ELECTRIC CIRCUITS ECSE-2010

Lecture 21.1

Rensselaer

LECTURE 21.1

- AC Power
  - Average Power
  - Complex Power
  - Real Power
  - Reactive Power
  - Apparent Power
- Power Factor

AC POWER

- **AC Steady State Circuit Analysis:**
  - **Find <u>V</u>, <u>I</u> => Write down v(t), i(t)**
- **Will Now Focus on AC Power:**
  - **RMS Values**
  - **Average Power**
  - **Reactive Power**
  - **Complex Power**
  - **Power Factor**

RMS VALUES

$$\text{Consider } x(t) = |X| \cos\omega t \qquad \text{Periodic Function}$$

$$x_{ave} = \frac{1}{T}\int_0^T |X| \cos\omega t \, dt = 0$$

$$(x^2)_{ave} = \frac{1}{T}\int_0^T |X|^2 \cos^2\omega t \, dt = \frac{|X|^2}{2}$$

$$\text{Define } X_{RMS} = \sqrt{(x^2)_{ave}}$$

$$\text{For Sinusoids: } X_{RMS} = \frac{|X|}{\sqrt{2}} = .707|X|$$

RMS VALUES

$$\text{AC Voltage, } v(t) = |V|\cos(\omega t + \phi)$$

$$\Rightarrow \; V_{RMS} = \frac{|V|}{\sqrt{2}} \text{ Volts (RMS)}$$

$$\text{AC Current, } i(t) = |I|\cos(\omega t + \psi)$$

$$\Rightarrow \; I_{RMS} = \frac{|I|}{\sqrt{2}} \text{ Amps (RMS)}$$

RMS VALUES

- **RMS Value is a Scalar:**
  - **Not a Phasor!**
- **Convenient Way to Describe Current and Voltage in AC Steady State:**

RMS VALUES

- **Convenient Way to Calculate Power in AC Steady State:**
- **All Residential AC Electricity is Described in Terms of RMS Values:**
- **120 V = 120 Volts, RMS**

$$\left|V\right| = \sqrt{2}V_{RMS} \approx 170 \text{ Volts (0 to Peak)}$$

Rensselaer

REACTIVE POWER

Recall:

$$p(t) = |I|^2 \frac{|Z|}{2}\left(\cos\theta + \cos(2\omega t - \theta)\right)$$

$$|Z| = \frac{V_m}{I_m} = \frac{V_{RMS}}{I_{RMS}}; \quad I_{RMS} = \frac{I_m}{\sqrt{2}}$$

$$\Rightarrow \; p(t) = V_{RMS}\ I_{RMS}\left(\cos\theta + \cos\,(2\omega t - \theta)\right)$$

REACTIVE POWER

Use a different Trignometric Identity:

$$\cos(2\omega t-\theta)=\cos 2\omega t\cos\theta+\sin 2\omega t\sin\theta$$

$$\Rightarrow\ p(t)=V_{RMS}\ I_{RMS}\left(\cos\theta+\cos\left(2\omega t-\theta\right)\right)$$

$$=V_{RMS}I_{RMS}\cos\theta\ (1+\cos 2\omega t)+V_{RMS}I_{RMS}\sin\theta\ \sin 2\omega t$$

$$P=\frac{1}{T}\int_0^T p(t)\,dt=V_{RMS}I_{RMS}\cos\theta+0+0$$

=> Same result as before

REACTIVE POWER

$$\text{Define P} = \text{"Real Power"} = V_{RMS} I_{RMS} \cos\theta$$

P is Measured in Watts

$$\text{Define Q} = \text{"Reactive Power"} = V_{RMS} I_{RMS} \sin\theta$$

Q is Measured in VAR's

(Volt-Amperes-Reactive)

REACTIVE POWER

- Q is a Measure of the Rate of Change of Energy Stored in the Reactive Elements (L, C):
  - Power companies must worry about Q since they supplied this energy
  - Supplied Q over their Lines => Real Cost
  - Power companies want customers to have Low Q

REACTIVE POWER

- **Q is a Measure of the Rate of Change of Energy Stored in the Reactive Elements (L, C):**
  - **Big issue for a large user - like Rensselaer**
  - **Fans all use Motors - Motors are Inductive**
  - **Large users are charged for Q, not just P**

Rensselaer

REACTIVE POWER

$$\text{Impedance Triangle: } Z = |Z|\underline{/\theta}$$

$$R(\omega) = |Z|\cos\theta \ ; \ X(\omega) = |Z|\sin\theta$$

$$=> \quad \text{Real Power} = P = V_{RMS} I_{RMS} \cos\theta$$
$$= I_{RMS}^2 R(\omega) \quad [\text{Watts}]$$

$$=> \quad \text{Reactive Power} = Q = V_{RMS} I_{RMS} \sin\theta$$
$$= I_{RMS}^2 X(\omega) \quad [\text{VAR's}]$$

REACTIVE POWER

$$P = I_{RMS}^2 |Z| \cos\theta$$

$$= I_{RMS}^2 \ R(\omega)$$

$$= V_{RMS} I_{RMS} \cos\theta$$

$\left\{ \begin{array}{l} \text{Equivalent ways of} \\ \text{expressing Real Power} \end{array} \right.$

[Watts]

$$Q = I_{RMS}^2 |Z| \sin\theta$$

$$= I_{RMS}^2 \ X(\omega)$$

$$= V_{RMS} I_{RMS} \sin\theta$$

$\left\{ \begin{array}{l} \text{Equivalent ways of} \\ \text{expressing Reactive Power} \end{array} \right.$

[VAR's]

REACTIVE POWER

- **Notes on Reactive Power:**
  - **Real Power = P is always $\geq$ 0**
  - **Reactive Power = Q can be $\geq$ 0 or $\leq$ 0**
  - **For Inductive Load, X > 0 => Q > 0**
  - **For Capacitive Load, X < 0 => Q < 0**

COMPLEX POWER

$$\text{Define "Complex Power"} = \underline{S} = P + jQ$$

$\underline{S}$ is a Complex Number, but not a Phasor

$\underline{S}$ is a Convenient Way to Keep Track of P and Q

$$\text{Real}\{\underline{S}\} = P = V_{RMS} I_{RMS} \cos\theta \Rightarrow \text{Watts}$$

$$\text{Imag}\{\underline{S}\} = Q = V_{RMS} I_{RMS} \sin\theta \Rightarrow \text{VAR's}$$

COMPLEX POWER

- **Significance of $\underline{S}$ = Complex Power:**
  - **For Several Loads => $\underline{S}_{Total} = \underline{S}_1 + \underline{S}_2 + \underline{S}_3 + \ldots$**
  - **$\underline{S}_{Total} = (P_1 + P_2 + \ldots) + j\,(Q_1 + Q_2 + \ldots)$**
  - **Regardless of how loads are connected! Just Add P's and Q's**

APPARENT POWER

$$\text{Magnitude of } \underline{S} = |S| = \sqrt{P^2 + Q^2} = V_{RMS} I_{RMS}$$

$$|S| = \text{"Apparent Power"} \Rightarrow [\text{Volt-Amperes}]$$

$$|S| = \text{Product of } V_{RMS} \text{ x } I_{RMS} \text{ at Terminals}$$

Rensselaer

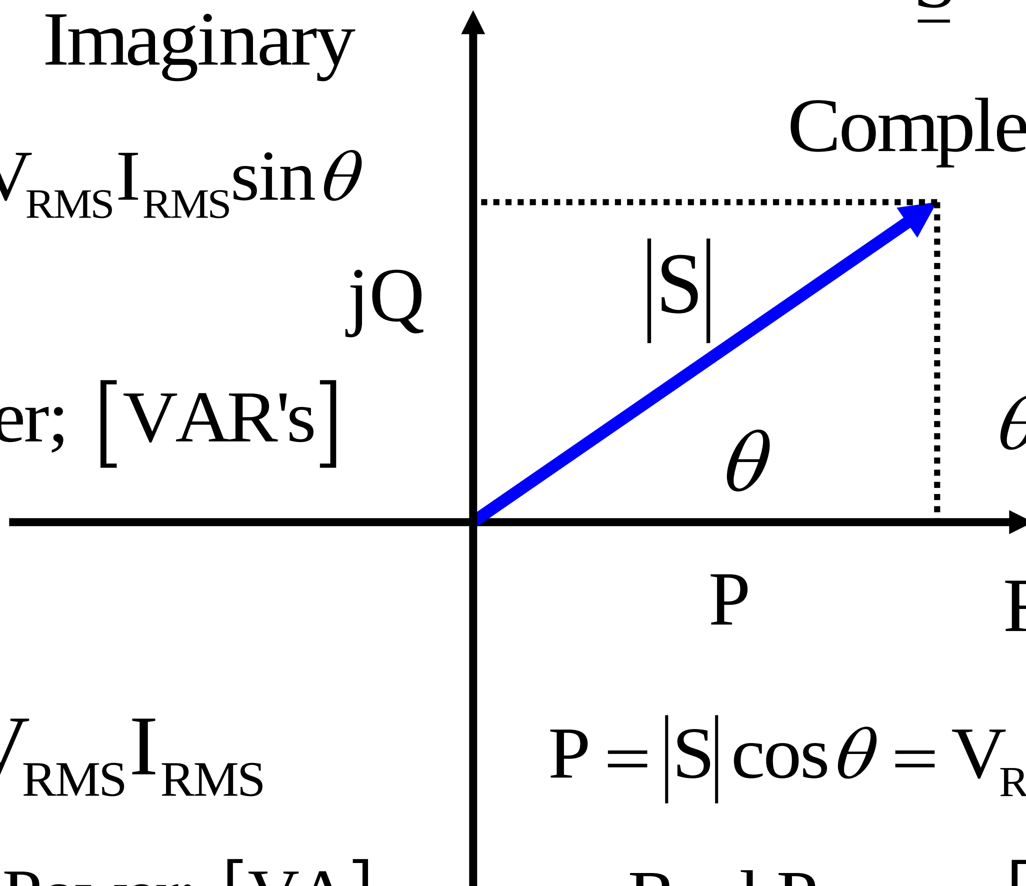
POWER TRIANGLE

$$\underline{S} = P + jQ$$

Complex Power, $\underline{S}$

$$Q = |S|\sin\theta = V_{RMS} I_{RMS} \sin\theta$$

Reactive Power; [VAR's]



θ = Angle of Z

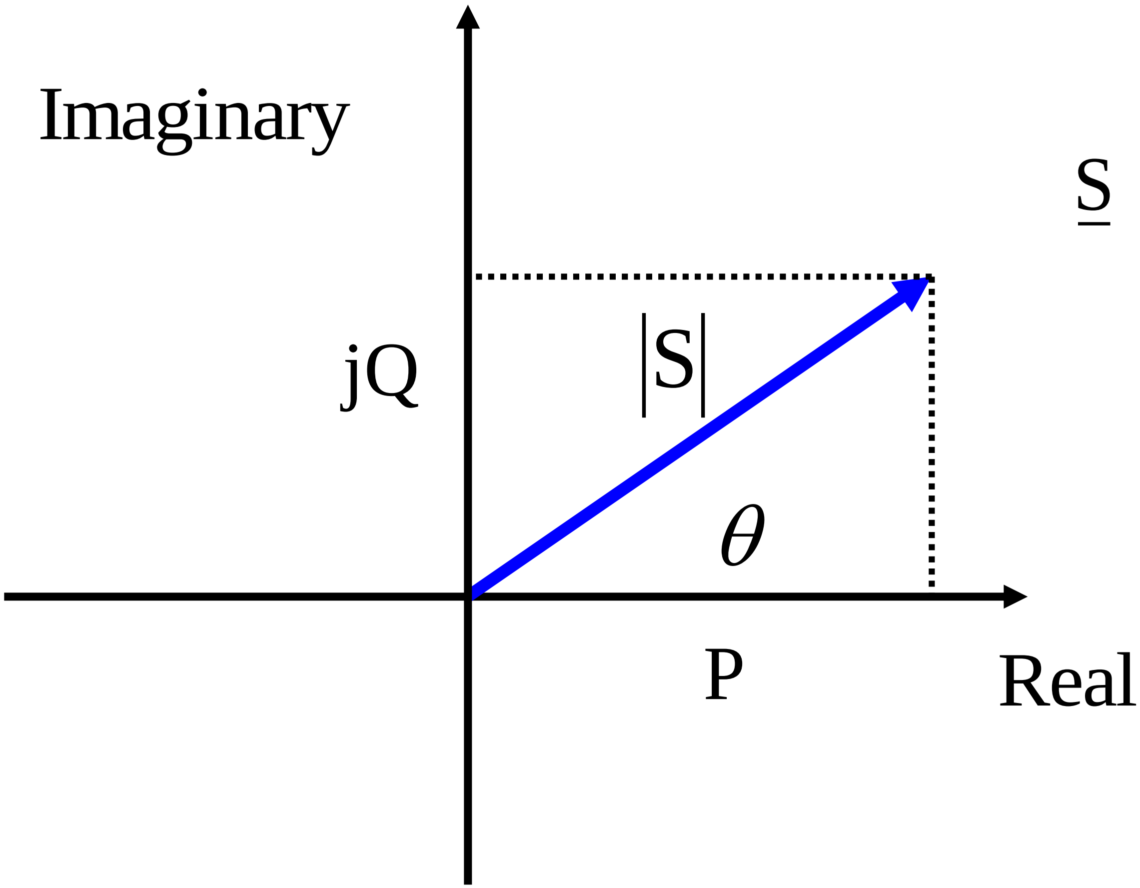
$$|S| = V_{RMS} I_{RMS}$$

Apparent Power; [VA]

$$P = |S|\cos\theta = V_{RMS} I_{RMS} \cos\theta$$

Real Power; [Watts]

POWER FACTOR



$$\cos\theta = \frac{P}{|S|} = \frac{\text{Real Power}}{\text{Apparent Power}} = \text{Power Factor} = \text{pf}$$

POWER FACTOR

$$\text{For Inductive Loads, } \theta > 0; \;\; \cos\theta > 0$$

$$\text{For Capacitive Loads, } \theta < 0; \;\; \cos\theta > 0$$

Need a Way to Distinguish

$$\underline{\mathrm{I}} = \frac{\underline{\mathrm{V}}}{\underline{\mathrm{Z}}} = \frac{|\mathrm{V}|\underline{/\phi}}{|\mathrm{Z}|\underline{\theta}} = \frac{|\mathrm{V}|}{|\mathrm{Z}|}\underline{/\phi - \theta}$$

$$\text{If } \theta > 0; \Rightarrow \text{Lagging Power Factor } (\underline{\mathrm{I}} \text{ lags } \underline{\mathrm{V}})$$

$$\text{If } \theta < 0; \Rightarrow \text{Leading Power Factor } (\underline{\mathrm{I}} \text{ leads } \underline{\mathrm{V}})$$

Rensselaer

POWER FACTOR

Power Factor:

Define $\text{pf} = \cos\theta; \quad 0 \le \text{pf} \le 1$

Must distinguish between $\theta \ge 0, \ \theta \le 0$:

$\theta \ge 0; \ X \ge 0; \ Q \ge 0; \ \underline{I}$ lags $\underline{V}$; lagging pf

$\theta \le 0; \ X \le 0; \ Q \le 0; \ \underline{I}$ leads $\underline{V}$; leading pf

e.g: pf $= .8$ lagging => Inductive Load

pf $= .8$ leading => Capacitive Load