ELECTRIC CIRCUITS
ECSE-2010

Lecture 24 continued

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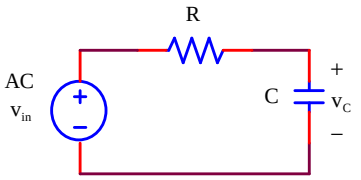
LECTURE 19.1 AGENDA

- First order low pass filter
- First order high pass filter

FIRST ORDER FILTERS



As $\omega \to 0$ (DC): $Z_C \to \infty$; $v_C = v_{in}$

As $\omega \to \infty$: $Z_C \to 0$; $v_C = 0$

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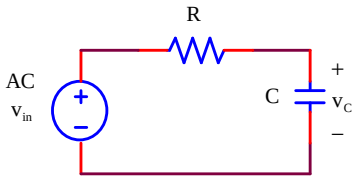
FIRST ORDER FILTERS



Let's look at this in the s-Domain

$$\text{Find } H(s) = \left.\frac{V_C(s)}{V_{in}(s)}\right|_{\text{No Initial Stored Energy}}$$

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FIRST ORDER FILTERS

$$H(s) = \frac{V_C(s)}{V(s)} = \frac{1/RC}{s + 1/RC} = \frac{1/\tau}{s + 1/\tau} = \frac{\omega_c}{s + \omega_c}$$

$$H(j\omega) = \frac{\omega_c}{\omega_c + j\omega} = |H(j\omega)| \underline{/\theta}$$

$$|H(j\omega)| = \frac{\omega_c}{\sqrt{\omega^2 + \omega_c^2}} = \text{Gain} = \sqrt{\frac{1}{1 + \frac{\omega^2}{\omega_c^2}}}$$

$$\theta = -\tan^{-1}\frac{\omega}{\omega_c} = \text{Phase Shift}$$

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1st ORDER LOW PASS FILTER

$$|H(j\omega)| = \frac{\omega_c}{\sqrt{\omega^2 + \omega_c^2}} = \text{Gain}$$

varying

At $\omega = 10\omega_c$, Gain $\approx \frac{1}{10}$; $20\log_{10}(\text{Gain}) \approx -20$ dB

At $\omega = 100\omega_c$, Gain $\approx \frac{1}{100}$, $20\log_{10}(\text{Gain}) \approx -40$ dB

At $\omega = 1000\omega_c$, Gain $\approx \frac{1}{1000}$, $20\log_{10}(\text{Gain}) \approx -60$ dB

Gain decreases at a Slope $= -20$ dB/decade

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1st ORDER LOW PASS FILTER

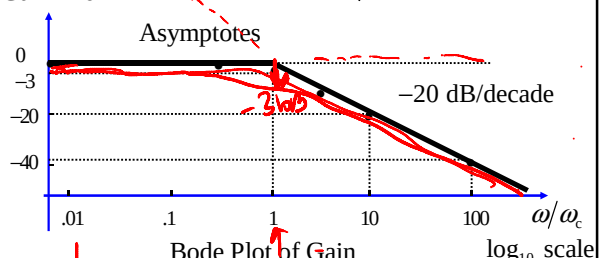
$$|H(j\omega)| = \frac{\omega_c}{\sqrt{\omega^2 + \omega_c^2}} = \text{Gain}$$

$\frac{1}{\sqrt{2}}$

$20\log_{10}|H|$

Gain in dB

-3dB



Bode Plot of Gain

$\log_{10}$ scale

$\frac{1}{\sqrt{2}}$ = $\omega = \omega_c$

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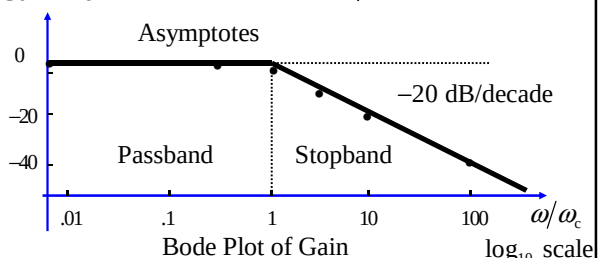
1st ORDER LOW PASS FILTER

$$|H(j\omega)| = \frac{\omega_c}{\sqrt{\omega^2 + \omega_c^2}} = \text{Gain}$$

$20\log_{10}|H|$

Gain in dB



Bode Plot of Gain

$\log_{10}$ scale

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1st ORDER LOW PASS FILTER

$$\theta = -\tan^{-1}\frac{\omega}{\omega_c} = \text{Phase Shift}$$

For "low frequencies" ($\omega << \omega_c$): $\theta \to 0$

For "high frequencies" ($\omega >> \omega_c$): $\theta \to -90^0$

For $\omega = \omega_c$: $\theta = -45^0$

Most of the change in θ occurs for $.1 \le \frac{\omega}{\omega_C} \le 10$

Corner Frequencies at $\frac{\omega}{\omega_C} = .1,\ 10$

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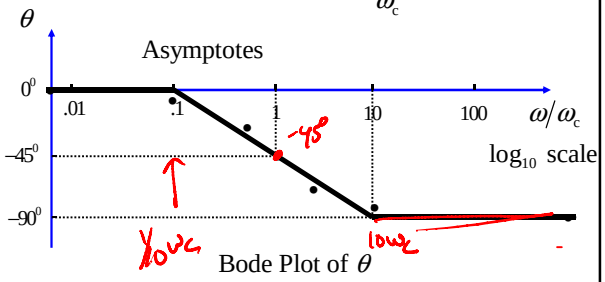
1st ORDER LOW PASS FILTER

$$\theta = -\tan^{-1}\frac{\omega}{\omega_c} = \text{Phase Shift}$$



Bode Plot of θ

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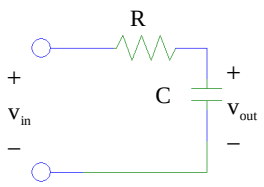
1st ORDER LOW PASS FILTER



$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{1/RC}{s + 1/RC} = \frac{\omega_c}{s + \omega_c} \quad \Rightarrow \text{Low Pass Filter}$$

Low Frequencies "Pass"; High Frequencies "Stopped"

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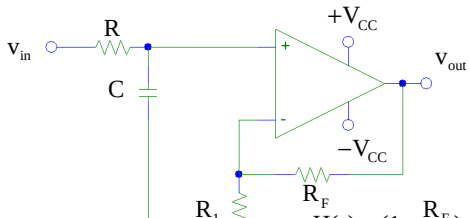
BETTER 1st ORDER LOW PASS



$$H(s) = (1 + \frac{R_F}{R_1})\frac{1/RC}{s + 1/RC}$$

$$= (1 + \frac{R_F}{R_1})\frac{\omega_c}{s + \omega_c}$$

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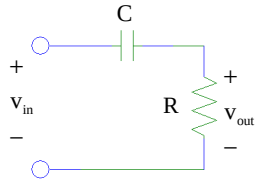
1st ORDER HIGH PASS FILTER



$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{s}{s + 1/RC} = \frac{s}{s + \omega_c} \quad \Rightarrow \text{High Pass Filter}$$

High Frequencies "Pass"; Low Frequencies "Stopped"

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1st ORDER HIGH PASS FILTER

$$|H(j\omega)| = \frac{\omega}{\sqrt{\omega^2 + \omega_c^2}} = \text{Gain}$$

As $\omega \to 0$, Gain $\to 0$, $20\log_{10}(\text{Gain}) \to -\infty$

At $\omega = .01\omega_c$, Gain $= \dfrac{.01\omega_c}{\sqrt{(.01)^2 + \omega_c^2}} \approx .01$

$$20\log_{10}(\text{Gain}) = -40 \text{ dB}$$

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1st ORDER HIGH PASS FILTER

$$|H(j\omega)| = \frac{\omega}{\sqrt{\omega^2 + \omega_c^2}} = \text{Gain}$$

At $\omega = .01\omega_c$, Gain $\approx .01$; $20\log_{10}(\text{Gain}) \approx -40$ dB

At $\omega = .1\omega_c$, Gain $\approx .1$, $20\log_{10}(\text{Gain}) \approx -20$ dB

Gain increases at a Slope $= +20$ dB/decade

At $\omega = \omega_c$, Gain $= \dfrac{1}{\sqrt{2}}$, $20\log_{10}(\text{Gain}) = -3$ dB

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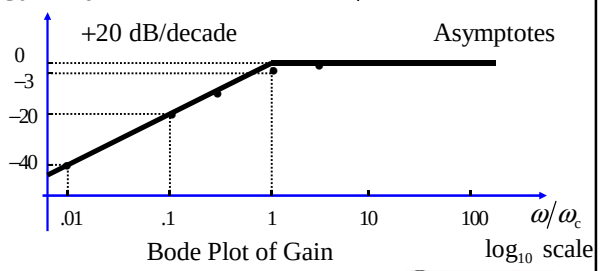
1st ORDER HIGH PASS FILTER

$20\log_{10}|H|$

Gain in dB

$$|H(j\omega)| = \frac{\omega}{\sqrt{\omega^2 + \omega_c^2}} = \text{Gain}$$



Bode Plot of Gain

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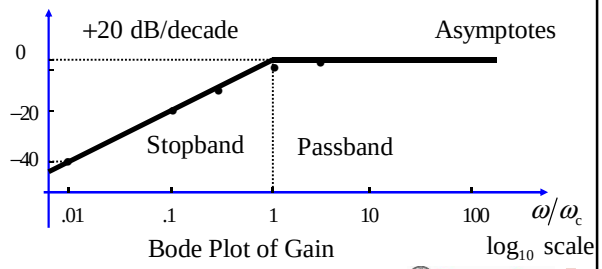
1st ORDER HIGH PASS FILTER

$20\log_{10}|H|$

Gain in dB

$$|H(j\omega)| = \frac{\omega}{\sqrt{\omega^2 + \omega_c^2}} = \text{Gain}$$



Bode Plot of Gain

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1st ORDER HIGH PASS FILTER

$$\theta = 90^0 - \tan^{-1}\frac{\omega}{\omega_c} = \text{Phase Shift}$$

For "low frequencies" ($\omega << \omega_c$): $\theta \to 90^0$

For "high frequencies" ($\omega >> \omega_c$): $\theta \to 0^0$

For $\omega = \omega_c$: $\theta = +45^0$

Most of the change in θ occurs for $.1 \le \frac{\omega}{\omega_C} \le 10$

Corner Frequencies at $\frac{\omega}{\omega_C} = .1,\ 10$

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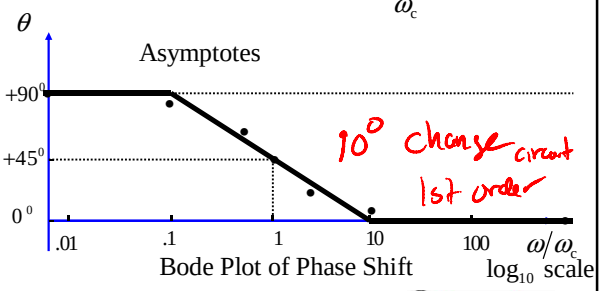
1st ORDER HIGH PASS FILTER

$$\theta = 90^0 - \tan^{-1}\frac{\omega}{\omega_c} = \text{Phase Shift}$$



Bode Plot of Phase Shift

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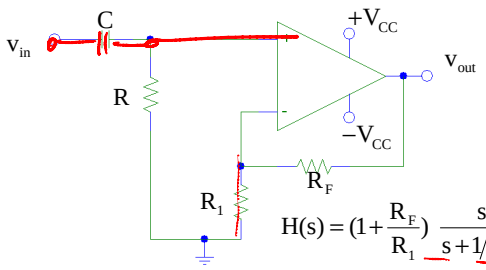
BETTER 1st ORDER HIGH PASS



$$H(s) = (1 + \frac{R_F}{R_1})\frac{s}{s + 1/RC}$$

$$= (1 + \frac{R_F}{R_1})\frac{s}{s + \omega_c}$$

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USING STRAIGHT LINE APPROXIMATIONS
MAGNITUDE BODE PLOTS

1. Find poles, zeros
2. In each region between sequential poles/zeros, can use H(jω) or H(s)
   a) Draw line in first region based on three possibilities based on frequency range
   b) Repeat for every region
3. Add corrections at poles/zeros
   a) n*pole n(-3db) correction at that pole
   b) n*zero n(+3db) correction at that zero

USING STRAIGHT LINE APPROXIMATIONS
MAGNITUDE BODE PLOTS

We have three possibilities

Note: Can ALSO analyze using H(s) instead of H(jω)

$H(j\omega) \propto \omega^n$ $\quad +n \cdot 20 \frac{db}{dec}$ slope

$H(j\omega) \propto \text{constant}, K$ $\quad$ constant $\quad 20 \cdot \log \cdot |K|$

$H(j\omega) \propto \frac{1}{\omega^n}$ $\quad -n \cdot 20 \frac{db}{dec}$ slope

USING STRAIGHT LINE APPROXIMATIONS
PHASE BODE PLOTS

1. Crossing a n*pole is a change of phase of –n*90 deg (absolute change)
   a) Changing over approximately two decades
   b) Specifically, 0.1 and 10 times ω_c
2. Crossing a n*zero is a change of phase +n*90 deg (absolute change)
   a) Changing over approximately two decades
   b) Specifically, 0.1 and 10 times ω_c

USING STRAIGHT LINE APPROXIMATIONS
PHASE BODE PLOTS

1. Crossing a n*pole is a change of phase of –n*90 deg (absolute change)
   a) Changing over approximately two decades
   b) Specifically, 0.1 and 10 times ω_c
2. Crossing a n*zero is a change of phase +n*90 deg (absolute change)
   a) Changing over approximately two decades
   b) Specifically, 0.1 and 10 times ω_c

SLA have slopes of +/- n*45 deg per decade slope (+ is a zero, - is a pole)

ELECTRIC CIRCUITS
ECSE-2010

Lecture 19.2

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LECTURE 19.2 AGENDA

- Cascaded filters/Parallel filters
- First order Bandpass filter
- First order Bandstop (Notch filter)

LOW PASS + HIGH PASS

$$H(s) = K\left(\frac{s}{s+\omega_{cH}}\right)\left(\frac{\omega_{cL}}{s+\omega_{cL}}\right)$$

$$|H(j\omega)| = |K|\left(\frac{\omega}{\sqrt{\omega^2+\omega_{cH}^2}}\right)\left(\frac{\omega_{cL}}{\sqrt{\omega^2+\omega_{cL}^2}}\right)$$

High Pass Low Pass

Let's Design Such that $\omega_{cL} > \omega_{cH}$

$\Rightarrow R_H C_H > R_L C_L$

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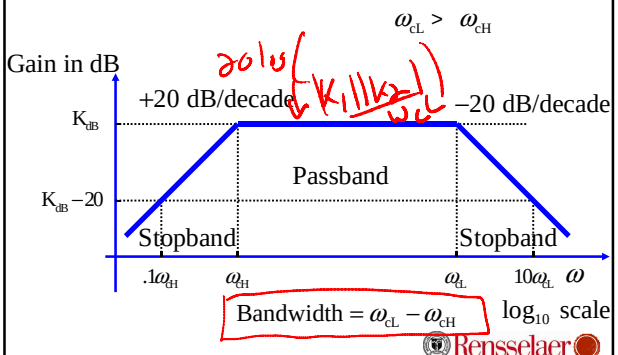
BANDPASS FILTER

$\omega_{cL} > \omega_{cH}$



Bandwidth = $\omega_{cL} - \omega_{cH}$

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LOW PASS + HIGH PASS

$$H(s) = H_L(s) + H_H(s) = \left(1 + \frac{R_A}{R_B}\right)\left(\frac{\omega_{cL}}{s+\omega_{cL}} + \frac{s}{s+\omega_{cH}}\right)$$

For Low Frequencies $\Rightarrow$ Looks Like a 1st Order Low Pass

For High Frequencies $\Rightarrow$ Looks Like a 1st Order High Pass

Let's Design Such that $\omega_{cH} > \omega_{cL}$

$\Rightarrow R_L C_L > R_H C_H$

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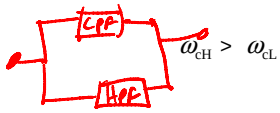
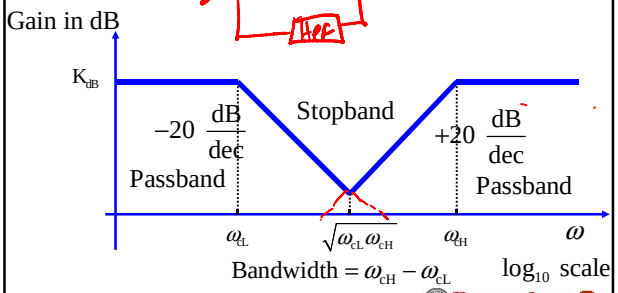
BANDGAP OR NOTCH FILTER

stop

$H(s) = H_1(s) + H_2(s)$



$\omega_{cH} > \omega_{cL}$



Bandwidth $= \omega_{cH} - \omega_{cL}$

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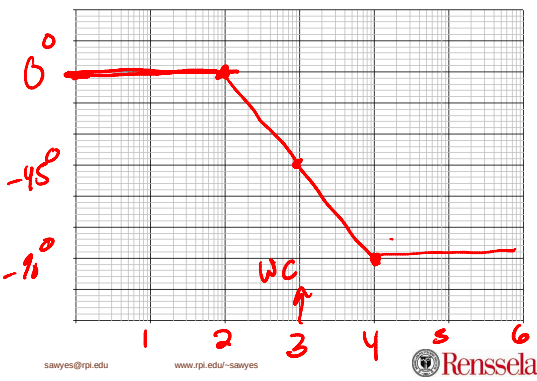
a) $H(s) = \frac{1000}{(s+1000)}$



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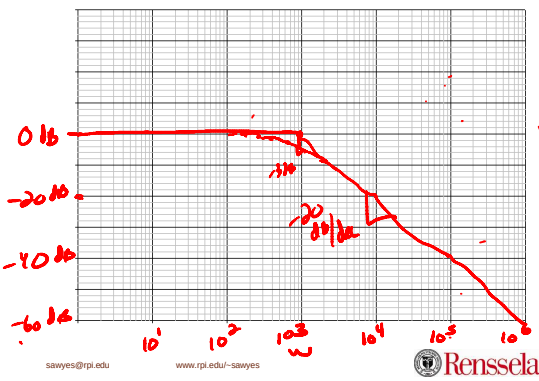
a) $H(s) = \frac{1000}{(s+1000)}$

$\omega_c = 10^3$



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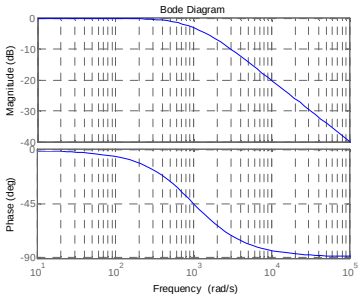
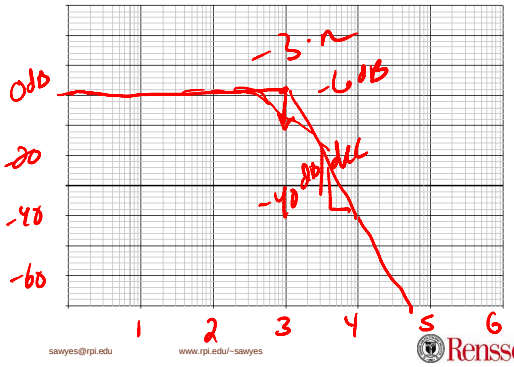
```
>> sys=tf([1000],[1 1000]);
>> h=bodeplot(sys);grid
>>
```

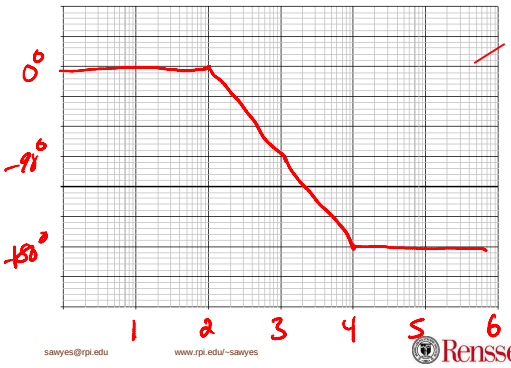


b) $H(s)=\dfrac{1000^2}{(s+1000)^2}$

$-n\cdot 20 = -40$ dB/dec



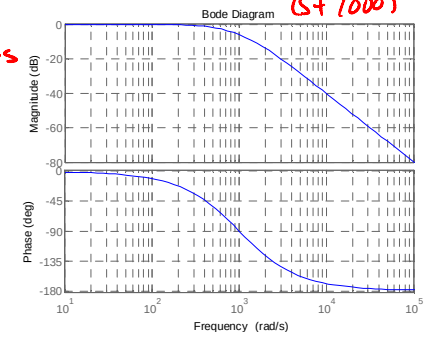
b) $H(s)=\dfrac{1000^2}{(s+1000)^2}$



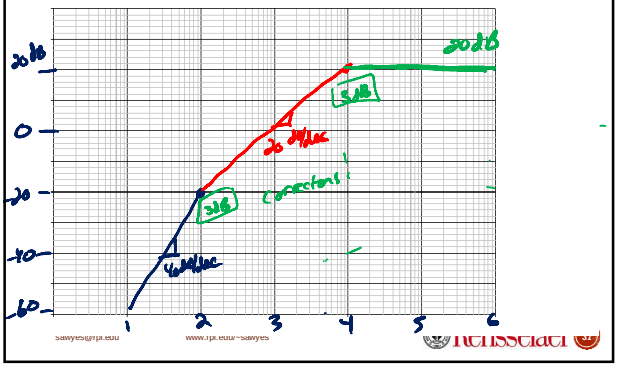

```
>> sys=tf([1000000],[1 2000 1000000]);
>> h=bodeplot(sys);grid
```

$\dfrac{1000^2}{(s+1000)^2}$

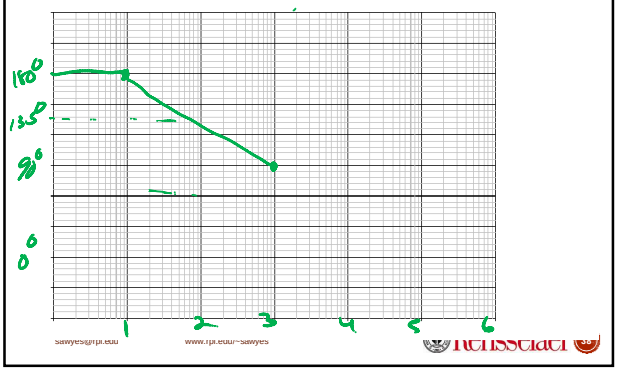
$s^2+2000s+10^6$



c) $H(s)=10\frac{s^2}{(s+100)(s+10000)}$ Corrections



c) $H(s)=10\frac{s^2}{(s+100)(s+10000)}$



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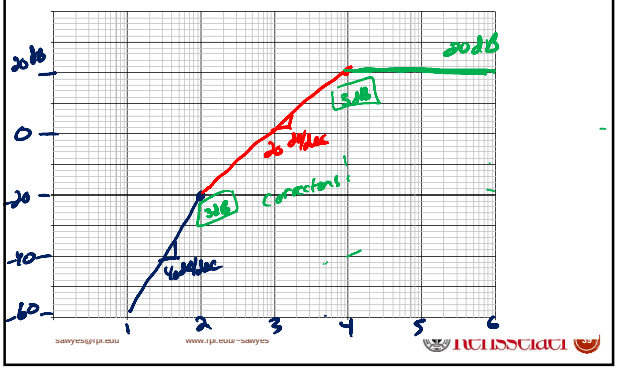
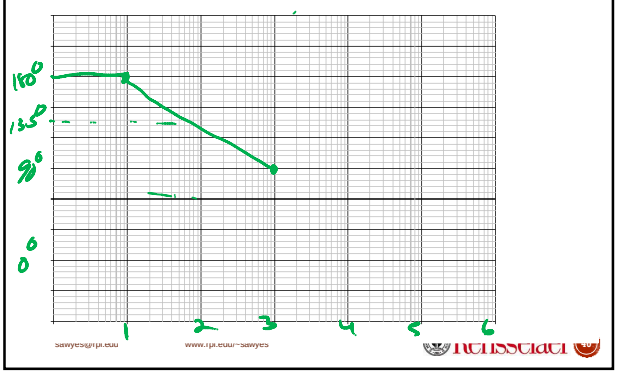
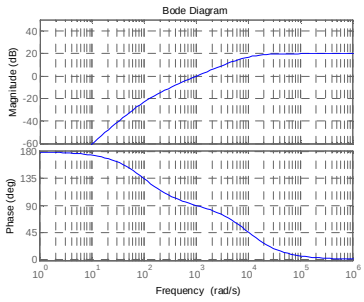


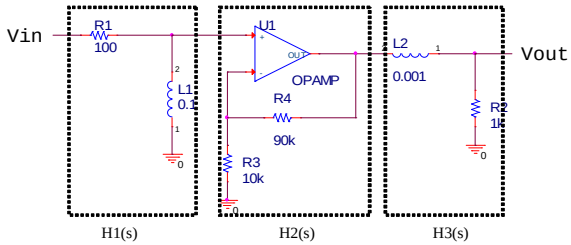
c) $H(s)=10\frac{s^2}{(s+100)(s+10000)}$



```
>> sys=tf([10 0 0],[1 10100 1000000]);
>> h=bodeplot(sys);grid
>> setoptions(h,'MagLowerLimMode','manual','MagLowerLim',-60)
```



2) Bode plot-multiple stages



```
>> sys=tf([10000000 0],[1 1001000 1000000000]);
>> h=bodeplot(sys);grid
```