ELECTRIC CIRCUITS ECSE-2010

Lecture 20.1

Rensselaer

Lecture 20.1 Agenda

- AC Thevenin/Norton circuits
- AC node equations
- AC mesh equations
- AC bridge circuits

FREQUENCY DEPENDENCE

- Usually Interested in the Steady State Behavior of a Circuit as the Frequency of the Input is Varied:
  - Frequency Response of a Circuit
- Since $Z = R(\omega) + j\,X(\omega)$ is Frequency Dependent:
  - Behavior of a Circuit in AC Steady State Varies Considerably with Frequency

FREQUENCY DEPENDENCE

$$Z_C = -\frac{j}{\omega C};$$

$Z_C \rightarrow \infty$ as $\omega \rightarrow 0$ Open Circuit

$Z_C \rightarrow 0$ as $\omega \rightarrow \infty$ Short Circuit

$$Z_L = j\omega L;$$

$Z_L \rightarrow 0$ as $\omega \rightarrow 0$ Short Circuit

$Z_L \rightarrow \infty$ as $\omega \rightarrow \infty$ Open Circuit

AC SS CIRCUIT ANALYSIS

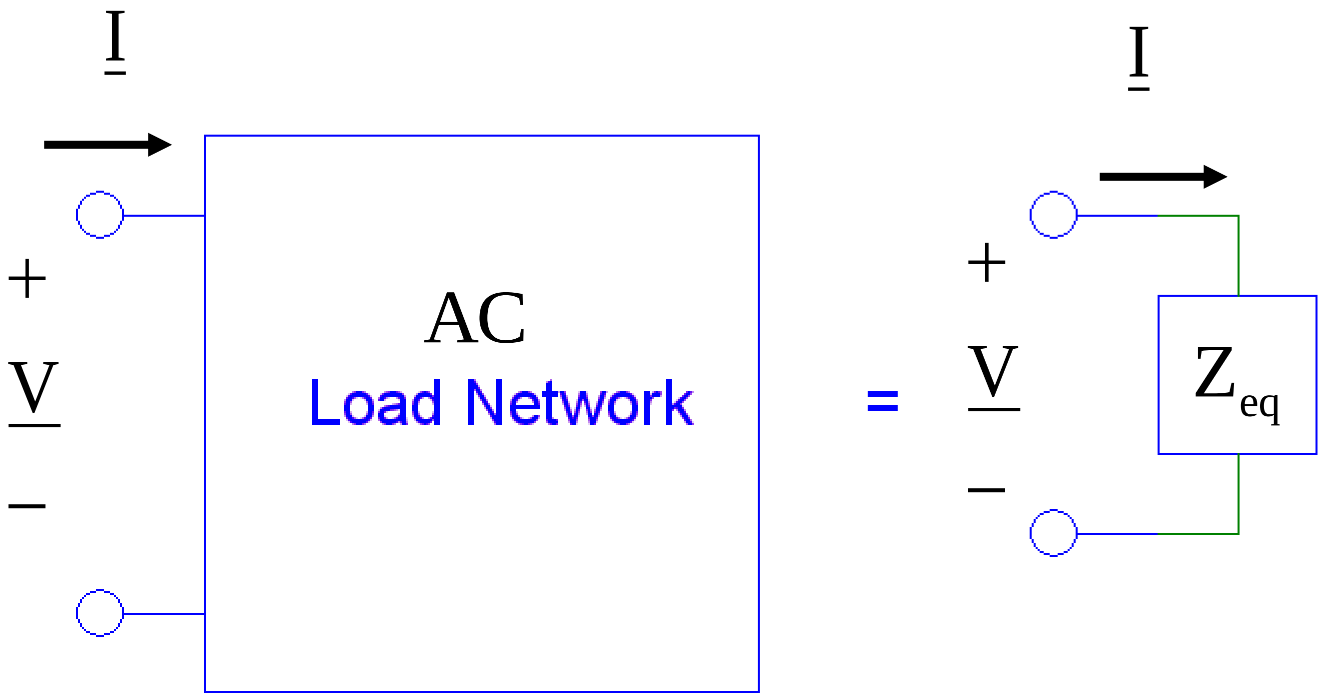
- Now have all the tools we need to solve circuits in the AC Steady State
- Transform to Frequency Domain

$$=> v(t), i(t) \rightarrow \underline{V},\ \underline{I}$$

$$=> R, L, C \rightarrow R, j\omega L, \frac{-j}{\omega C}$$

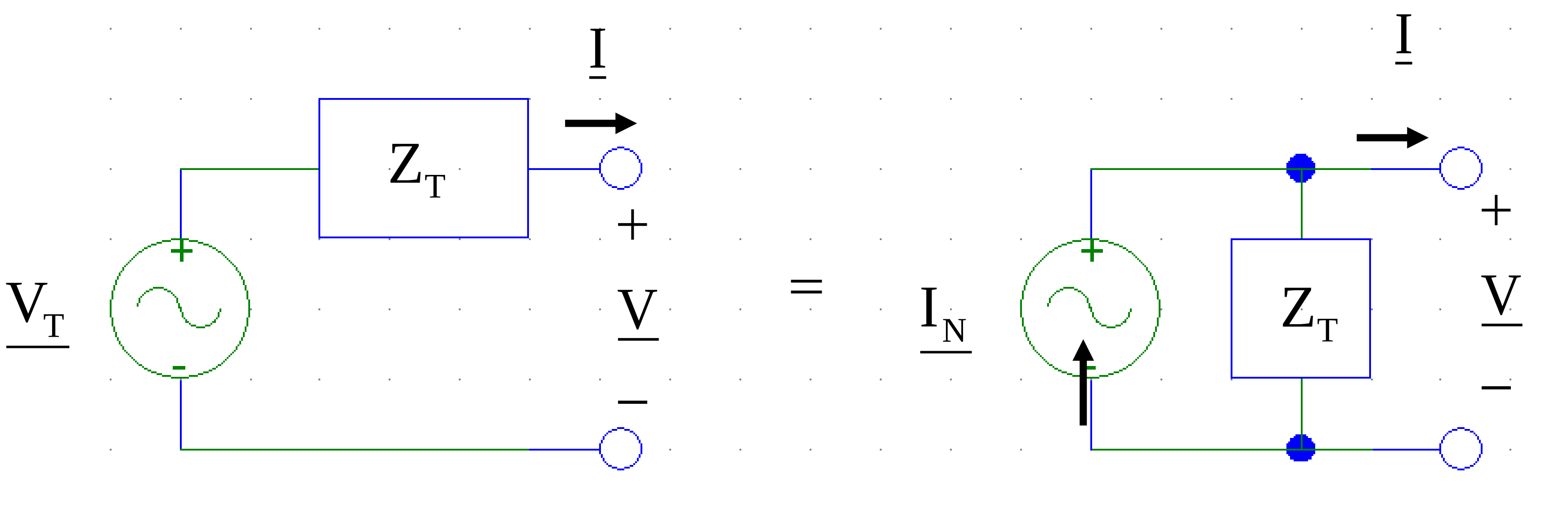
- Find Output $\underline{Y}$ as a Phasor - Unit I Techniques
- Observe Frequency Response

EQUIVALENT IMPEDANCE



$$Z_{eq} = \frac{\underline{V}}{\underline{I}}$$

AC THEVENIN/NORTON



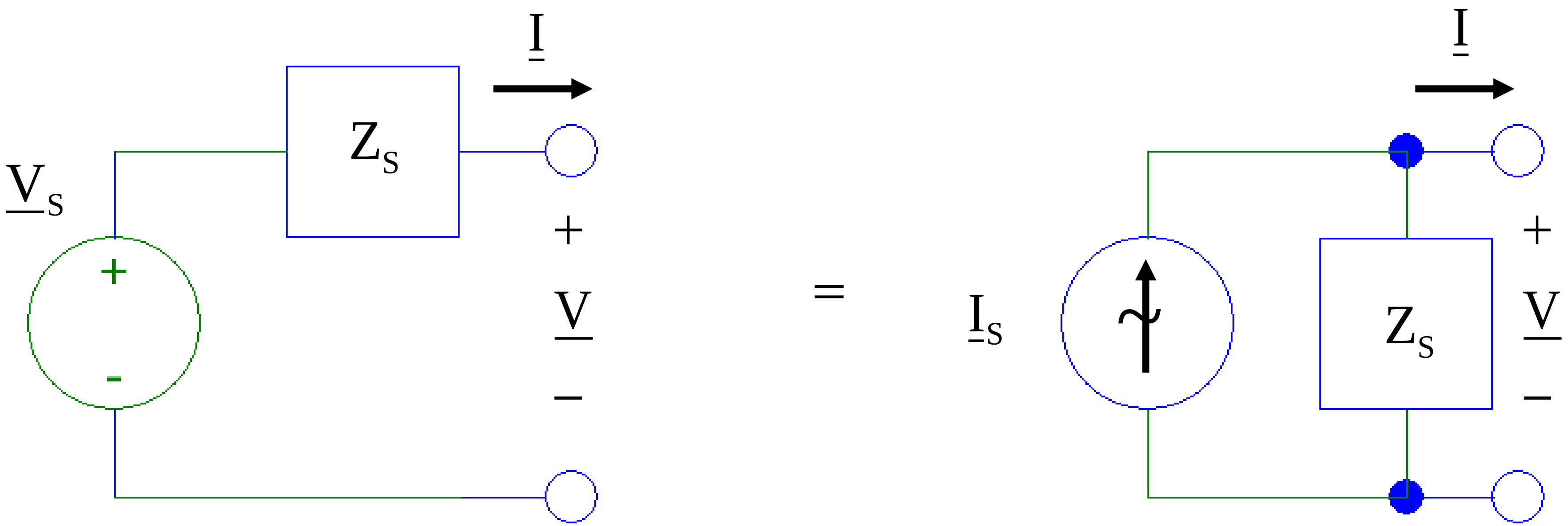
AC Thevenin Circuit

AC Norton Circuit

$$\underline{V}_T = \underline{I}_N \; Z_T$$

$Z_T = Z_{eq}$ of Dead Source Network

AC Source Conversions



$$\underline{I}_S = \frac{\underline{V}_S}{Z_S}$$

AC NODE EQUATIONS

Technique to Solve Any AC Steady State Circuit

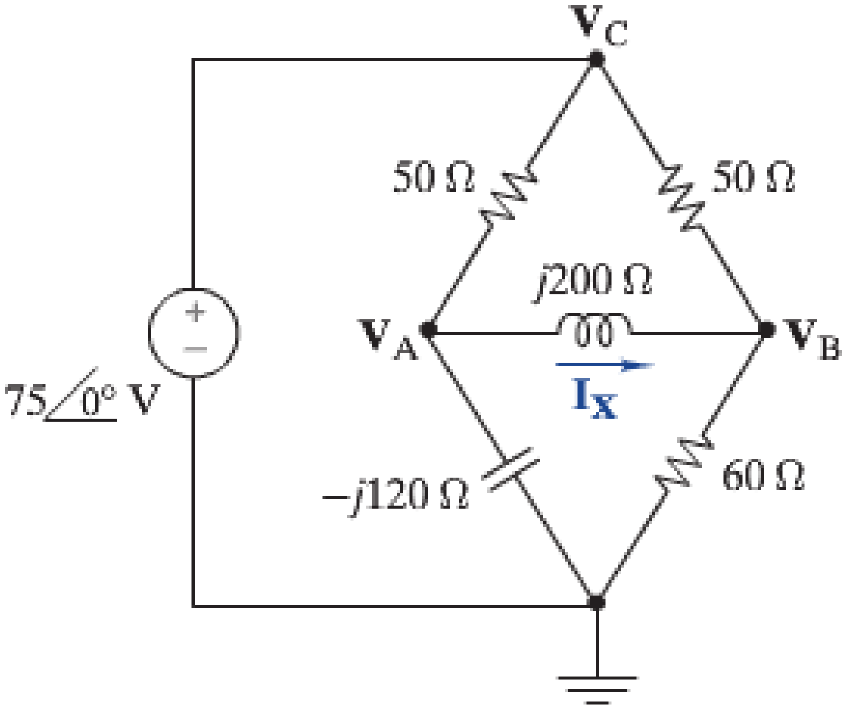
1. Label Unknown Phasor Node Voltages, $\underline{V}_1$, $\underline{V}_2$, etc.
2. # Unknown Nodes = # Nodes - # Voltage Sources – 1 (Reference)
3. Write a KCL at Each Unknown Node
4. Sum of Phasor Currents OUT of Node = 0
5. Relate Phasor Currents to Phasor Node Voltages using Ohm's Law for AC Steady State
6. Will Always Get the Same Number of Equations as Unknowns
7. Solve Complex Linear Equations for $\underline{V}_1$, $\underline{V}_2$, etc.

AC MESH EQUATIONS

Technique to Solve Any AC Steady State Circuit

1. Define All Phasor Mesh Currents
   1. Unknown Mesh Currents ($\underline{I}_1$, $\underline{I}_2$, $\underline{I}_3$, etc.) and Current Sources (Independent and Controlled)
2. Write KVL around Each Unknown Mesh
3. Sum of Phasor Voltages around Mesh = 0
4. Relate Phasor Voltages to Phasor Mesh Currents using Ohm's Law for AC Steady State
5. Will Always get Same Number of Equations as Unknowns
6. Solve Complex Linear Equations for $\underline{I}_1$, $\underline{I}_2$, $\underline{I}_3$, etc.

EXAMPLE PROBLEM

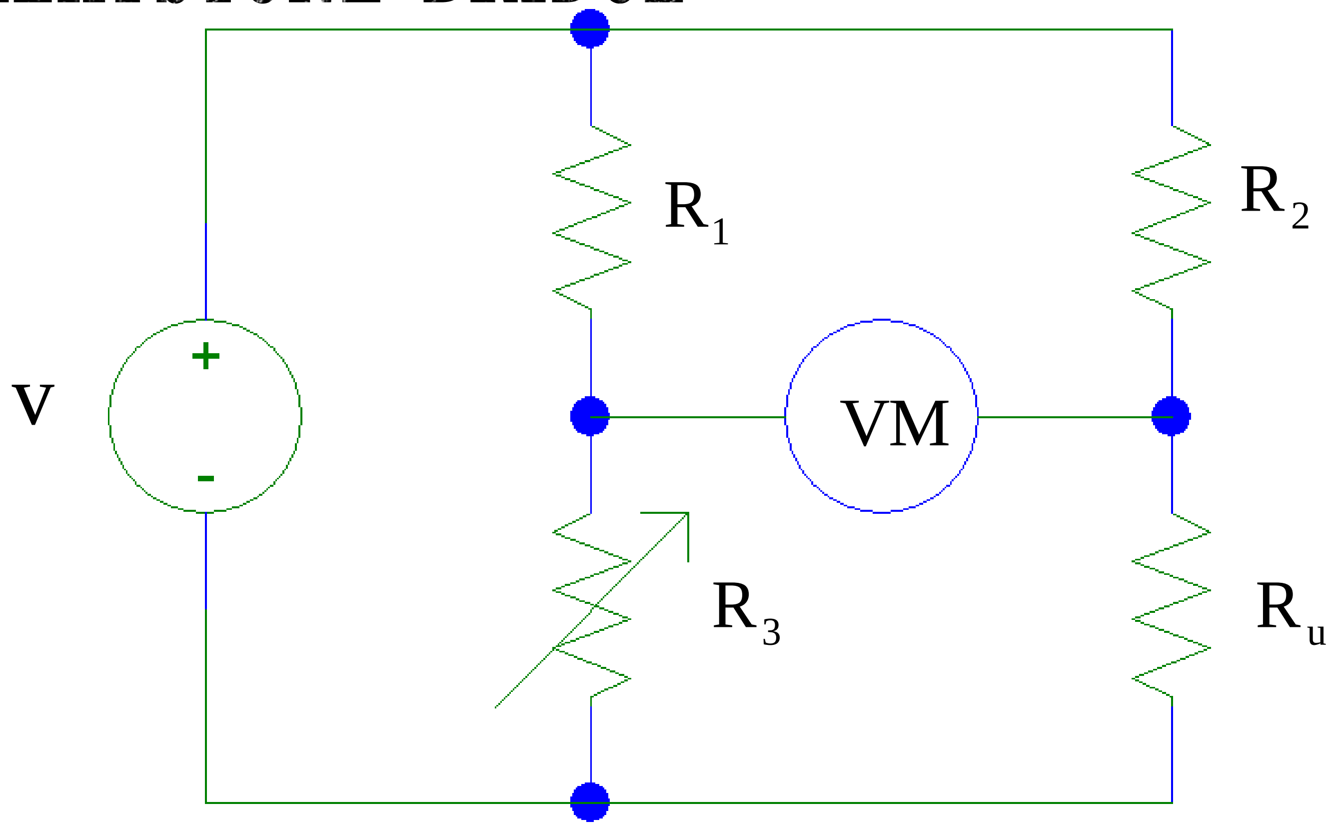


Use node analysis to find the current Ix.

AC BRIDGE CIRCUITS

- AC Bridge Circuits are often used to Accurately Measure R, L and C's:
  - Often called Impedance Bridges
  - Wheatstone Bridge Measures R (AC/DC)
    - Experiment #2b used DC
  - Maxwell Bridge Measures L (AC Only)
  - There are Several Other Types of AC Bridges

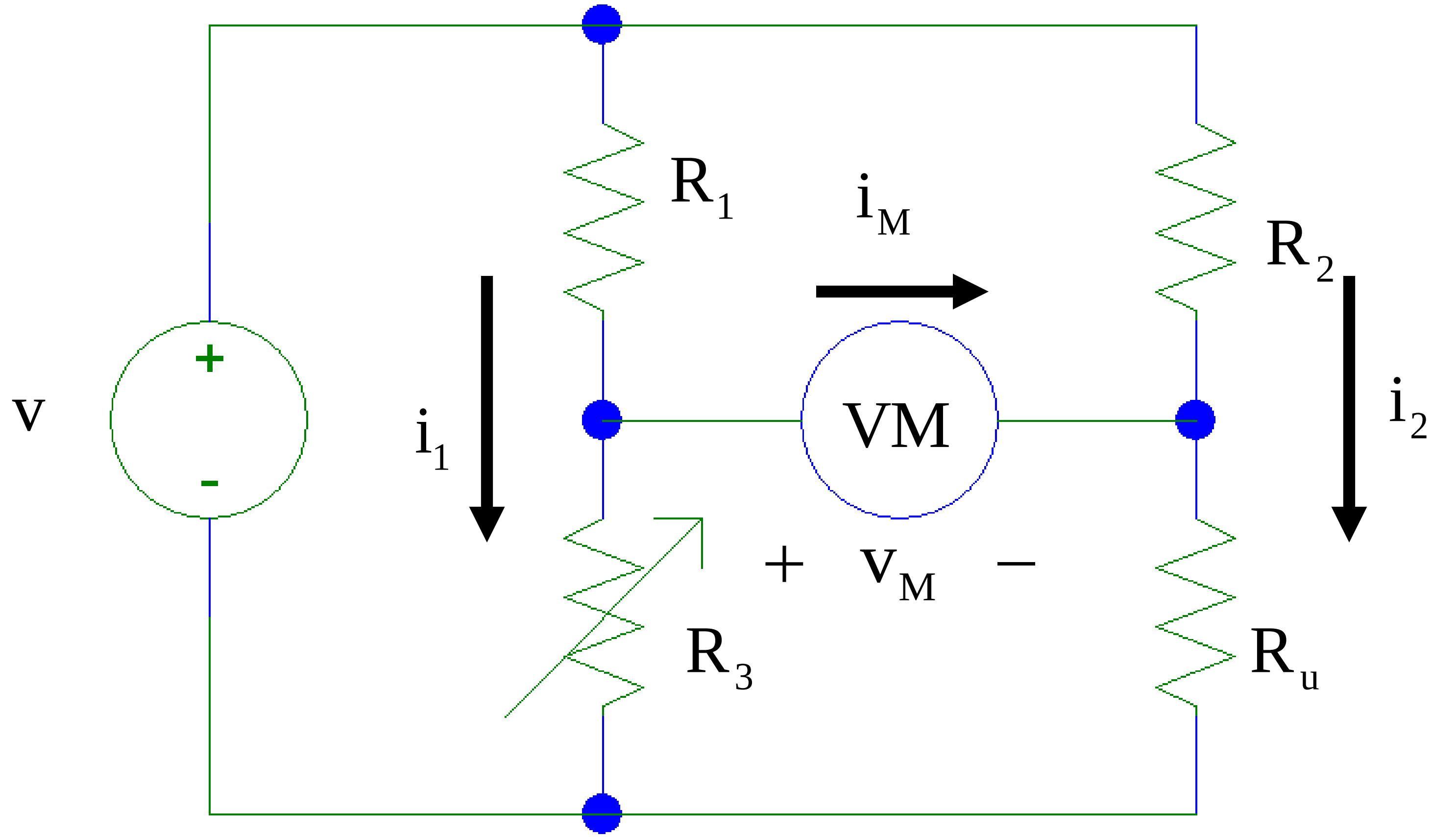
Wheatstone Bridge



Connect Voltmeter across "Bridge"   "Balancing the Bridge"

Adjust R_3 such that VM reads 0   Accurate Measurement of R_u

WHEATSTONE BRIDGE

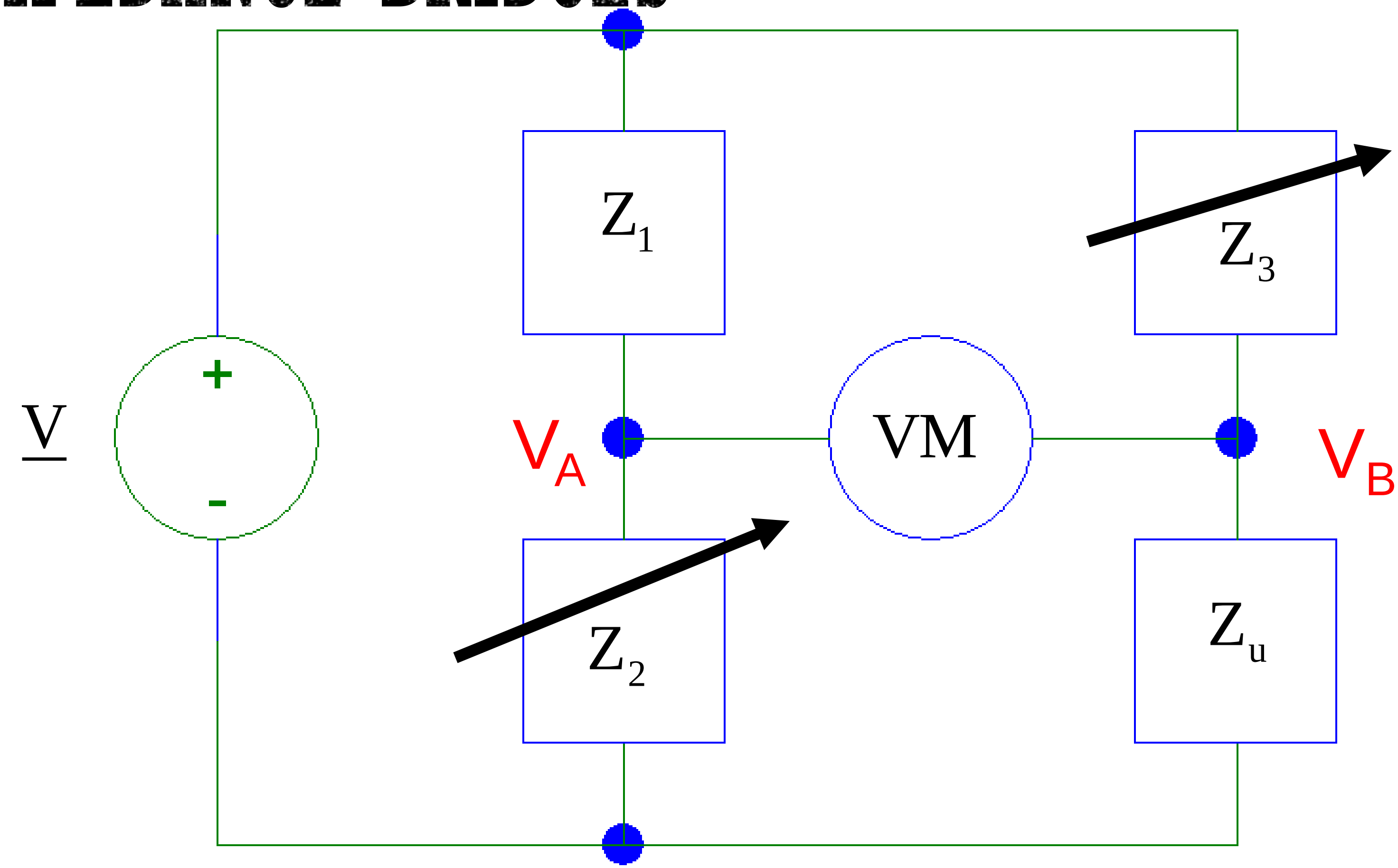


If Balanced:

$$i_M = v_M = 0$$

$$i_1 = i_2 \qquad \Rightarrow R_u = \frac{R_2 R_3}{R_1}$$

IMPEDANCE BRIDGES



Balance the Bridge

Accurate Measurement of Z_u

IMPEDANCE BRIDGES

Parallel voltage dividers

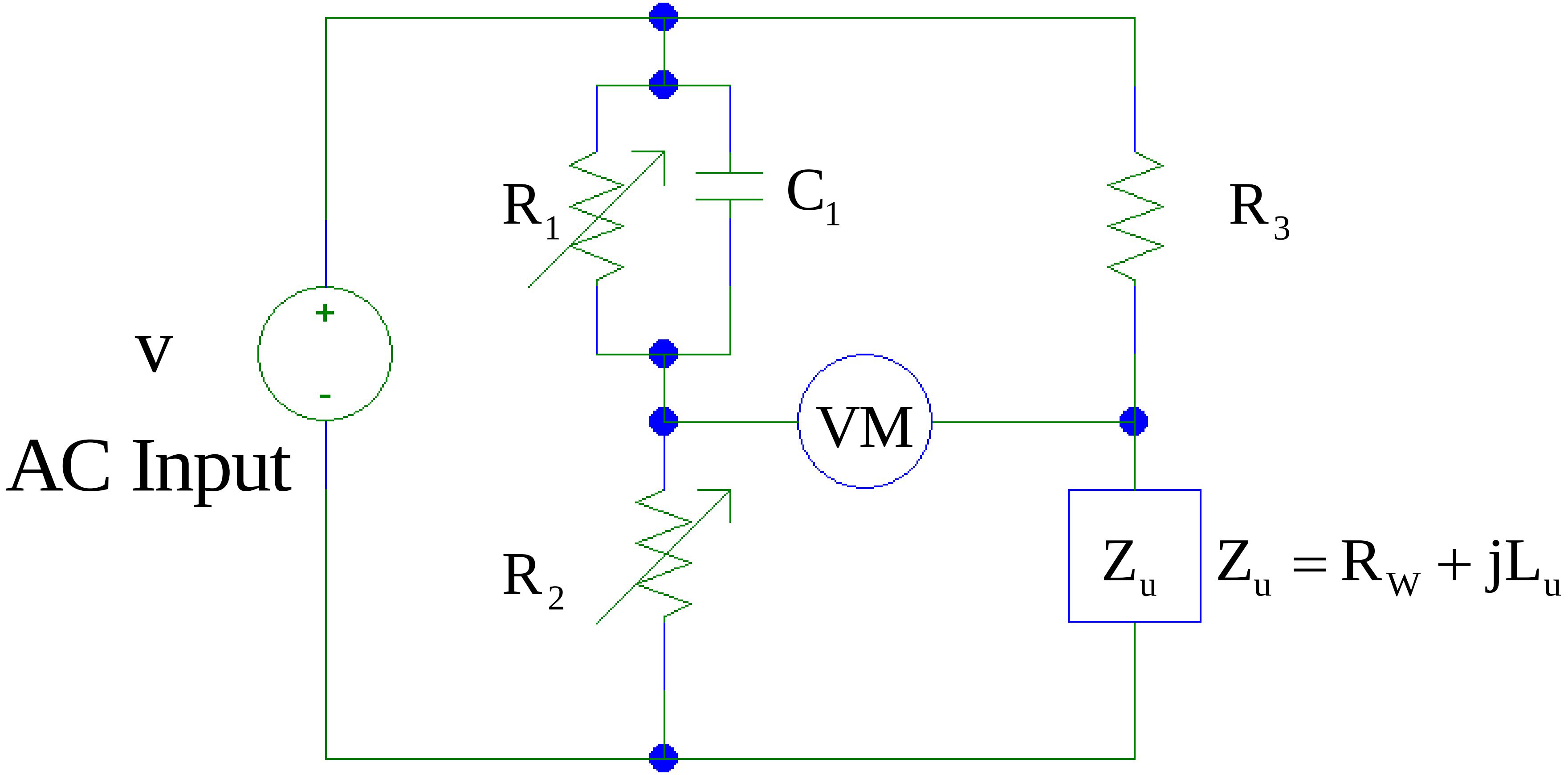
$$V_M = V_A - V_B = \left(\frac{Z_2}{Z_1 + Z_2}\right)\cdot V_s - \left(\frac{Z_u}{Z_3 + Z_u}\right)\cdot V_s$$

$$V_M = \left[\frac{Z_2\cdot Z_3 - Z_1\cdot Z_u}{(Z_1 + Z_2)\cdot(Z_3 + Z_u)}\right]\cdot V_s$$

VM is zero when
$Z_2 Z_3 = Z_1 Z_u$

$$Z_u = \frac{Z_2\cdot Z_3}{Z_1} = R_X + jX_X$$

MAXWELL BRIDGE

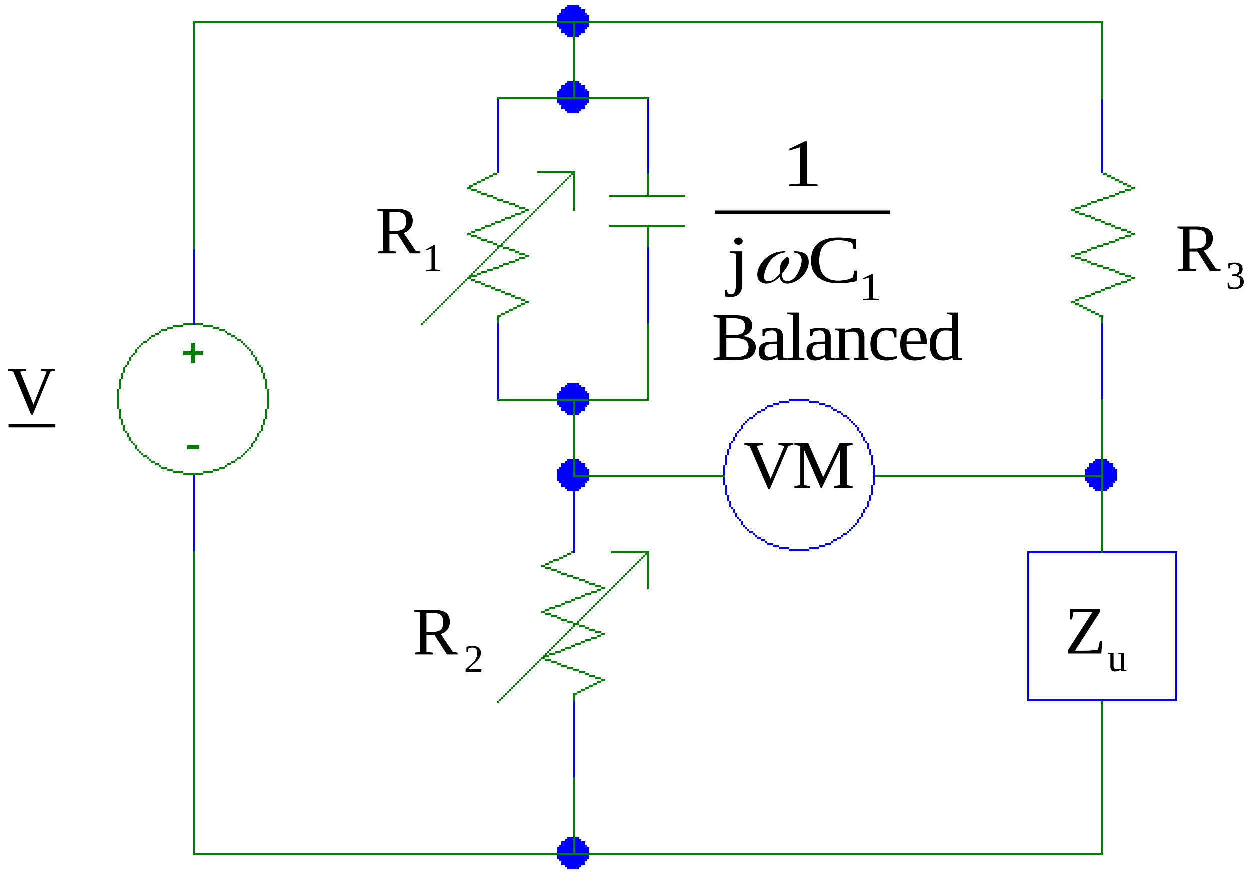


Measures an Inductive Impedance

IMPEDANCE BRIDGES

- Why 2 Variable Impedances?:
  - Must balance Resistance and Reactance of the Circuit
    - Amplitude and Phase of $\underline{I}_m$, $\underline{V}_m$
    - Real and Imaginary Parts of $\underline{I}_m$, $\underline{V}_m$

MAXWELL BRIDGE



$$R_{.w} + j\omega L_{.u} = \frac{R_{.2} \cdot R_{.3}}{R_{.1}} + j\omega C_{.1} \cdot R_{.2} \cdot R_{.3}$$

$$R_{.w} = \frac{R_{.2} \cdot R_{.3}}{R_{.1}}$$

$$L_{.u} = R_{.2} \cdot R_{.3} \cdot C_{.1}$$