ELECTRIC CIRCUITS ECSE-2010

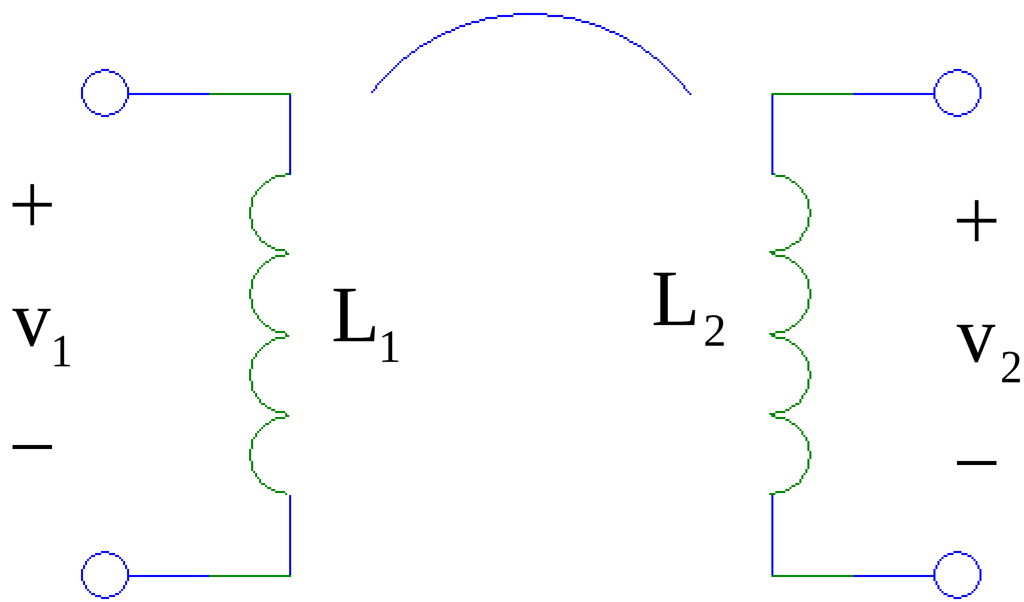
Lecture 22.1

Rensselaer

LECTURE 22.1

- Coupled Inductors
- Ideal Transformer
- Transformer Circuit
- Power Transfer
- Impedance Matching
- Mutual Inductance (Tee Model)

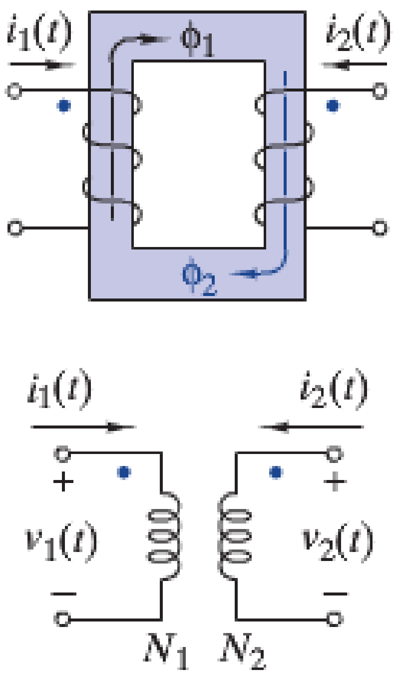
COUPLED INDUCTORS



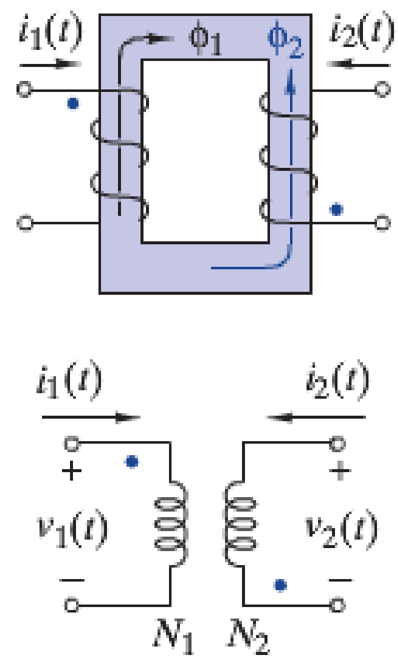
Magnetic Fields of Inductors Link Each Other

If Magnetic Field is Time Varying => Creates Voltages

COUPLED INDUCTORS



(a) Additive



(b) Subtractive

$$\text{Inductor 1: } v_1(t) = L_1 \frac{di_1(t)}{dt} \pm M \frac{di_2(t)}{dt}$$

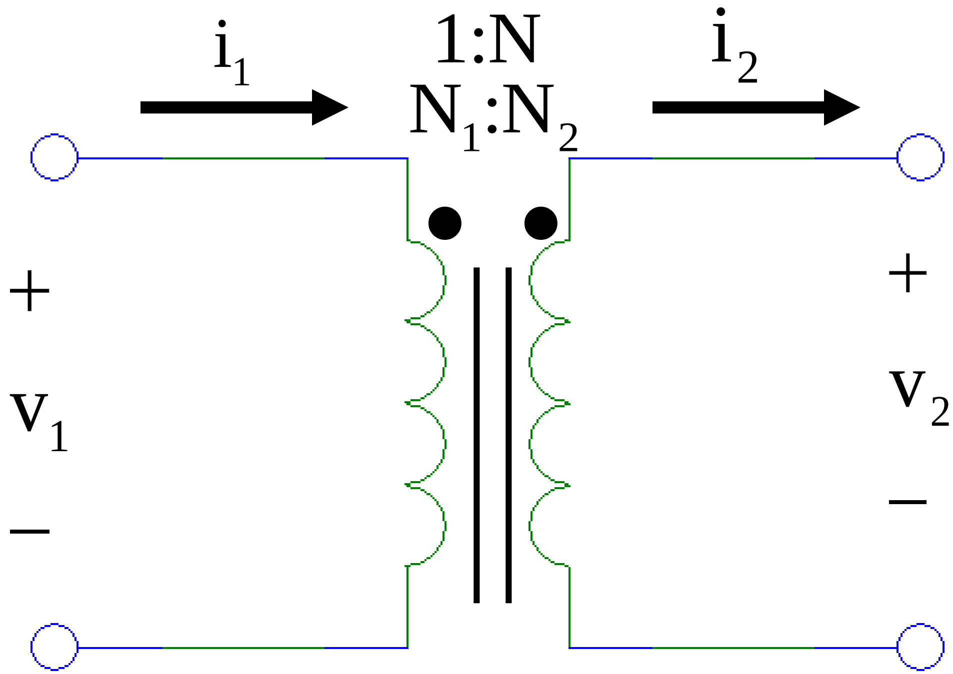
$$\text{Inductor 2: } v_2(t) = \pm M \frac{di_1(t)}{dt} + L_2 \frac{di_2(t)}{dt}$$

Thomas, Rosa, Toussaint, The Analysis and Design of Linear Circuits, 7th edition.

TRANSFORMERS

- **Transformers are Easy to Make:**
- **Transformers Can Be Used to Easily Change AC Voltages:**
- **One of the Key Reasons why World Runs on AC, not DC:**
- **Can Model Most "Real" Transformers with "Ideal" Transformers:**

IDEAL TRANSFORMER



Primary Winding N_1 Turns

Secondary Winding N_2 Turns

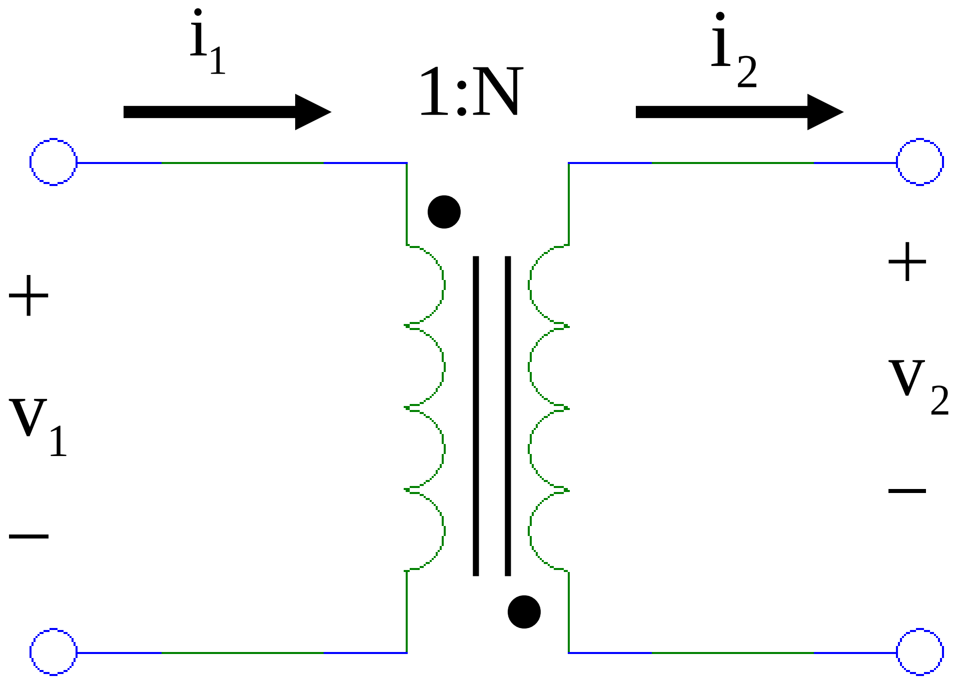
$$N = \frac{N_2}{N_1}$$

Turns Ratio

Dots are on Same Side or Opposite Sides

Dots indicate direction of windings

IDEAL TRANSFORMER



$$N = \frac{N_2}{N_1}$$

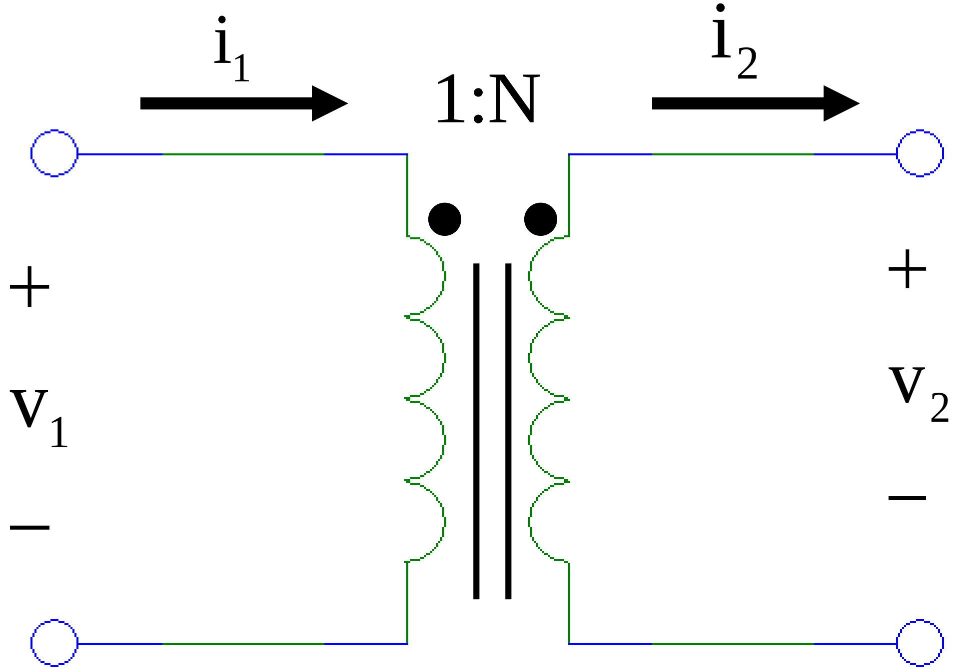
Turns Ratio

Dots on Opposite Sides

Primary Winding N_1 Turns

Secondary Winding N_2 Turns

IDEAL TRANSFORMER

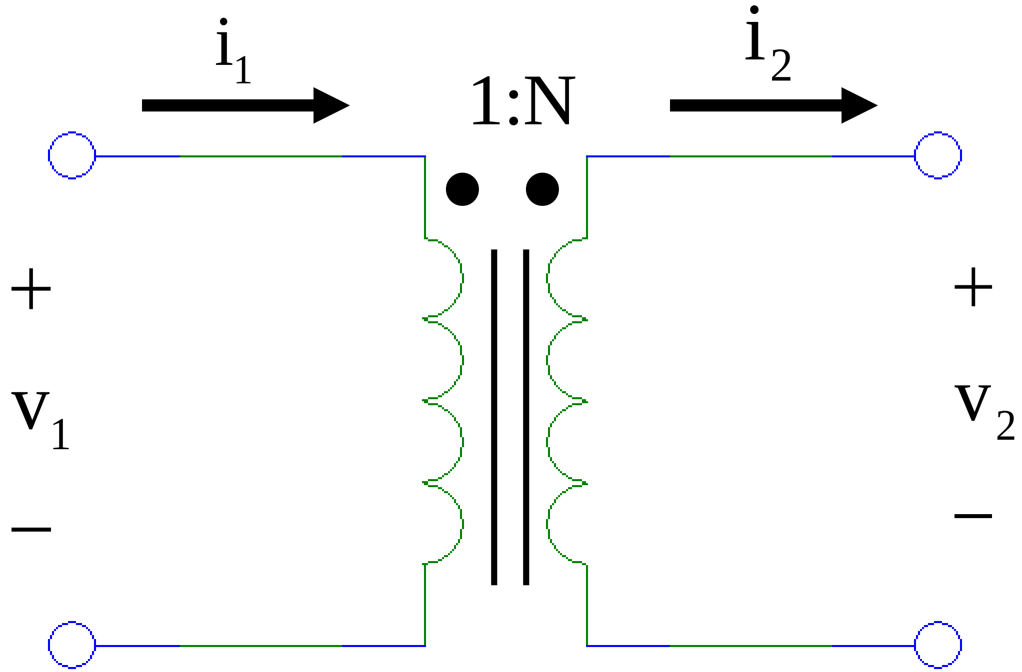


For an Ideal Tranformer

there is No Energy Loss from

Primary to Secondary

IDEAL TRANSFORMER



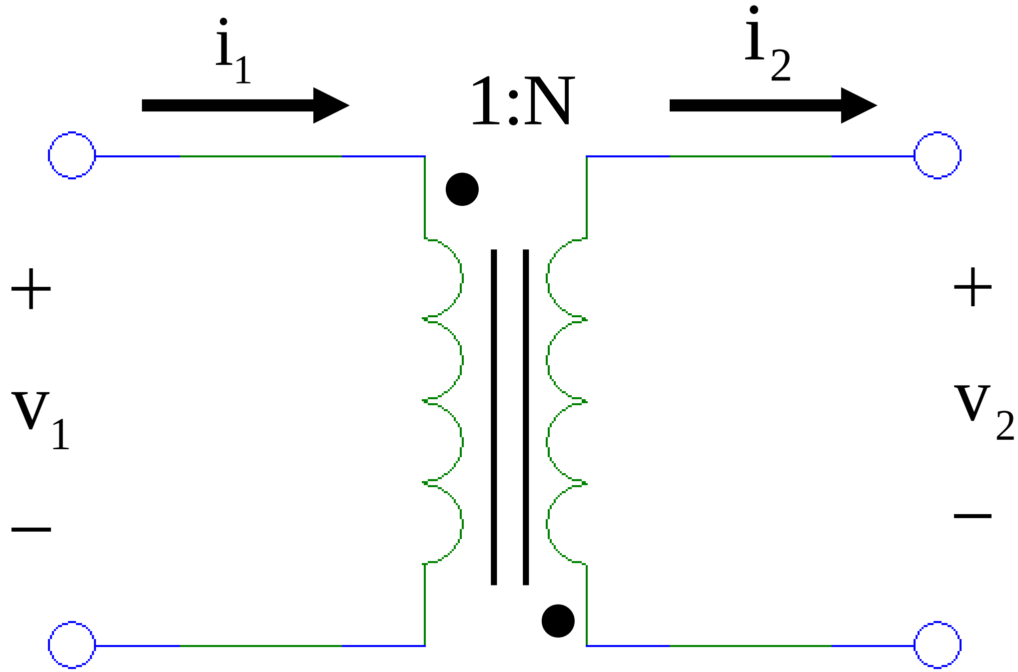
Dots on Same Sides

Transformer "Rules":

$$v_2 = N\, v_1 \qquad i_2 = \frac{i_1}{N}$$

$$p_2 = v_2 i_2 = p_1 = v_1 i_1 \Rightarrow \text{No Energy Loss}$$

IDEAL TRANSFORMER



Dots

on Opposite

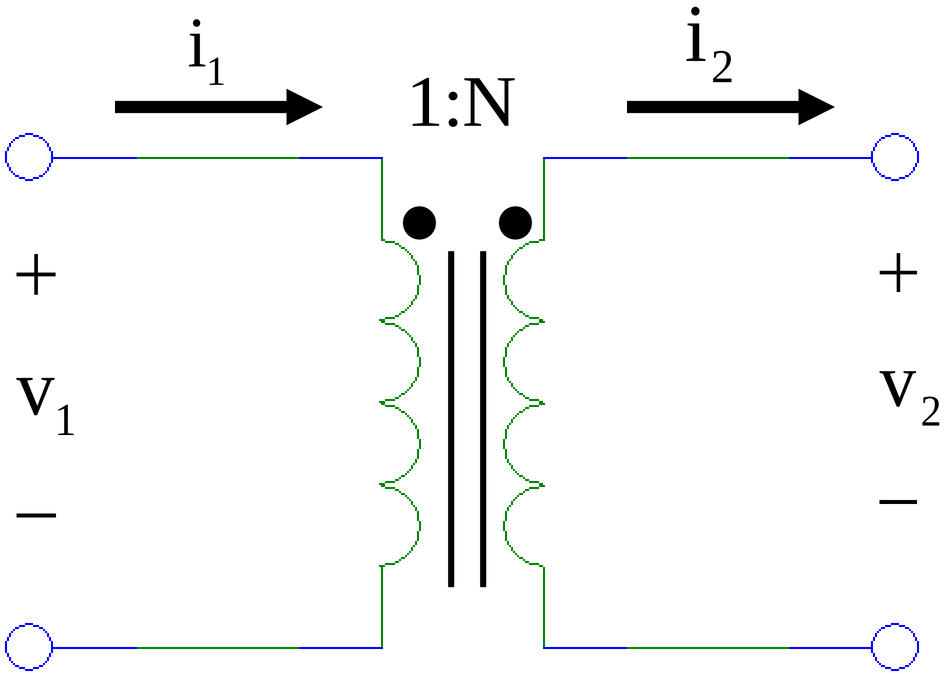
Sides

Transformer "Rules":

$$v_2 = -N\, v_1 \qquad i_2 = -\frac{i_1}{N}$$

OR: Treat N as a Negative Number

IDEAL TRANSFORMER

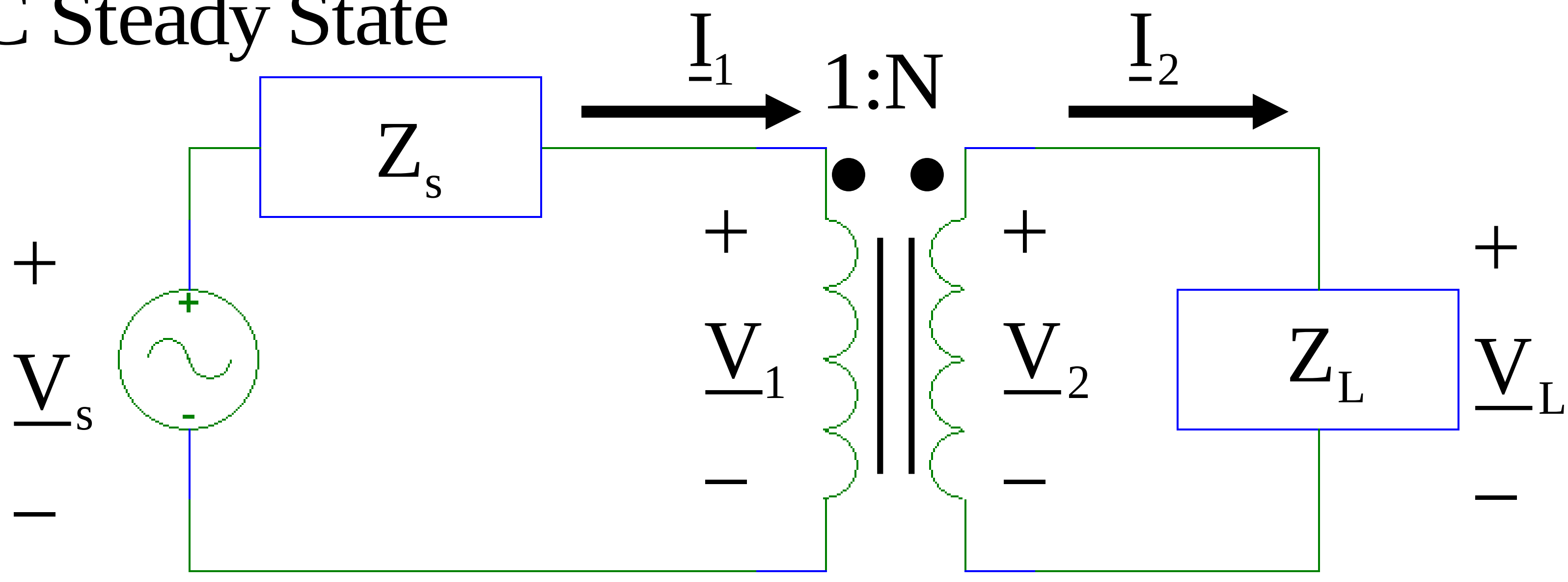


Uses for Transformers:

"Step Up" or "Step Down" Voltages
Isolate Load from Source
Impedance Matching

TRANSFORMER CIRCUIT

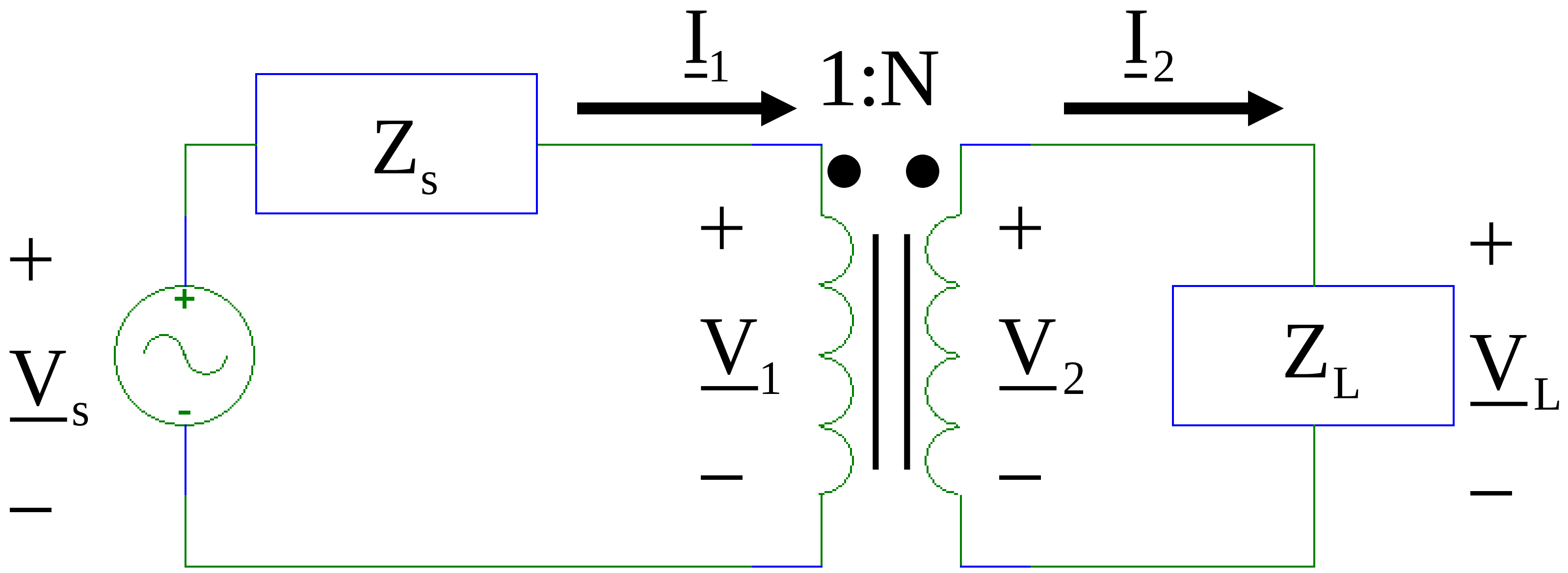
AC Steady State



To do Circuit Analysis, Want to Replace this Circuit with an Equivalent Circuit that has No Transformer!

Can Then Use AC Steady State Circuit Analysis

TRANSFORMER CIRCUIT



2 Choices for the Equivalent Circuit

Refer Secondary Circuit to the Primary

OR

Refer Primary Circuit to the Secondary

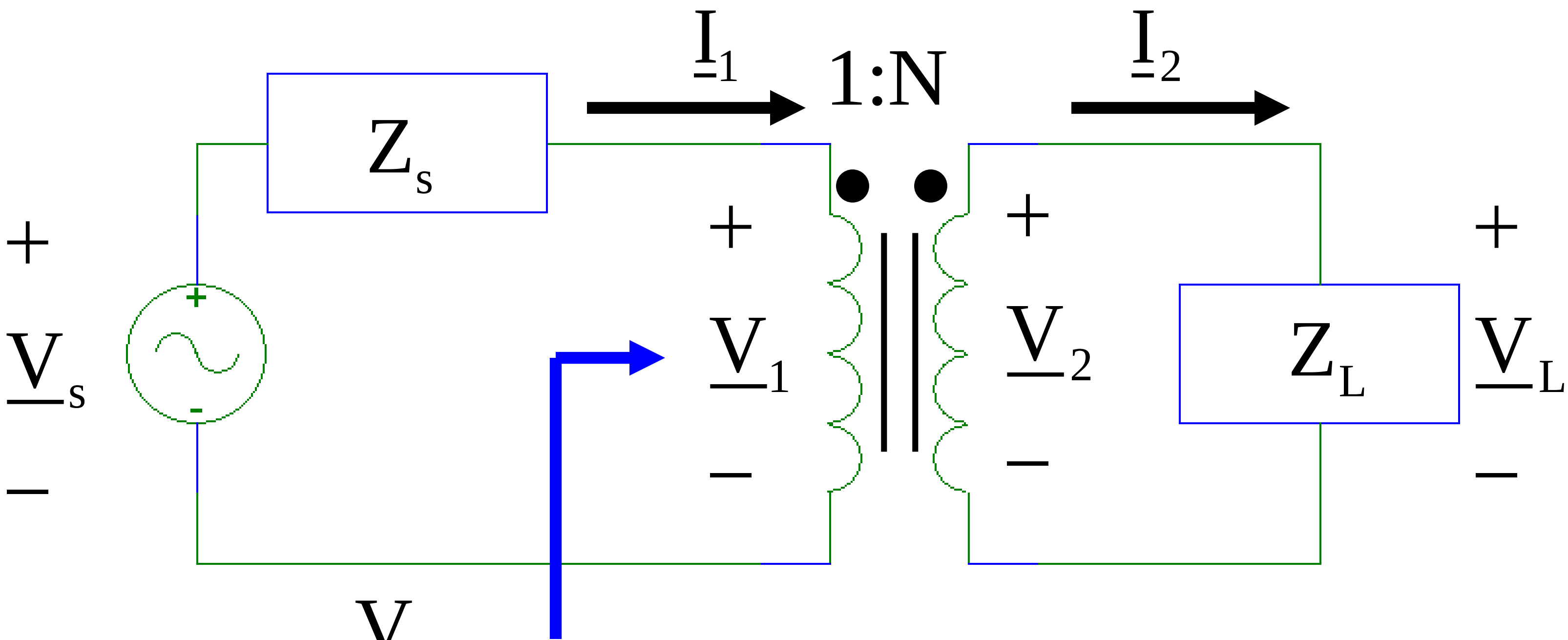
IDEAL TRANSFORMERS

- **Circuit Analysis with Transformers:**
  - **Want to Replace Ideal Transformer with a Circuit that Does NOT Contain an Ideal Transformer so We Can Use our Regular AC Steady State Techniques for Circuit Analysis**

IDEAL TRANSFORMERS

- **Two Choices for Finding the Equivalent Circuit:**
  - **Refer Secondary Circuit to Primary**
  - **Refer Primary Circuit to Secondary**
  - **Can Use Either; Choose the Referral Method that is Easiest for the Particular Problem**

REFERRAL TO PRIMARY

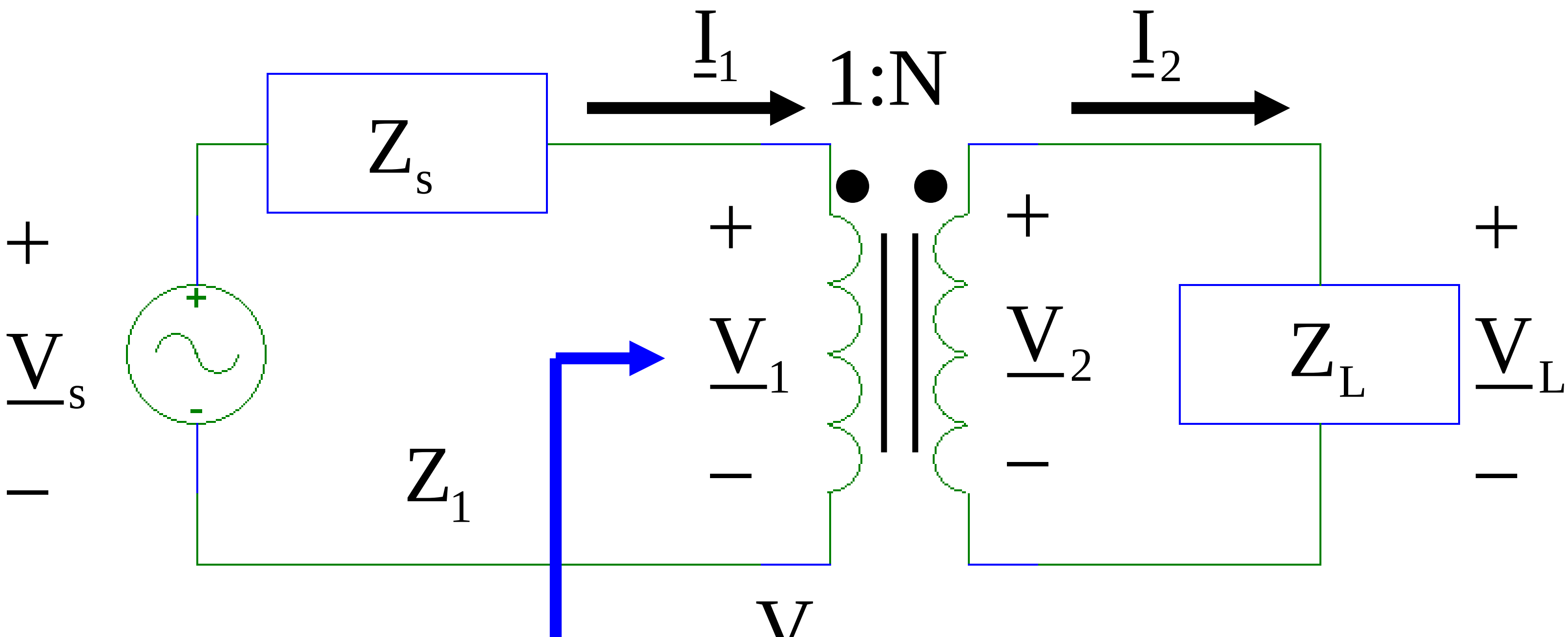


$$Z_1 = \frac{\underline{V}_1}{\underline{I}_1}$$

Find Equivalent Impedance Seen

Looking IN to the Primary

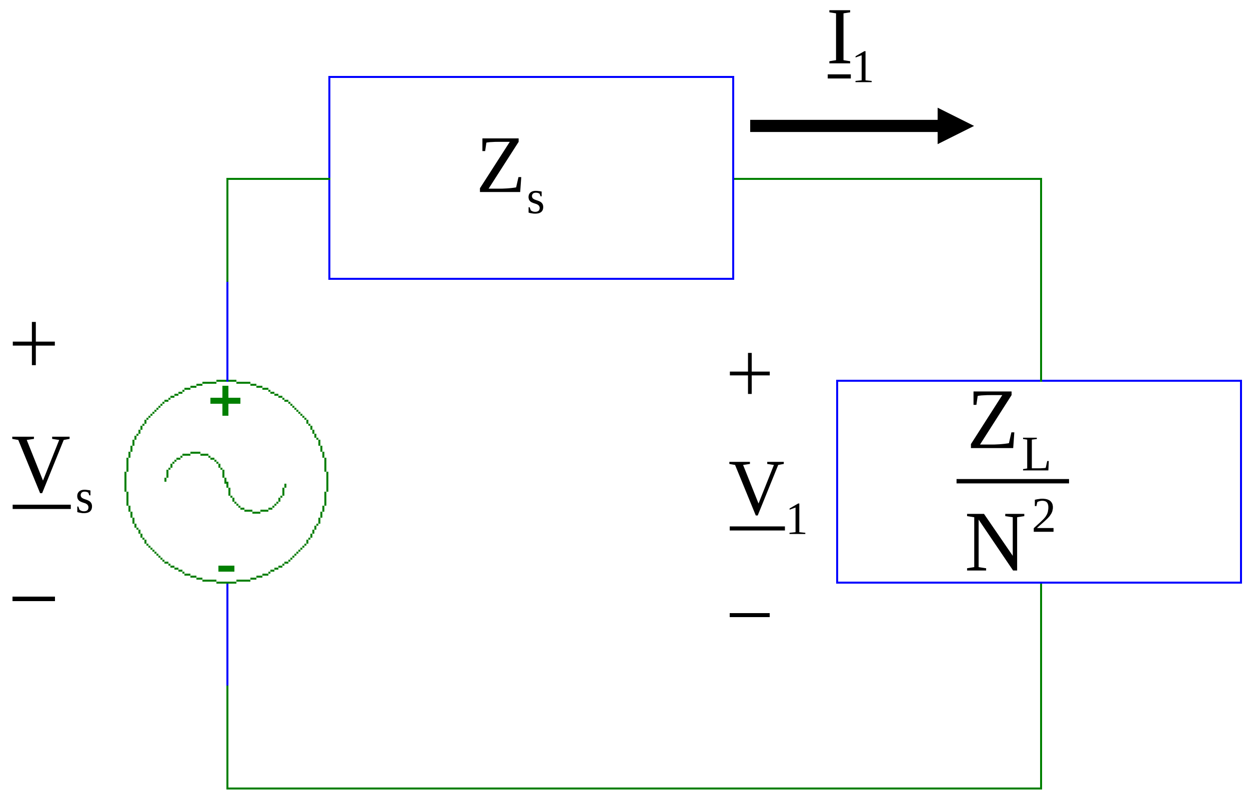
REFERRAL TO PRIMARY



$$\underline{V}_1 = \frac{\underline{V}_2}{N} \qquad \underline{I}_1 = N\,\underline{I}_2$$

$$Z_1 = \frac{\underline{V}_1}{\underline{I}_1} = \frac{\underline{V}_2 / N}{N\,\underline{I}_2} = \frac{\underline{V}_2}{\underline{I}_2}\frac{1}{N^2} = \frac{Z_L}{N^2}$$

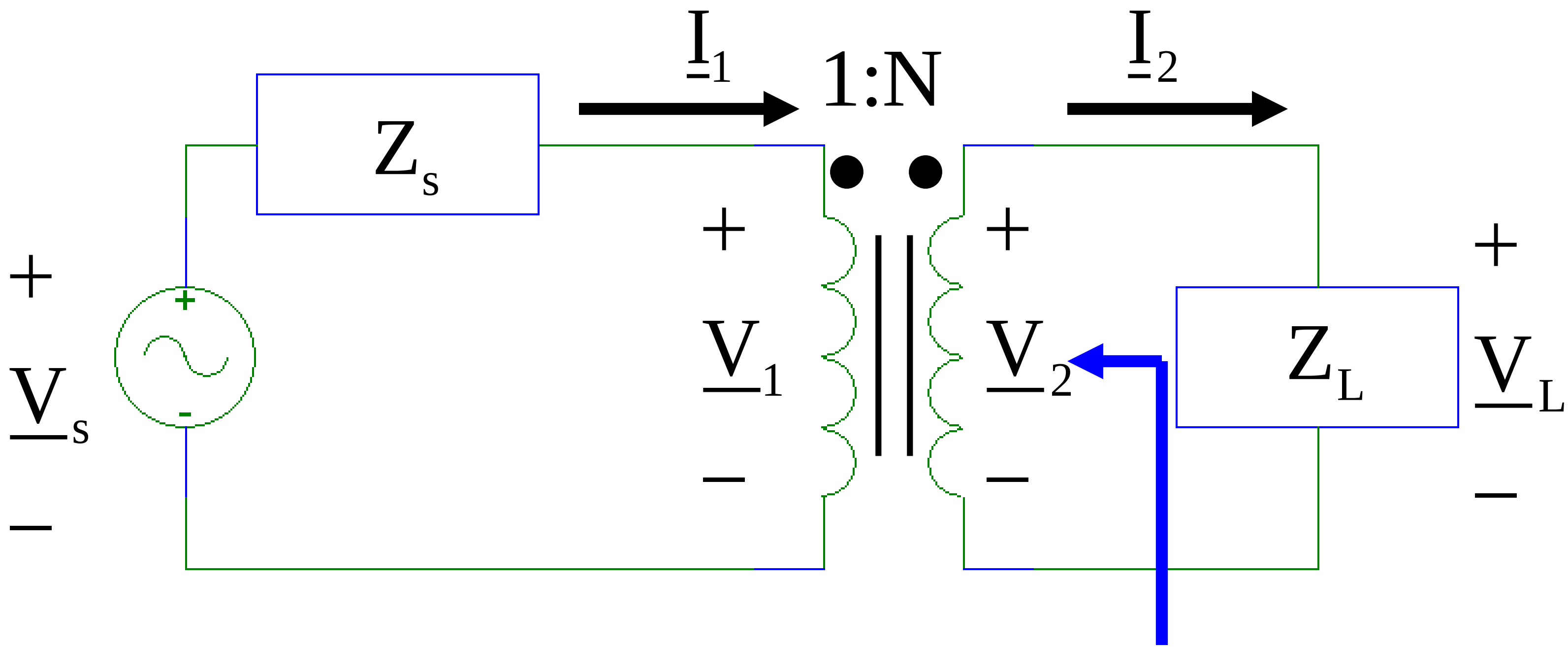
REFERRAL TO PRIMARY



Equivalent to Basic Transformer Circuit

Can Now Do AC Steady State Circuit Analysis

REFERRAL TO SECONDARY

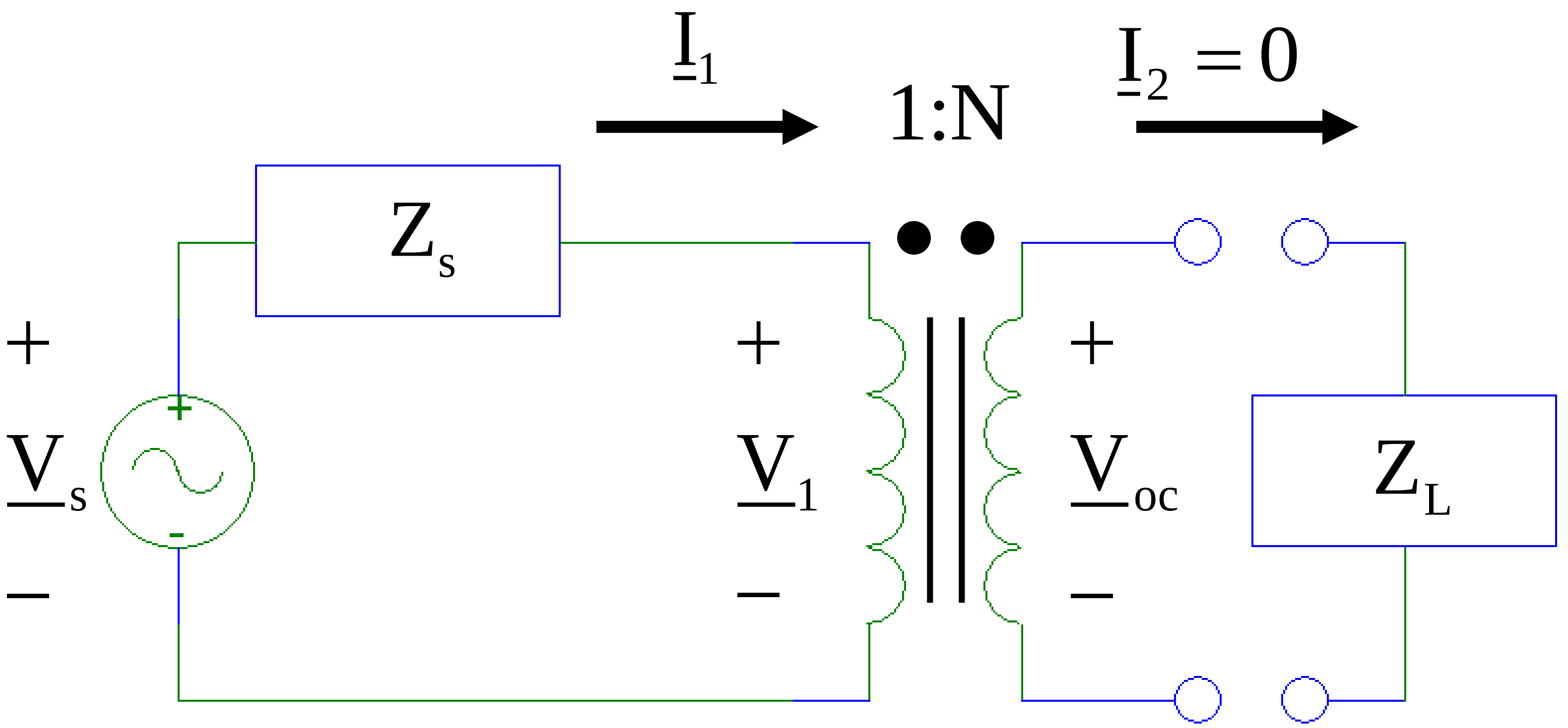


Find Thevenin Equivalent Circuit Seen

Looking BACK IN to the Secondary

$\underline{V}_{oc}$; Z_T

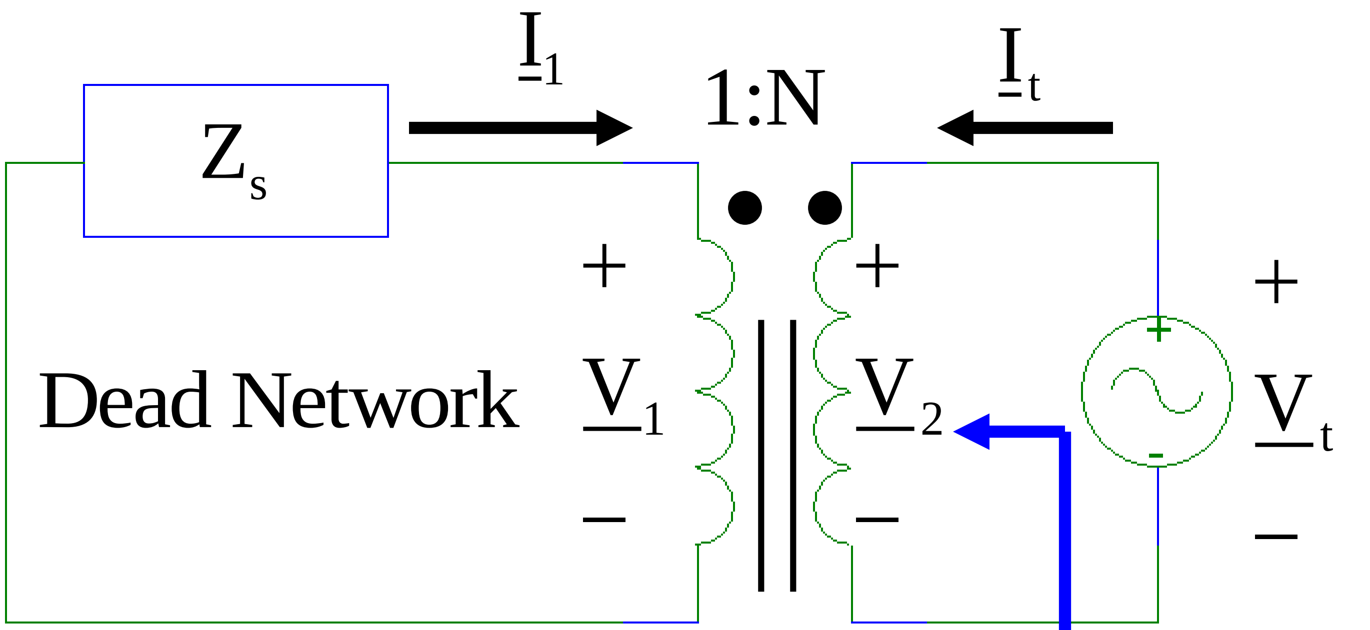
REFERRAL TO SECONDARY



$$\underline{I}_2 = 0 \Rightarrow \underline{I}_1 = N\underline{I}_2 = 0$$

$$\underline{V}_1 = \underline{V}_s \qquad \Rightarrow \underline{V}_{oc} = N\underline{V}_1 = N\underline{V}_s$$

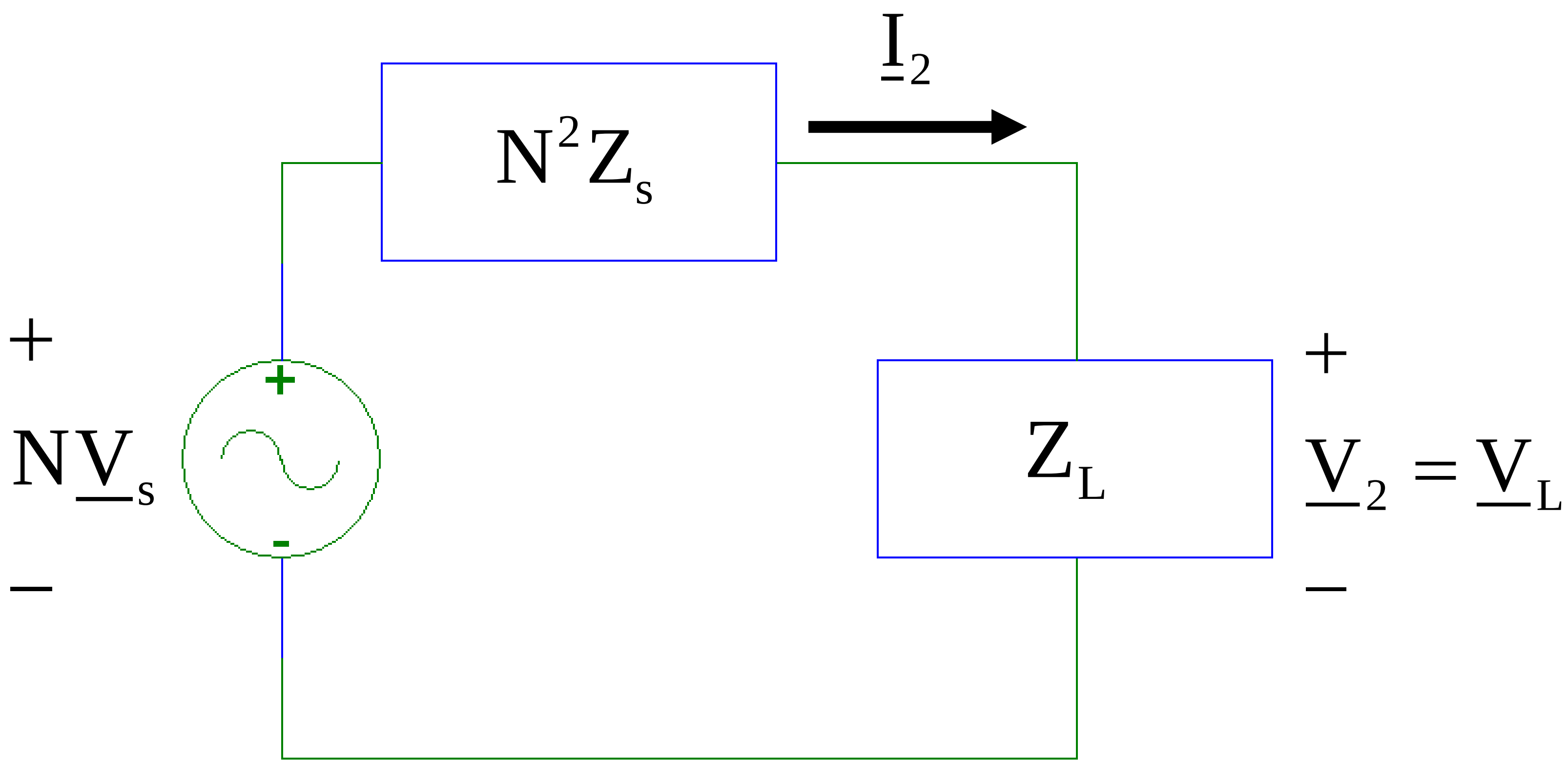
REFERRAL TO SECONDARY



$$\underline{V}_2 = \underline{V}_t = N\underline{V}_1 \qquad \underline{I}_t = -\underline{I}_2 = -\frac{\underline{I}_1}{N} \qquad Z_T = \frac{\underline{V}_t}{\underline{I}_t}$$

$$=> Z_T = \frac{\underline{V}_t}{\underline{I}_t} = \frac{N\underline{V}_1}{-\underline{I}_1/N} = N^2\frac{\underline{V}_1}{-\underline{I}_1} = N^2 Z_s$$

REFERRAL TO SECONDARY



Equivalent to Basic Transformer Circuit

Can Now Do AC Steady State Circuit Analysis

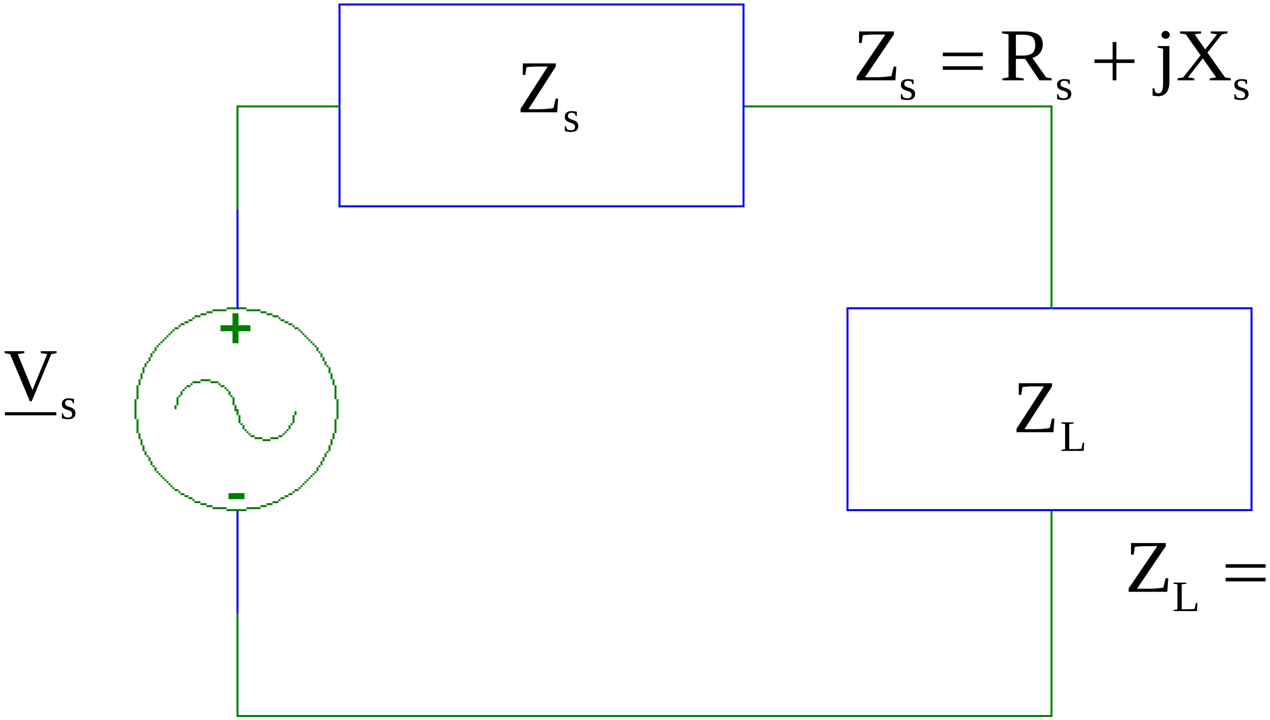
REFERRAL METHODS

- **Can Replace Ideal Transformer and Secondary Circuit with Z_L/N^2 in the Primary Circuit:**
  - **Referral to Primary**
- **Can Replace Primary Circuit and Ideal Transformer with N $\underline{V}_s$, $N^2 Z_s$ in the Secondary Circuit:**
  - **Referral to Secondary**

REFERRAL METHODS

- **Can Refer to Primary OR Refer to Secondary => Choose the Easiest for Particular Problem:**

POWER TRANSFER



$$Z_s = R_s + jX_s$$

$$Z_L = R_L + jX_L$$

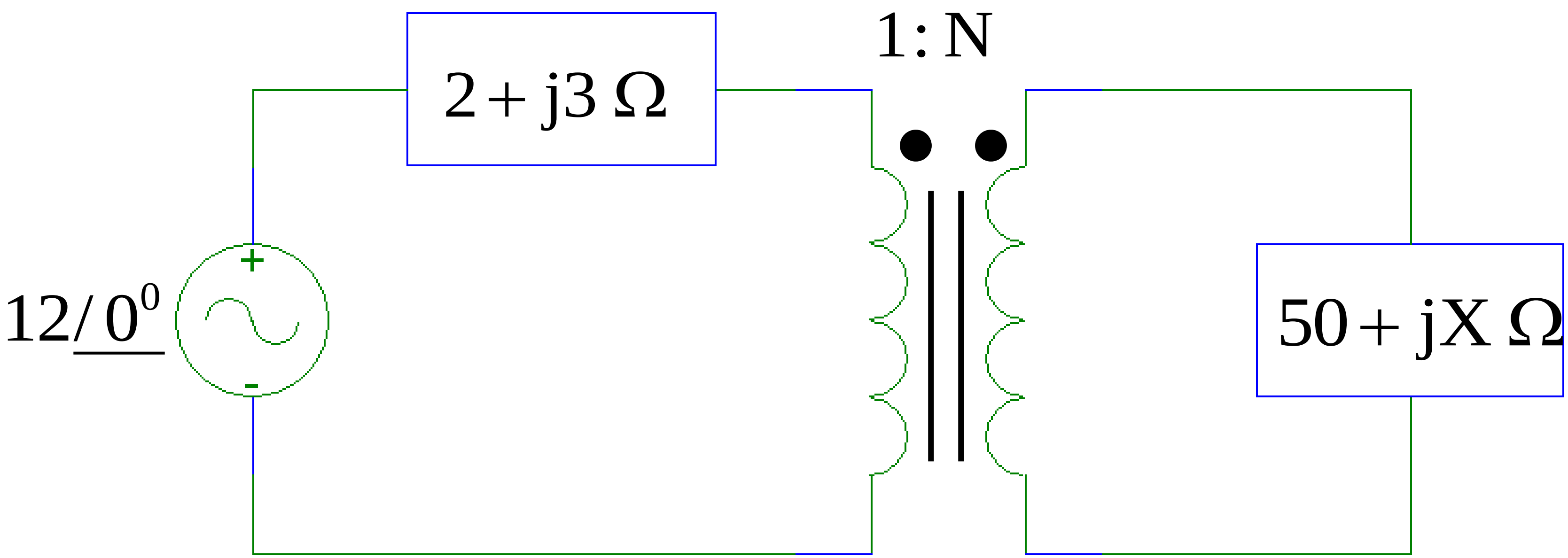
For Maximum Power to $Z_{L,}$ Choose $Z_L = Z_s^*$

$$\Rightarrow R_L = R_s \text{ and } X_L = -X_s$$

IMPEDANCE MATCHING

- For P_{MAX} to Z_L => Want $Z_L = Z_s^*$
  - For Fixed Z_L, Helps to Use Transformer
  - Make $Z_L / N^2 = Z_s^*$
  - Provides an Additional Knob
  - Most Power Amps use a Transformer to Couple the Output to the Speakers
  - Provides both Isolation and Impedance Matching
  - Let's Look at this with an Example

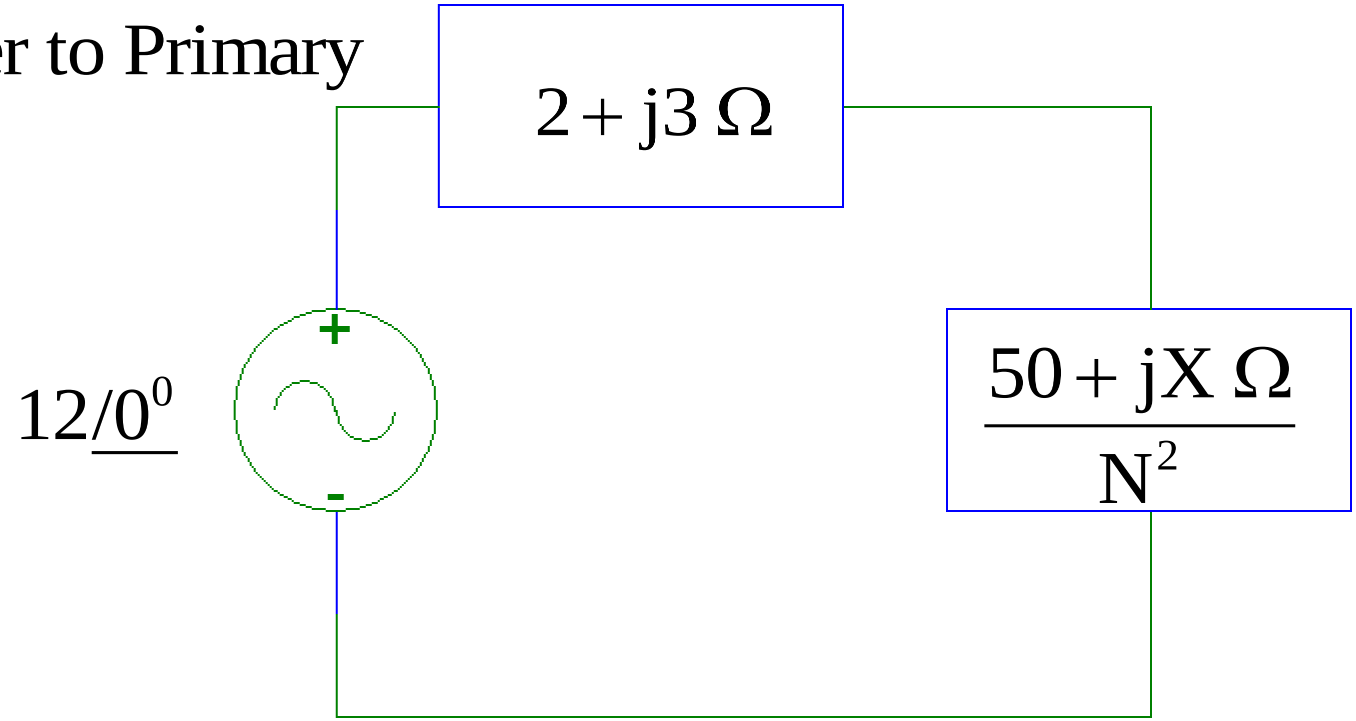
Example



Choose Values for X and N to Maximize Power to Load

Example

Refer to Primary

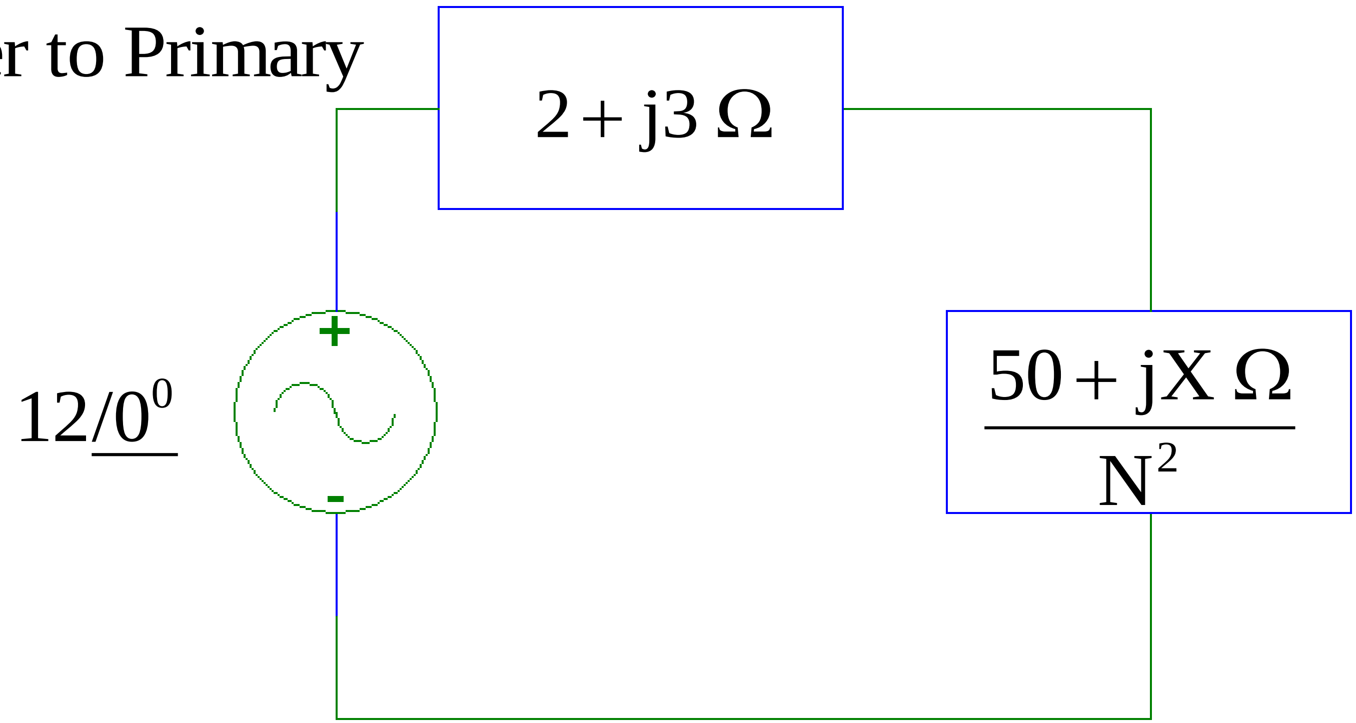


For Maximum Power, Choose $Z_L = Z_s^*$

$$=> \frac{50}{N^2} = 2 \text{ and } \frac{X}{N^2} = -3$$

Example

Refer to Primary



For Maximum Power to Load

$$\frac{50}{N^2} = 2 \quad => \quad N = 5 \qquad\qquad \frac{X}{N^2} = \frac{X}{25} = -3 => X = -75\ \Omega$$

MUTUAL INDUCTANCE

- **Consider 2 Inductors, L_1, L_2:**
- **Magnetic Field of L_1 can Link with L_2 and Vice Versa:**
  - **If Magnetic Field changes with Time => Creates Voltages in L_1, L_2**
  - **Refer to this as Mutual Inductance, M**
  - **M is also measured in Henries**
  - **=>M =** $k\sqrt{L_1 L_2}$ **; $0 \leq k \leq 1$**

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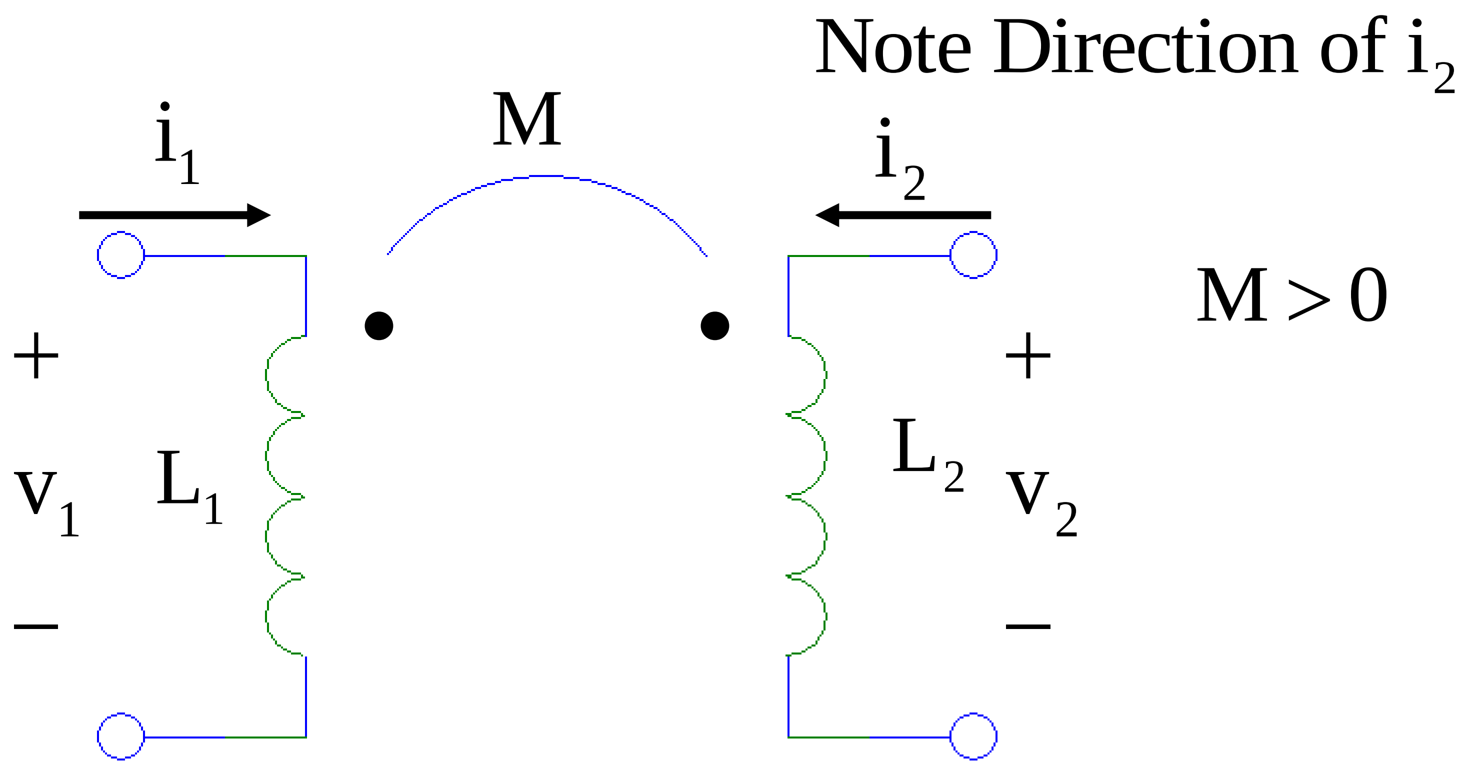
MUTUAL INDUCTANCE

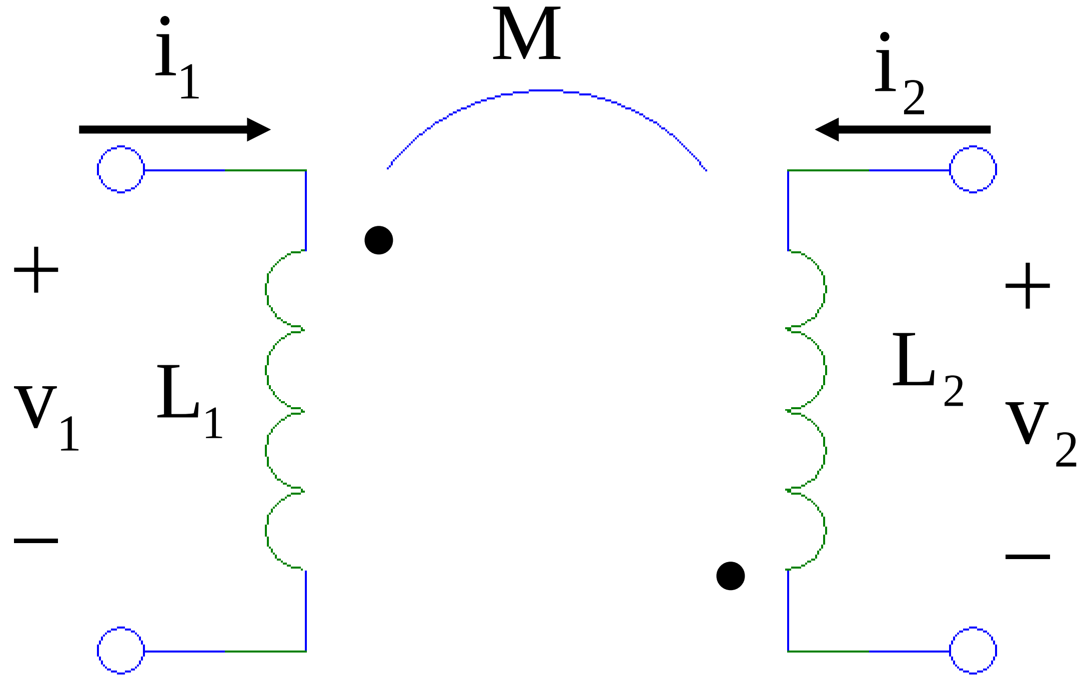


Must Keep Track of How Inductors are Wound

=> Dot Convention

MUTUAL INDUCTANCE

Note Direction of i_2

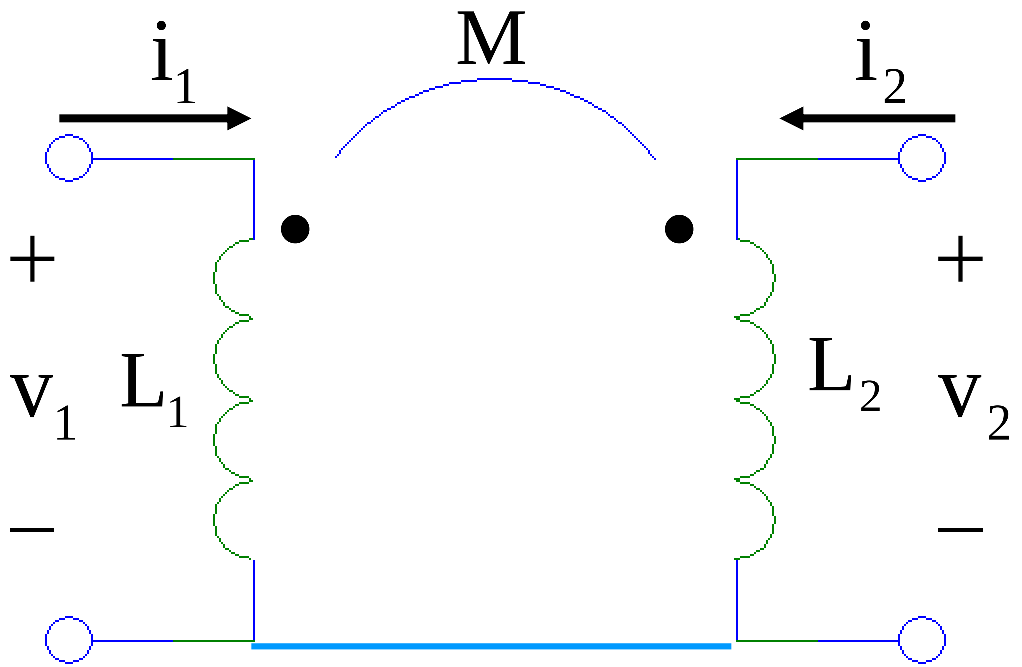


$M < 0$

Must Keep Track of How Inductors are Wound

=> Dot Convention

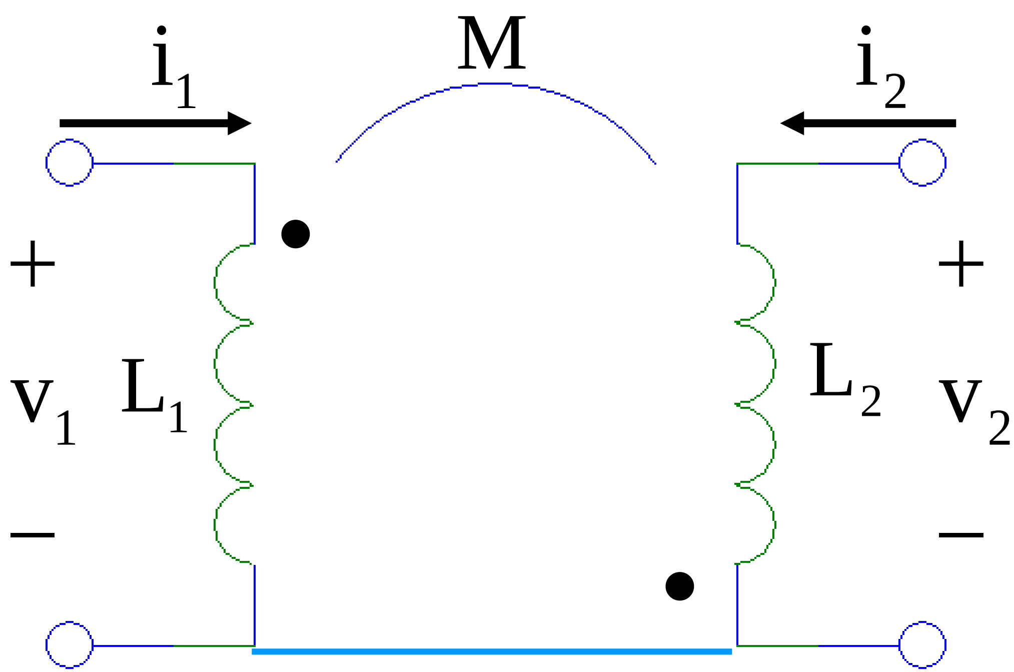
MUTUAL INDUCTANCE



$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

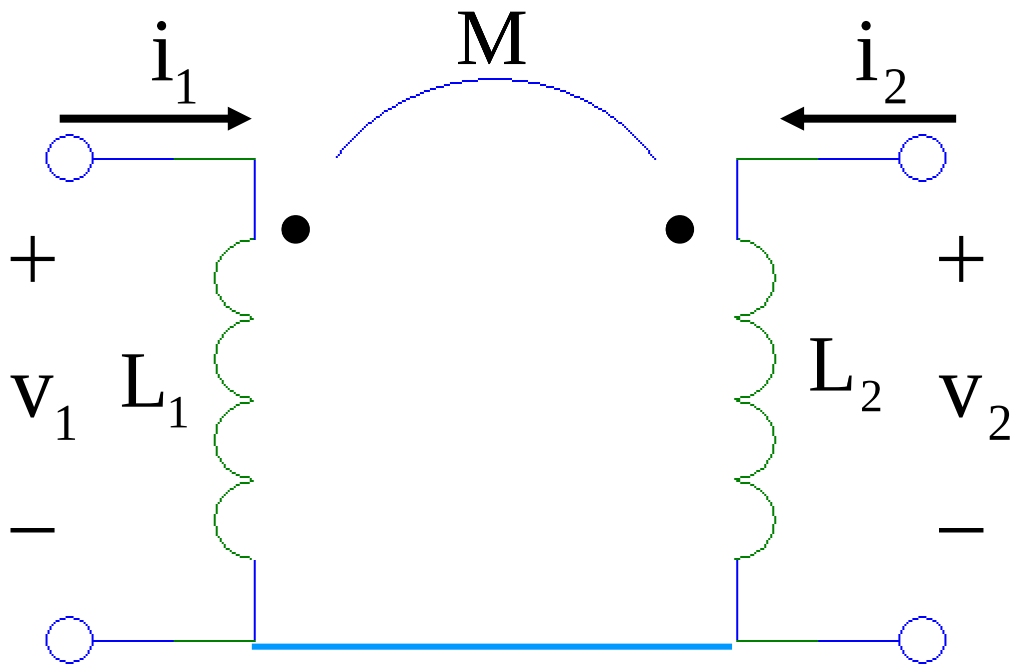
MUTUAL INDUCTANCE



$$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

$$v_2 = -M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

MUTUAL INDUCTANCE



Would Like to Replace with

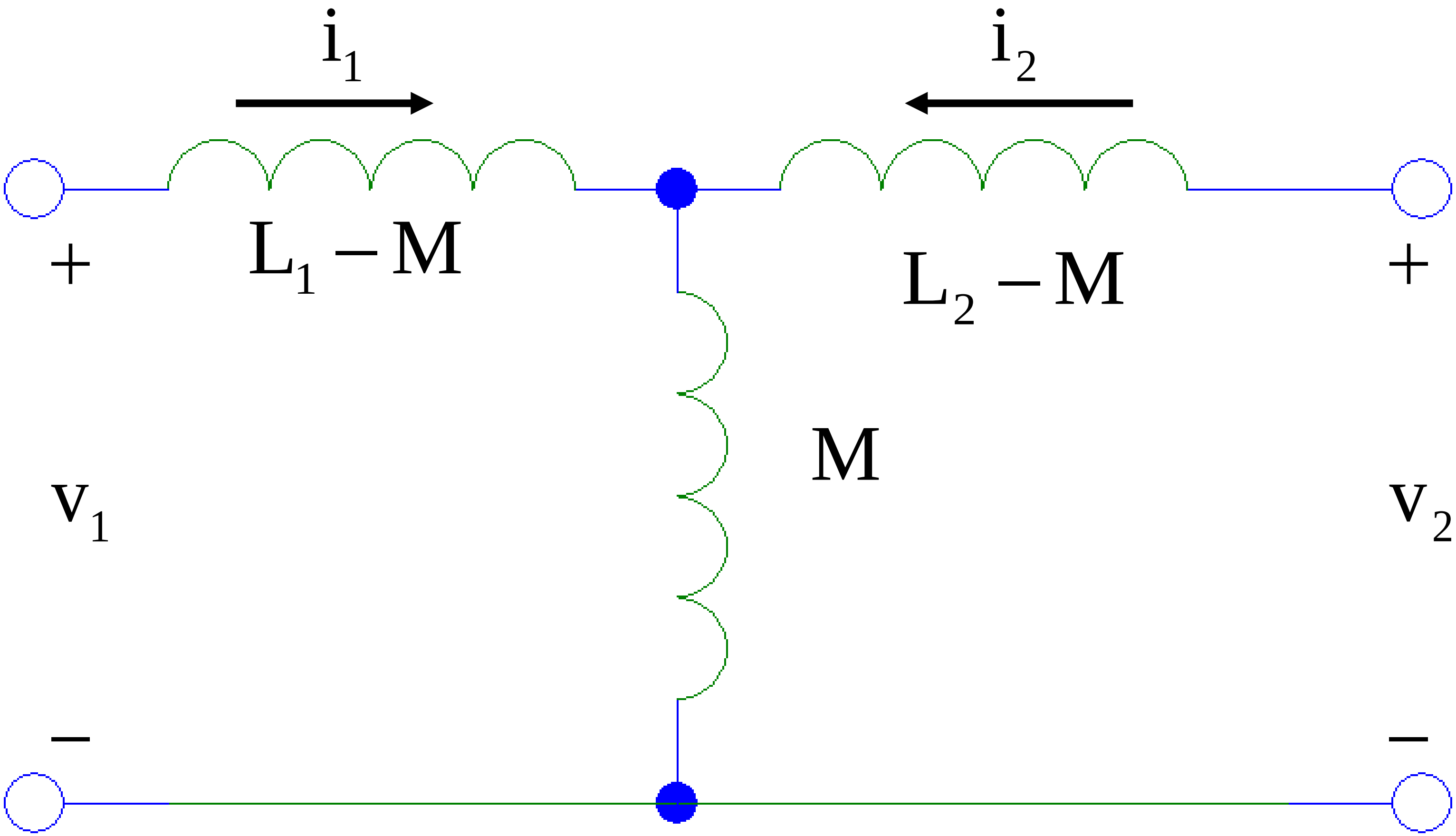
an Equivalent Circuit that does

NOT have any Mutual Inductance

TEE MODEL

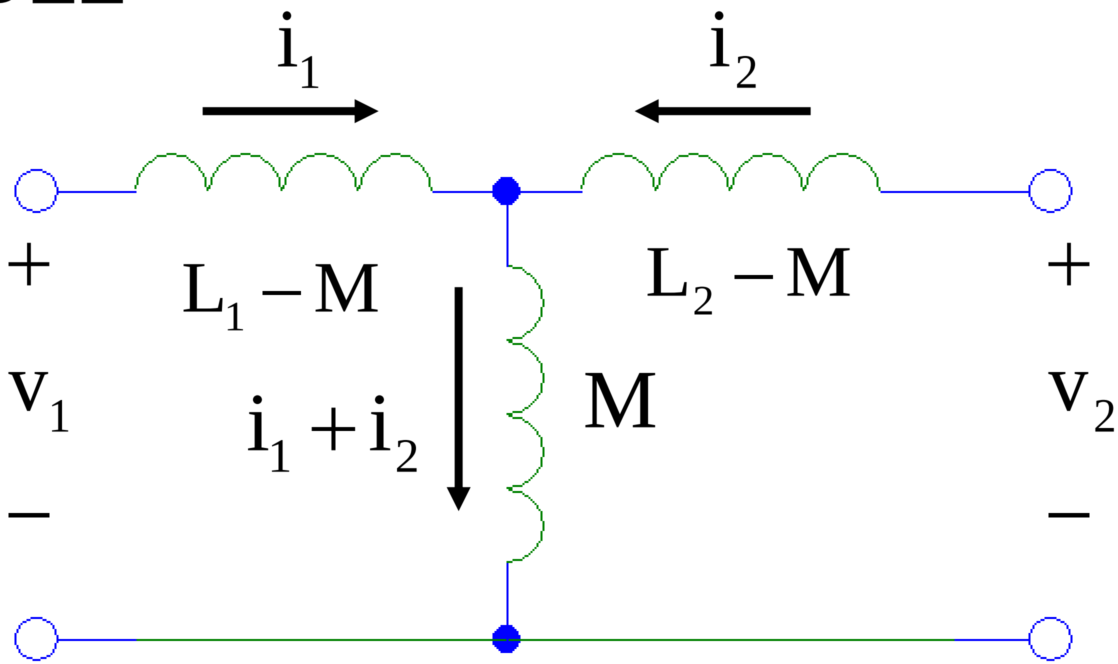
- **Consider a Circuit That Looks Like a "Tee":**
  - **3 Ideal Inductors**
  - **No Mutual Inductance**

TEE MODEL



No Coupling Between Inductors

TEE MODEL



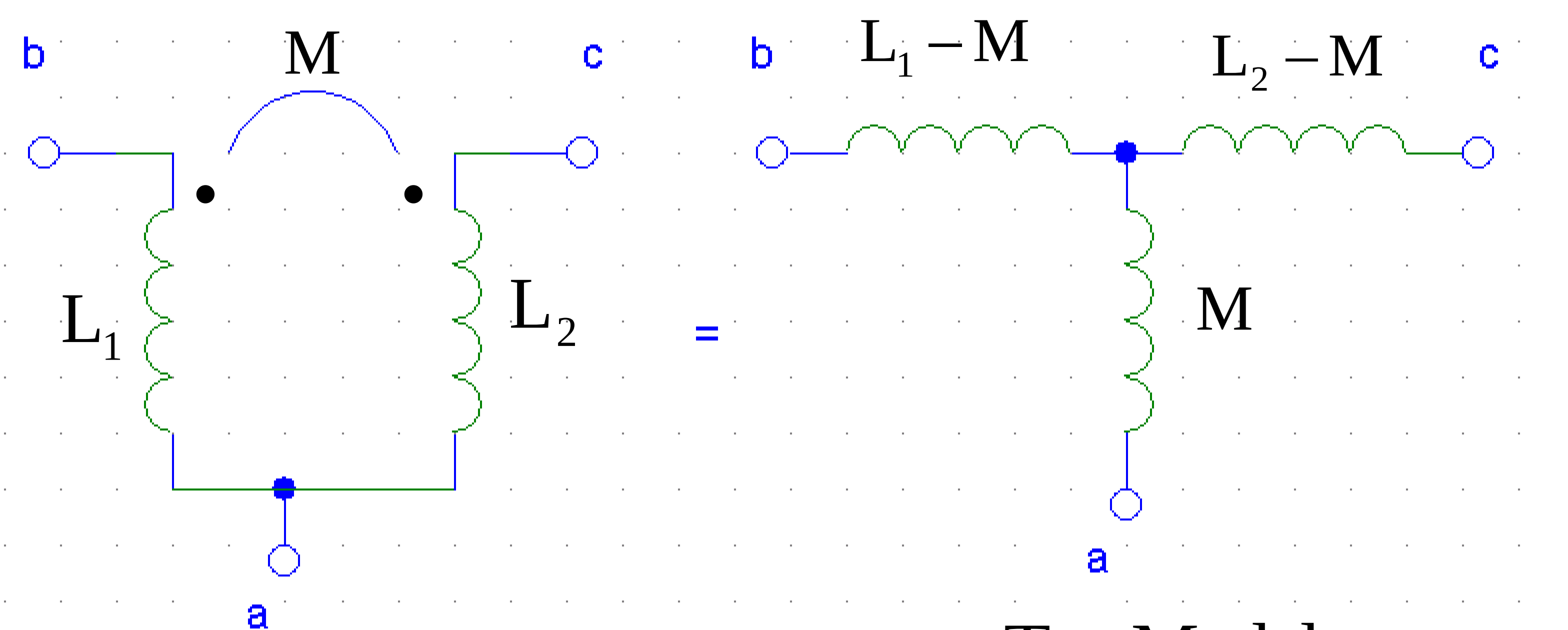
$$v_1 = M\frac{d}{dt}(i_1 + i_2) + (L_1 - M)\frac{di_1}{dt} = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$v_2 = M\frac{d}{dt}(i_1 + i_2) + (L_2 - M)\frac{di_2}{dt} = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

TEE MODEL

- For a "Tee" Circuit:
  - 3 Ideal Inductors, No Mutual Inductance
  - Same Equations as Before
- => Can Replace Inductors exhibiting Mutual Inductance with "Tee Model" and then do AC Steady State Circuit Analysis :

TEE MODEL



Transformer-like Model

Tee Model

If Dots on Opposite Sides => $M \rightarrow -M$

Some Inductors in Tee Model May Be Negative!