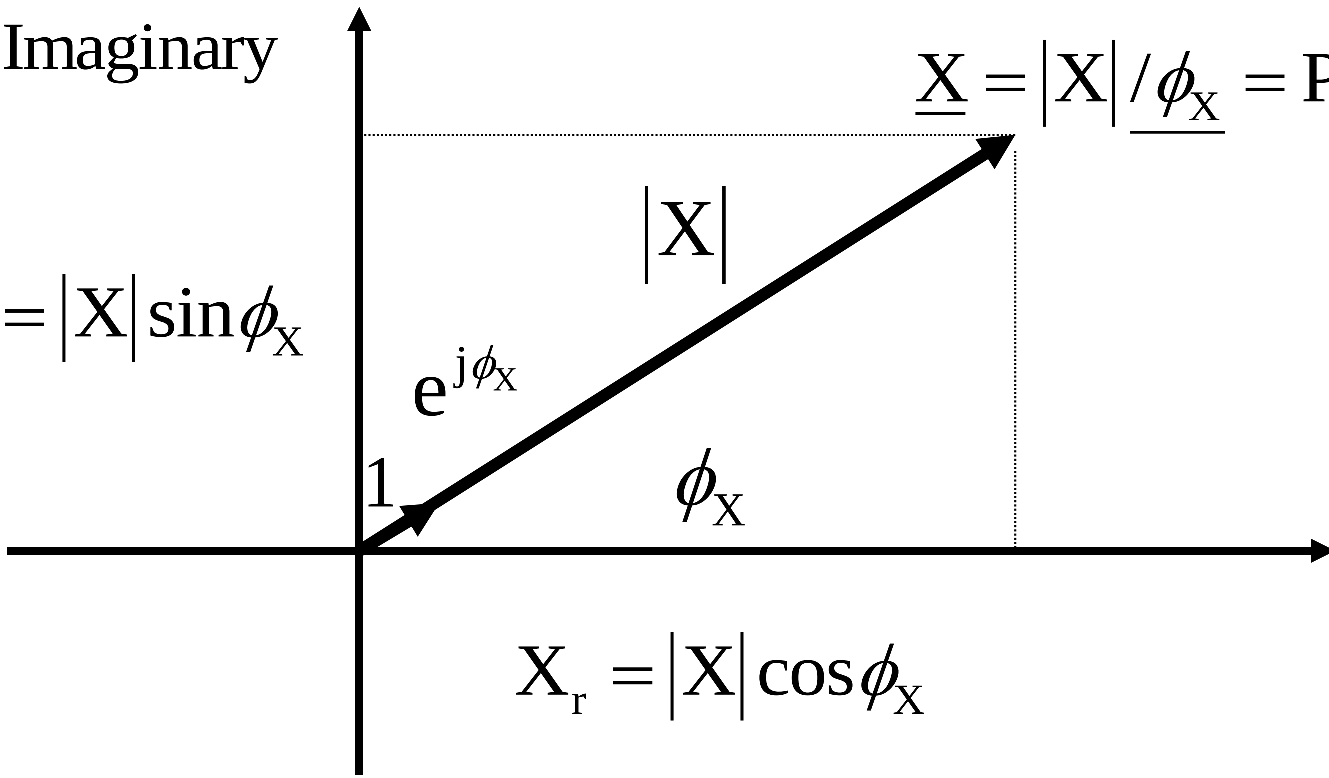
Can Easily Convert from Polar (P) to Rectangular (R) forms using Calculator

Need to be able to do this by next class

PHASORS

Complex Space

$\underline{X} = |X| \underline{/\phi_X} = \text{Polar Form}$



$X_i = |X| \sin\phi_X$

$X_r = |X| \cos\phi_X$

$\underline{X} = X_r + jX_i = \text{Rectangular Form}$

$\underline{X} = |X| e^{j\phi_X} = \text{Euler Form}$

EULER FORM

Euler's Formula:

$$e^{j\phi_X} = \cos\phi_X + j\sin\phi_X$$

$$e^{j\phi_X} = \text{Unit Vector in Direction of } \underline{X}$$

$$\Rightarrow \underline{X} = |X| e^{j\phi_X} \Rightarrow \text{ Euler Form for } \underline{X}$$

PHASORS

- **Phasors are Complex Numbers:**
  - **Need to Use Complex Math**
  - **Will Use Complex Math Instead of Solving Differential Equations or Using Laplace Transforms to Find Amplitude and Phase Changes between AC Input and AC Steady State Output**

PHASORS

- **Phasors are Complex Numbers:**
  - **Will Find that Equations Relating Current Phasors, $\underline{I}$, and Voltage Phasors, $\underline{V}$ for R, L and C will be Linear and Algebraic**
  - **Can Use All Techniques from Unit I to Solve Circuits in the AC Steady State**

PHASORS

- 3 Ways to Express Phasors

Rectangular Form; $\underline{X} = X_r + jX_i$

Polar Form; $\underline{X} = |X| \underline{/\,\phi_X}$

Euler Form; $\underline{X} = |X| e^{j\phi_X}$

- Will Need to Be Able to Easily Convert Between the 3 Different Forms

COMPLEX MATH

- **Addition:**
  - $\underline{A} + \underline{B} = (a_r + j\, a_i) + (b_r + j\, b_i)$
    $= (a_r + b_r) + j\,(a_i + b_i)$
- **Subtraction:**
  - $\underline{A} - \underline{B} = (a_r + j\, a_i) - (b_r + j\, b_i)$
    $= (a_r - b_r) + j\,(a_i - b_i)$
- **=> Do Addition/Subtraction in Rectangular Form**

COMPLEX MATH

- **Multiplication:**
  - **Difficult to do in Rectangular Form**
  - **Use Euler or Polar Form**

$$\underline{A} \text{ x } \underline{B} = Ae^{j\phi_1} \text{ x } Be^{j\phi_2} = AB\, e^{j(\phi_1+\phi_2)}$$

$$= A\underline{/\phi_1} \text{ x } B\underline{/\phi_2} = AB\underline{/(\phi_1+\phi_2)}$$

COMPLEX MATH

- **Division:**

$$\underline{A} \div \underline{B} = Ae^{j\phi_1}/Be^{j\phi_2} = \frac{A}{B}e^{j(\phi_1-\phi_2)}$$

$$= A\underline{/\phi_1} / B\underline{/\phi_2} = \frac{A}{B}\underline{/(\phi_1-\phi_2)}$$

- **Do Multiplication/Division in Polar Form or Euler Form**

RATIONALIZATION

- **Division in Rectangular Form:**

$$\underline{C} = \underline{A} \div \underline{B} = \frac{a_r + ja_i}{b_r + jb_i}$$

$$\text{Want to express as } \underline{C} = C_r + jC_i$$

$$\text{Multiply} \frac{a_r + ja_i}{b_r + jb_i} \text{ by } \frac{b_r - jb_i}{b_r - jb_i}$$

$$\Rightarrow \underline{C} = \frac{a_r b_r + a_i b_i}{b_r^2 + b_i^2} + j \frac{a_i b_r - a_r b_i}{b_r^2 + b_i^2}$$

$$= \qquad C_r \qquad + j \qquad C_i$$

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COMPLEX CONJUGATES

- $\underline{A} = a_r + j\, a_i$

- $\underline{A}^* = a_r - j\, a_i$ = Complex Conjugate of $\underline{A}$

- $\underline{A} \times \underline{A}^* = a_r^2 + a_i^2 = |A|^2$
- Angle of $\underline{A}^*$ = - (Angle of $\underline{A}$)

$$\underline{\underline{/A}}^* = -\underline{\underline{/A}}$$

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ACTIVITY 24-1

$\underline{A} = -j5 \qquad \underline{B} = -4 + j2 \qquad \underline{C} = 1 + j3$

Find $\underline{D} = \underline{A} \text{ x } \underline{B}$

Find $\underline{E} = \dfrac{\underline{D}}{\underline{C}^*}$

ACTIVITY 24-1

$\underline{A} = -j5 \qquad \underline{B} = -4 + j2 \qquad \underline{C} = 1 + j3$

Find $\underline{D} = \underline{A} \text{ x } \underline{B} = \left(-j5\right) \text{ x } \left(-4 + j2\right) = j20 - j^2 10$

$j^2 = -1 \qquad \underline{D} = 10 + j20$

ACTIVITY 24-1

Find $\underline{E} = \dfrac{\underline{D}}{\underline{C}^*}$

$\underline{D} = 10 + j20$

$\underline{C} = 1 + j3 \quad => \underline{C}^* = 1 - j3$

$$\underline{E} = \frac{10 + j20}{1 - j3} \text{ x } \frac{1 + j3}{1 + j3} = \frac{10 + j30 + j20 - 60}{1 + 9}$$

$$= \frac{-50 + j50}{10} = -5 + j5$$

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ACTIVITY 24-1

$$\underline{A} = 4e^{-j120^0} \qquad \underline{B} = 26 - j15$$

$$\underline{C} = 2\underline{/-150^0} \qquad \underline{D} = 2\underline{/30^0}$$

$$\text{Find } \underline{E} = \frac{\underline{A}^* \text{ x } \underline{B}}{\underline{C}}$$

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ACTIVITY 24-1

Convert All to Polar Form

$$\underline{A} = 4\underline{/-120^{o}}, \Rightarrow \underline{A}^{*} = 4\underline{/+120^{0}}$$

$$\underline{B} = 26 - j15 = 30\ \underline{/-30^{o}}$$

$$\underline{E} = \frac{\underline{A}^{*}\underline{B}}{\underline{C}} = \frac{4\underline{/120^{0}}\ 30\underline{/-30^{0}}}{2\underline{/-150^{0}}}$$

$$\underline{E} = 60\ (\text{angle of } 240^{0}) = 60\underline{/240^{o}}$$

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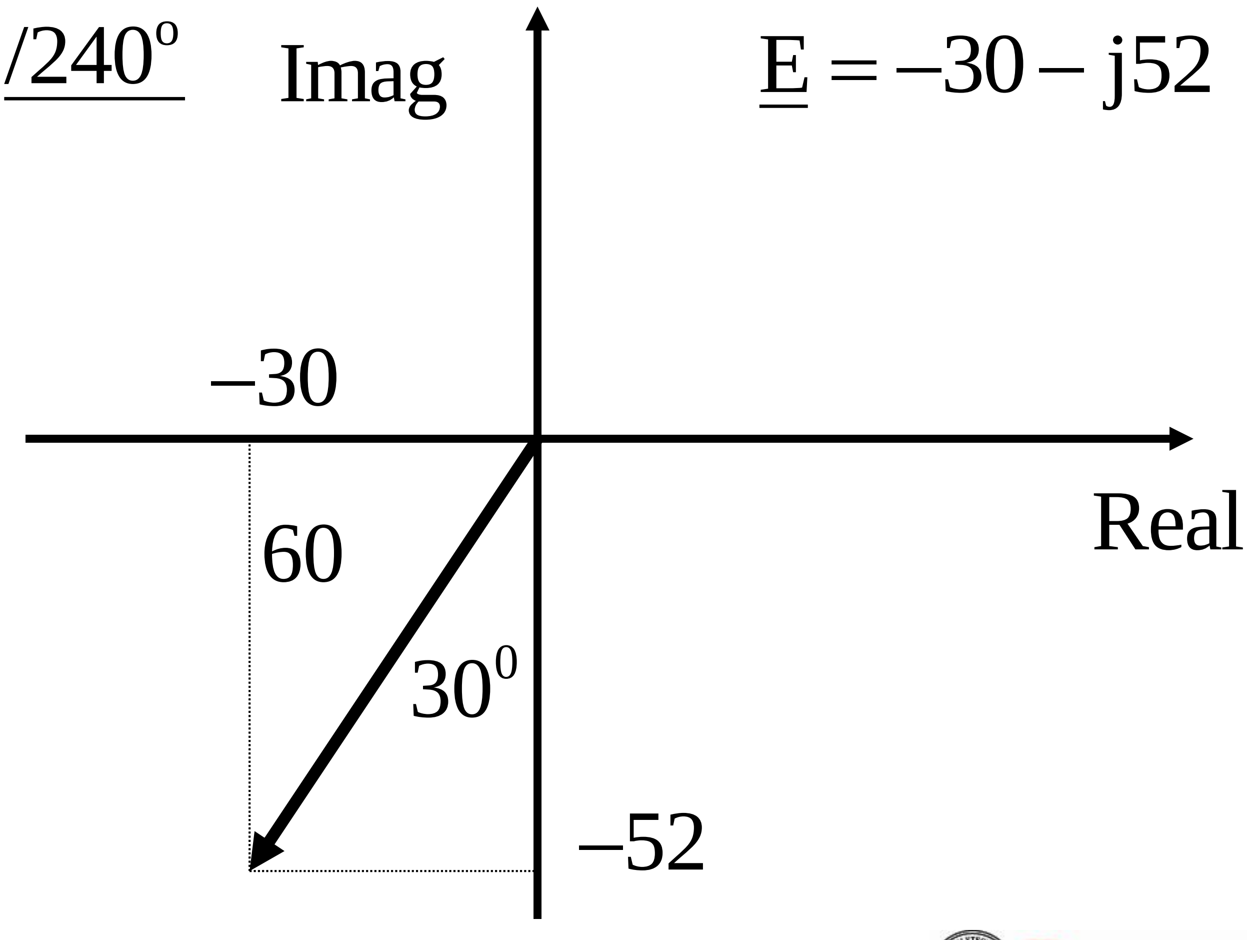
ACTIVITY 24-1

$\underline{E} = 60\underline{/240^o}$

$\underline{E} = -30 - j52$



ACTIVITY 24-1

$$\underline{F} = \underline{D}^3 - \underline{E} - 55$$

Convert All to Rectangular Form

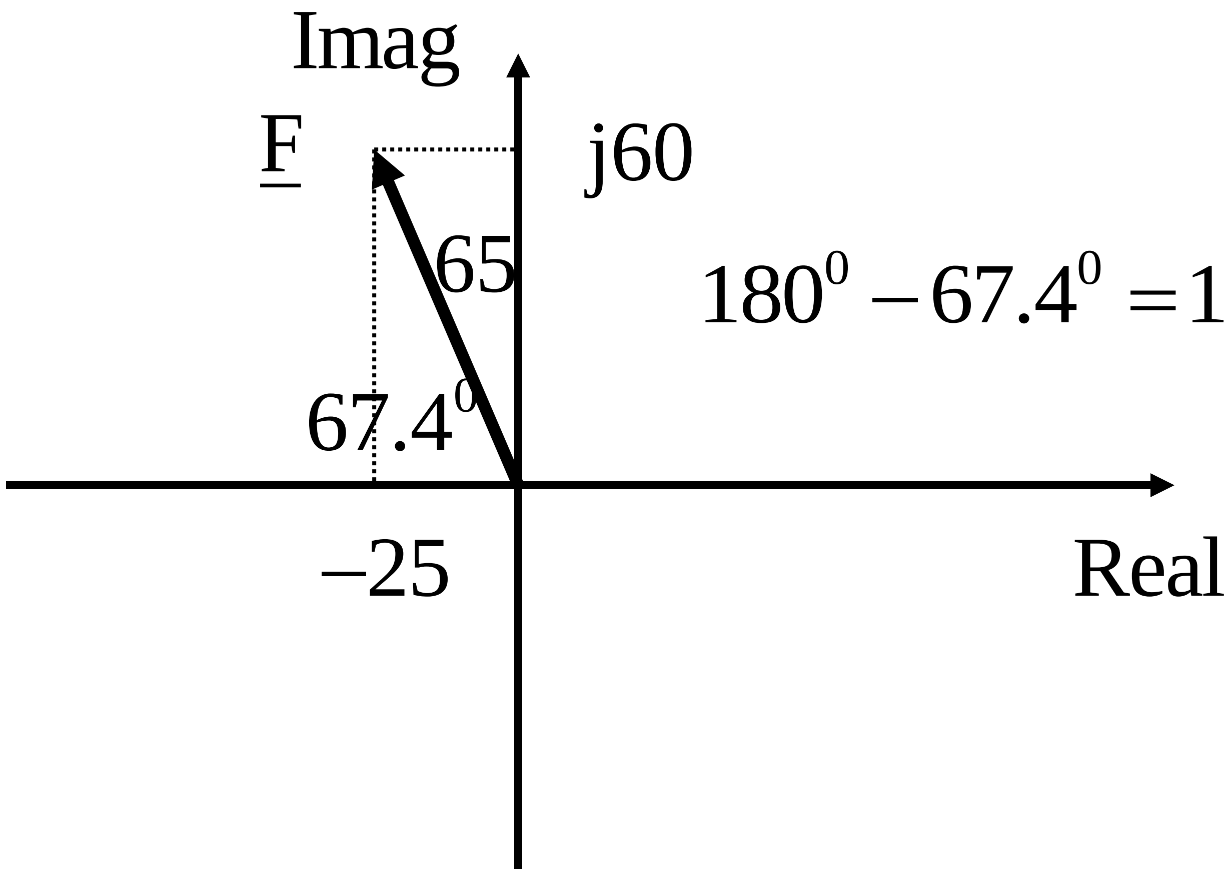
$$\underline{D}^3 = (2\underline{/30^0})^3 = 8\underline{/90^0} = j8$$

$$\underline{E} = 60\underline{/240^0} = -30 - j52$$

$$55 = 55 + j0$$

$$\Rightarrow \underline{F} = \underline{D}^3 - \underline{E} - 55 = -25 + j60 = 65\ \underline{/112.6^o}$$

ACTIVITY 24-1



$$180^0 - 67.4^0 = 112.6^0$$

$$\underline{F} = -25 + j60 = 65\ \underline{/112.6^o}$$