ELECTRIC CIRCUITS ECSE-2010

Lecture 19.1

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LECTURE 19.1 AGENDA

- Kirkoff's laws for phasors
- AC steady state impedence
- Examples

PHASORS

Phasors module

Phasor = Rotating Vector in Complex Space

Phasor $\underline{X}$ Rotates Counterclockwise at

Angular Frequency ω

Amplitude (or Magnitude) of $\underline{X} = |X|$

Angle (or Phase) of $\underline{X} = \underline{/\,X} = \phi_X$

$\Rightarrow$ Can Express as $\underline{X} = |X|\underline{/\phi_X}$ $\Rightarrow$ "$|X|$ at an angle of ϕ_X"

$\underline{X} = |X|\underline{/\phi_X} =$ "Polar Form for $\underline{X}$"

K'S LAWS FOR PHASORS

- Circuit in AC steady state:

  $$\text{Input} = x(t) = |X|\cos(\omega t + \phi_X)$$

  $$\text{Express } x(t) \text{ as a Phasor } \underline{X} = |X|\underline{/\,\phi_X}$$

- Can express all v's and i's in circuit as phasors:

  $$v_1(t) \rightarrow \underline{V_1};\ v_2(t) \rightarrow \underline{V_2}$$

  $$i_1(t) \rightarrow \underline{I_1};\ \ i_2(t) \rightarrow \underline{I_2}$$

AC STEADY STATE

- Time Domain:
  - Currents, Voltages expressed as Sinusoids; i(t), v(t)
- Frequency Domain:
  - Currents, Voltages expressed as Phasors; $\underline{I}$, $\underline{V}$

K'S LAWS FOR PHASORS

- KCL:
  - If $i_1 + i_2 = i$; $\Rightarrow$ $\underline{I}_1 + \underline{I}_2 = \underline{I}$
- KVL:
  - If $v_1 + v_2 = v$; $\Rightarrow$ $\underline{V}_1 + \underline{V}_2 = \underline{V}$
- K's Laws Work for Phasors!
  - Complex Addition, not Simple Addition

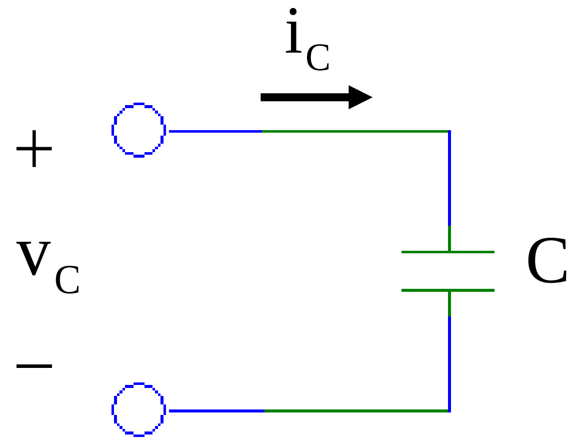
AC STEADY STATE

- Phasor Diagram:
  - Plot of Phasors in Complex Space
  - Same as Plotting Vectors in Real Space

AC Steady State Impedance

- **Capacitor:**

$$i_C = C\,\frac{dv_C}{dt}$$



$$v_C(t) = V\cos(\omega t + \phi)$$

$$= \text{Real}\left[V\,e^{j(\omega t+\phi)}\right]$$

$$= \text{Real}\left[V\,e^{j\phi}\;e^{j\omega t}\right] = \text{Real}\left[\underline{V_C}\,e^{j\omega t}\right]$$

AC STEADY STATE IMPEDANCE

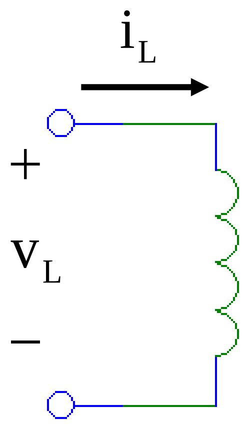
- **Inductor:**

$$v_L = L \frac{di_L}{dt}$$

$$i_L(t) = I\cos(\omega t + \phi)$$

$$= \text{Real}\left[I\, e^{j(\omega t + \phi)} \right]$$

$$= \text{Real}\left[I\, e^{j\phi} e^{j\omega t} \right] = \text{Real}\left[\underline{I_L}\, e^{j\omega t} \right]$$



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AC STEADY STATE IMPEDANCE

$$Z_R = R\ \Omega$$

$$Z_L = j\omega L\ \Omega$$

$$Z_C = -\frac{j}{\omega C} = \frac{1}{j\omega C}\ \Omega$$

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AC STEADY STATE IMPEDANCE

Note:

As $\omega \to 0$; $Z_L = j\omega L \to 0$

Inductor is a Short Circuit for DC

As $\omega \to \infty$; $Z_L = j\omega L \to \infty$

Inductor is an Open Circuit for Very High Frequencies

AC STEADY STATE IMPEDANCE

Note:

$$\text{As } \omega \to 0;\ Z_C = -\frac{j}{\omega C} \to \infty$$

Capacitor is an Open Circuit for DC

$$\text{As } \omega \to \infty;\ Z_C = -\frac{j}{\omega C} \to 0$$

Capacitor is a Short Circuit for

Very High Frequencies

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AC STEADY STATE IMPEDANCE

- In General, $\underline{V} = Z\ \underline{I}$ in AC Steady State:
  - Z = AC SS Impedance
  - Units of Ohms
  - Ohm's Law for AC Steady State
- Y = AC Steady State Admittance

  = 1/Z (Units of Siemens)

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AC STEADY STATE IMPEDENCE

$$\underline{V} = Z\underline{I}; \quad \text{Ohm's Law for AC Steady State}$$

$$Z = R(\omega) + jX(\omega) = \text{AC Steady State Impedance}$$

$$R(\omega) = \text{AC Steady State Resistance}$$

$$X(\omega) = \text{AC Steady State Reactance}$$

$$Y = G(\omega) + jB(\omega) = \text{AC Steady State Admittance}$$

$$G(\omega) = \text{AC Steady State Conductance}$$

$$B(\omega) = \text{AC Steady State Susceptance}$$

AC SS CIRCUIT ANALYSIS

- Time Domain $\rightarrow$ Frequency Domain

- Express all Voltages and Currents as Phasors

$$v_1(t) \rightarrow \underline{V}_1; \quad i_1(t) \rightarrow \underline{I}_1; \quad \text{etc.}$$

- Express R, L and C with AC Steady State Impedances

$$Z_R = R\ \Omega; \;\; Z_L = j\omega L\ \Omega; \;\; Z_C = \frac{-j}{\omega C}\ \Omega$$

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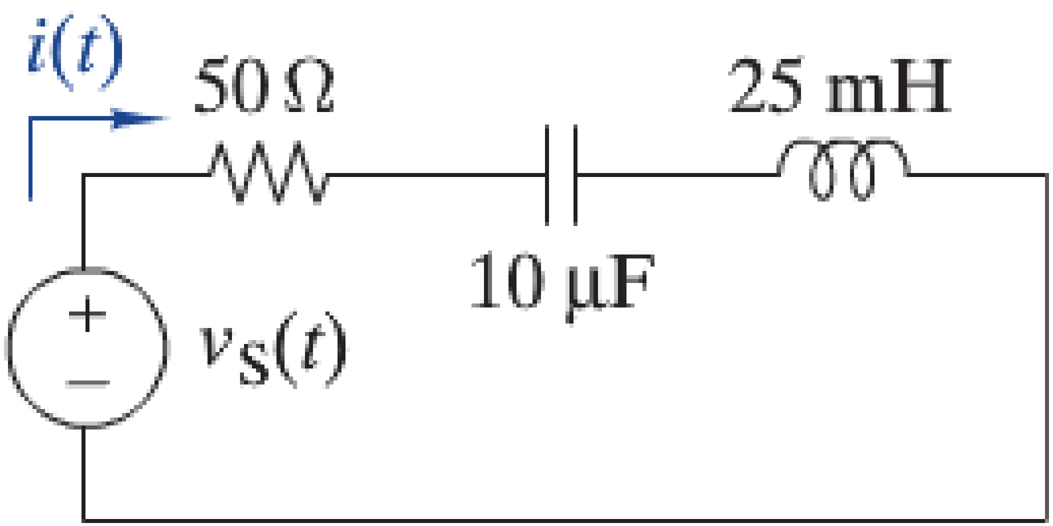
EXAMPLE 1A

The circuit in Figure 8-15(a) is operating in the sinusoidal steady state with $v_s(t) = 35 \cos 100t$ V



(a)

EXAMPLE 1A

The circuit in Figure 8-15(a) is operating in the sinusoidal steady state with $v_s(t) = 35 \cos 100t$ V

(a) Transform the circuit into the phasor domain
(b) Solve the phasor current I
(c) Solve for the phassor voltage across each element
(d) Find the waveforms corresponding to the phasors found in (b) and (c)

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EXAMPLE 1B

The circuit in Figure 8-15(a) is operating in the sinusoidal steady state with $v_s(t) = 100 \cos 2000t\text{-}45\text{deg}$ V

(a) Transform the circuit into the phasor domain
(b) Solve the phasor current I
(c) Solve for the phasor voltage across each element
(d) Find the waveforms corresponding to the phasors found in (b) and (c)
(e) Draw a phasor diagram of all three voltages and the current