ELECTRIC CIRCUITS
ECSE-2010

Lecture 25.1

LECTURE 20.1 AGENDA

- Series Resonance

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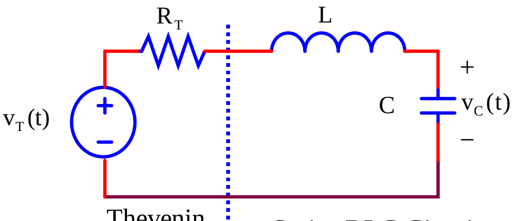
SERIES RESONANCE

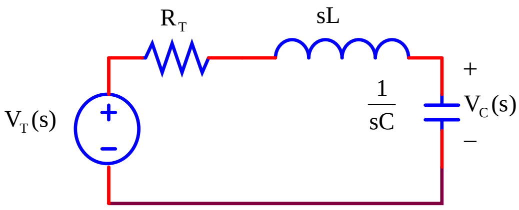


Thevenin Equivalent Circuit

Series RLC Circuit

$$\text{Let's Find } H(s) = \frac{V_C(s)}{V_T(s)}$$

SERIES RESONANCE



$$H(s) = \frac{V_C(s)}{V_T(s)} = \frac{1/sC}{R + sL + \frac{1}{sC}} = \frac{1/LC}{s^2 + \frac{R_T}{L}s + \frac{1}{LC}}$$

SERIES RESONANCE

$$H(s) = \frac{V_C(s)}{V_T(s)} = \frac{1/LC}{s^2 + \frac{R_T}{L}s + \frac{1}{LC}}$$

$$= \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

$$2\zeta\omega_0 = \frac{R_T}{L}, \ \omega_0 = \sqrt{\frac{1}{LC}}$$

$$\zeta = \frac{R_T}{2}\sqrt{\frac{C}{L}}$$

SERIES RESONANCE

$$\zeta = \frac{R_T}{2}\sqrt{\frac{C}{L}}$$

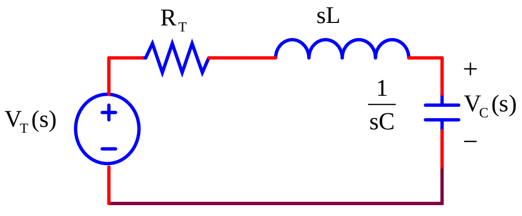
For $\zeta < 1$, Circuit is Underdamped

For Small R_T, Small C, Large L $\Rightarrow \zeta \ll 1$

Circuit is Very Underdamped

Let's Look at the Impedance of this Circuit

SERIES RESONANCE



$$Z(s) = R_T + sL + \frac{1}{sC}$$

SERIES RESONANCE

$$Z(s) = R_T + sL + \frac{1}{sC}$$

In AC Steady State:

$$Z(j\omega) = R_T + j\omega L - \frac{j}{\omega C}$$

$$= R_T + j(\omega L - \frac{1}{\omega C}) = R(\omega) + jX(\omega)$$

At $\omega^2 = \frac{1}{LC} = \omega_0^2 \qquad X(j\omega) \to 0$

SERIES RESONANCE

$$Z(j\omega) = R_T + j(\omega L - \frac{j}{\omega C}) = R(\omega) + jX(\omega)$$

$$|Z(j\omega)| = \sqrt{R^2 + X^2}$$

$$X = \text{AC Reactance} \rightarrow 0 \text{ at } \omega = \omega_0 = \sqrt{\frac{1}{LC}}$$

$$\omega_0 = \text{Resonant Frequency}$$

Series Resonant Circuit

$$|Z(j\omega_0)| = R_T = \text{Minimum at } \omega = \omega_0$$

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$$H(s) = \frac{V_C(s)}{V_T(s)} = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}; \quad \zeta = \frac{R_T}{2}\sqrt{\frac{C}{L}}$$

Let Input be a Sinusoid at $\omega = \omega_0; \Rightarrow s = j\omega_0$

$$H(j\omega_0) = \frac{\omega_0^2}{-\omega_0^2 + 2\zeta\omega_0(j\omega_0) + \omega_0^2} = \frac{\omega_0^2}{j2\zeta\omega_0^2}$$

$$|H(j\omega_0)| = \frac{1}{2\zeta} = \frac{2}{2R_T}\sqrt{\frac{L}{C}} = \frac{1}{R_T}\sqrt{\frac{L}{C}}$$

SERIES RESONANCE

$$|H(j\omega_0)| = \left|\frac{V_C(j\omega_0)}{V_T(j\omega_0)}\right| = \frac{1}{2\zeta} = \frac{1}{R_T}\sqrt{\frac{L}{C}}$$

For Small R_T, Small C, Large L $\Rightarrow \zeta = \frac{R_T}{2}\sqrt{\frac{C}{L}} \ll 1$

For Small R_T, Small C, Large L $\Rightarrow \left|\frac{V_C}{V_T}\right|_{\omega=\omega_0} = \frac{1}{R_T}\sqrt{\frac{L}{C}} \gg 1$

For $\omega = \omega_0 \Rightarrow |V_C| \gg |V_T|$

Series Resonance

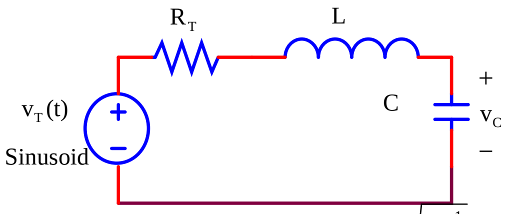
SERIES RESONANCE

For Small R_T, Small C, Large L $\Rightarrow \left|\frac{V_C}{V_T}\right|_{\omega=\omega_0} = \frac{1}{R_T}\sqrt{\frac{L}{C}} \gg 1$

For $\omega = \omega_0 \Rightarrow$ Output Voltage $\gg$ Input Voltage

For $\omega = \omega_0$: Voltage Gain $= \frac{|V_C|}{|V_T|} = Q_{series} = \frac{1}{2\zeta} = \frac{1}{R_T}\sqrt{\frac{L}{C}}$

SERIES RESONANCE



$R_T = 10\ \Omega$ (small)

$L = 0.1$ H (large)

$C = 1$ nF (small)

$$Q_{series} = \frac{1}{10}\sqrt{\frac{10^{-1}}{10^{-9}}} = 1000!!$$

$$|V_C| = 1000|V_T| \text{ at } \omega = \omega_0$$

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2ND ORDER FILTERS

General 2^{nd} Order Bandpass Filter

$$H_{BP}(s) = \frac{Ks}{s^2 + 2\zeta\omega_0 s + \omega_0^2};\ \text{1 Zero at Origin, 2 Poles}$$

General 2^{nd} Order Notch Filter or Bandgap Filter

$$H_{BG}(s) = K\frac{s^2 + 2\beta s + \omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2};\ \text{2 Zeros, 2 Poles}$$

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2ND ORDER FILTERS

- Let's Take a Closer Look at What Happens When We Have Real Poles, Repeated Poles and Complex Conjugate Poles for Low Pass, High Pass and Bandpass Filters
- All Will Exhibit Resonance if Poles are Complex Conjugates

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2ND ORDER LOW PASS

Low Pass: $H1_{LP}(s) = \frac{1000}{s^2 + 12s + 25};\quad \zeta = 1.2$

$H2_{LP}(s) = \frac{1000}{s^2 + 10s + 25};\quad \zeta = 1.0$

$\omega_0 = 5$ $\quad H3_{LP}(s) = \frac{1000}{s^2 + 8s + 25};\quad \zeta = .8$

$H4_{LP}(s) = \frac{1000}{s^2 + .8s + 25};\quad \zeta = .08$

$H5_{LP}(s) = \frac{1000}{s^2 + .08s + 25};\quad \zeta = .008$

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2ND ORDER LOW PASS

```
> with(plots):
Warning, the name changecoords has been redefined
> H1:=1000/(s^2+12*s+25):
> H2:=1000/(s^2+10*s+25):
> H3:=1000/(s^2+8*s+25):
> H4:=1000/(s^2+.8*s+25):
> H5:=1000/(s^2+.08*s+25):
> #Low Pass,  wo = 5, Damping Ratio = 1.2, 1.0, .8, .08, .008
> #Overdamped, Critically Damped, Underdamped, Very Underdamped
> s:=I*omega:
> g1:=20*log10(abs(H1)):
> g2:=20*log10(abs(H2)):
> g3:=20*log10(abs(H3)):
> g4:=20*log10(abs(H4)):
> g5:=20*log10(abs(H5)):
> theta1:=(180/Pi)*argument(H1):
> theta2:=(180/Pi)*argument(H2):
> theta3:=(180/Pi)*argument(H3):
> theta4:=(180/Pi)*argument(H4):
> theta5:=(180/Pi)*argument(H5):
> semilogplot([g1,g2,g3,g4,g5],omega=.1..50);
```

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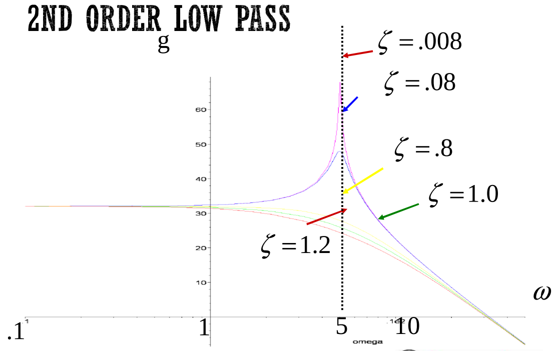
2ND ORDER LOW PASS



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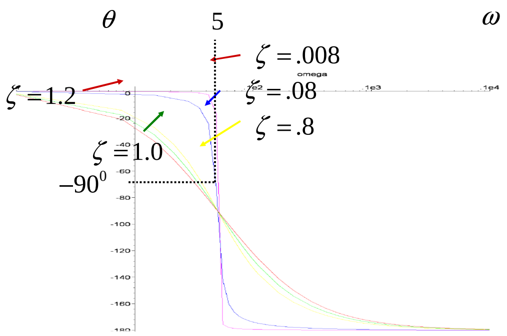
2ND ORDER LOW PASS



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2^{ND} ORDER HIGH PASS

High Pass: $H1_{HP}(s) = \dfrac{1000s^2}{s^2 + 12s + 25}$; $\zeta = 1.2$

$H2_{HP}(s) = \dfrac{1000s^2}{s^2 + 10s + 25}$; $\zeta = 1.0$

$\omega_0 = 5$ $\qquad H3_{HP}(s) = \dfrac{1000s^2}{s^2 + 8s + 25}$; $\zeta = .8$

$H4_{HP}(s) = \dfrac{1000s^2}{s^2 + .8s + 25}$; $\zeta = .08$

$H5_{HP}(s) = \dfrac{1000s^2}{s^2 + .08s + 25}$; $\zeta = .008$

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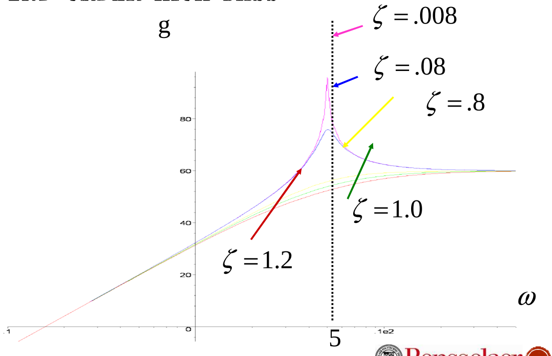
2ND ORDER HIGH PASS

```
> with(plots):
Warning, the name changecoords has been redefined
> H1:=(1000*s^2)/(s^2+12*s+25):
> H2:=(1000*s^2)/(s^2+10*s+25):
> H3:=(1000*s^2)/(s^2+8*s+25):
> H4:=(1000*s^2)/(s^2+.8*s+25):
> H5:=(1000*s^2)/(s^2+.08*s+25):
> #High Pass,  wo = 5, Damping Ratio = 1.2, 1.0, .8, .08, .008
> #Overdamped, Critically Damped, Underdamped, Very Underdamped
> s:=I*omega:
> g1:=20*log10(abs(H1)):
> g2:=20*log10(abs(H2)):
> g3:=20*log10(abs(H3)):
> g4:=20*log10(abs(H4)):
> g5:=20*log10(abs(H5)):
> theta1:=(180/Pi)*argument(H1):
> theta2:=(180/Pi)*argument(H2):
> theta3:=(180/Pi)*argument(H3):
> theta4:=(180/Pi)*argument(H4):
> theta5:=(180/Pi)*argument(H5):
> semilogplot([g1,g2,g3,g4,g5],omega=.1..50);
```


2ND ORDER HIGH PASS



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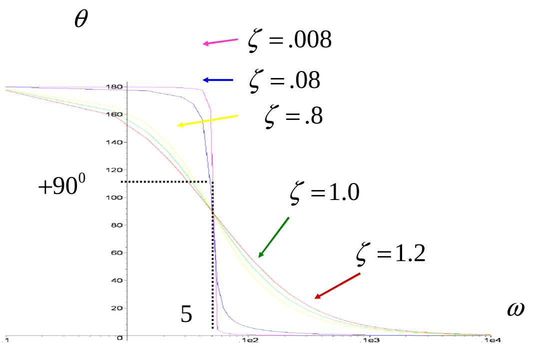
2ND ORDER HIGH PASS



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2ND ORDER BANDPASS

Bandpass: $H1_{HP}(s) = \dfrac{1000s}{s^2 + 12s + 25}; \quad \zeta = 1.2$

$H2_{HP}(s) = \dfrac{1000s}{s^2 + 10s + 25}; \quad \zeta = 1.0$

$\omega_0 = 5 \qquad H3_{HP}(s) = \dfrac{1000s}{s^2 + 8s + 25}; \quad \zeta = .8$

$H4_{HP}(s) = \dfrac{1000s}{s^2 + .8s + 25}; \quad \zeta = .08$

$H5_{HP}(s) = \dfrac{1000s}{s^2 + .08s + 25}; \quad \zeta = .008$

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2ND ORDER BANDPASS

```
> with(plots):
Warning, the name changecoords has been redefined
> H1:=(1000*s)/(s^2+12*s+25):
> H2:=(1000*s)/(s^2+10*s+25):
> H3:=(1000*s)/(s^2+8*s+25):
> H4:=(1000*s)/(s^2+.8*s+25):
> H5:=(1000*s)/(s^2+.08*s+25):
> #wo = 5, Damping Ratio = 1.2, 1.0, .8. .08, .008
> #Overdamped, Critically Damped, Underdamped
> #Bandpass Filter

> s:=I*omega:
>
> g1:=20*log10(abs(H1)):
> g2:=20*log10(abs(H2)):
> g3:=20*log10(abs(H3)):
> g4:=20*log10(abs(H4)):
> g5:=20*log10(abs(H5)):
> theta1:=(180/Pi)*argument(H1):
> theta2:=(180/Pi)*argument(H2):
> theta3:=(180/Pi)*argument(H3):
> theta4:=(180/Pi)*argument(H4):
> theta5:=(180/Pi)*argument(H5):
> semilogplot([g1,g2,g3,g4,g5],omega=.1..50);
```

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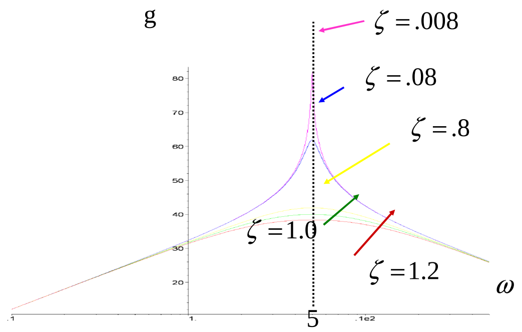
2ND ORDER BANDPASS



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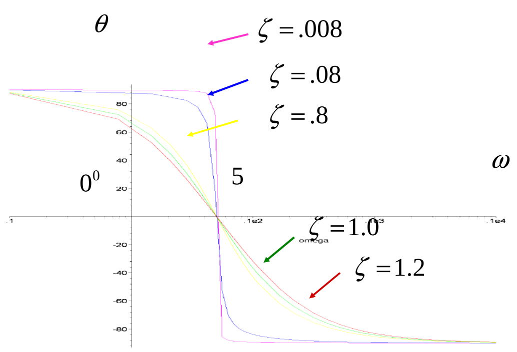
2ND ORDER BANDPASS



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2ND ORDER FILTERS

Always See Resonance When We Have Complex Conjugate Poles

$\zeta < 1 \Rightarrow$ Underdamping

Avoid Resonance for Good Low Pass and High Pass Filters

Use Resonance for Good Bandpass and Bandgap Filters

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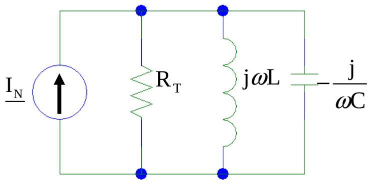
PARALLEL RESONANCE

Frequency Domain



$$Y_{eq} = \frac{1}{R_T} + \frac{1}{j\omega L} + \frac{\omega C}{-j} = \frac{1}{R_T} + j(\omega C - \frac{1}{\omega L})$$

$$= G(\omega) + jB(\omega)$$

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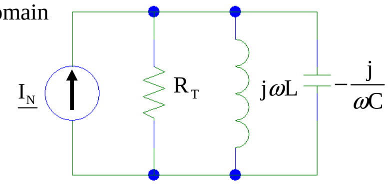
PARALLEL RESONANCE

Frequency Domain



$$Y_{eq} = \frac{1}{R_T} + j(\omega C - \frac{1}{\omega L}) = G(\omega) + jB(\omega)$$

AC Susceptance $= B(\omega) \rightarrow 0$ when $\omega = \omega_0 = \sqrt{\frac{1}{LC}}$

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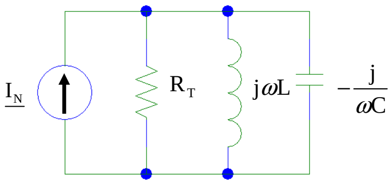
PARALLEL RESONANCE

Frequency Domain



$\omega_0 = \sqrt{\frac{1}{LC}}$ $\quad \omega_0$ = Resonant Frequency

=> Same as for Series Resonance

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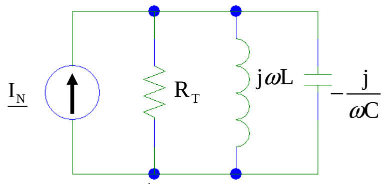
PARALLEL RESONANCE

Frequency Domain



$Y_{eq} = \frac{1}{R_T}$ at $\omega = \omega_0$

=> $Z_{eq}(\omega_0) = R_T$; Purely Resistive

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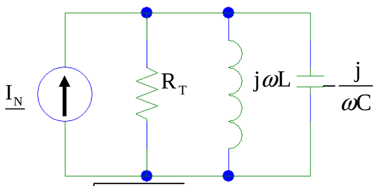
PARALLEL RESONANCE



$$\left|Y_{eq}(\omega)\right| = \sqrt{G^2 + B^2};\ \ B(\omega_0) \to 0$$

$$=> \left|Y_{eq}\right| \text{ is a MINIMUM at } \omega = \omega_0$$

$$=> \left|Z_{eq}\right| \text{ is a MAXIMUM at } \omega = \omega_0$$

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TYPE OF RESONANCE

Parallel Resonance:

$$=> \left|Z_{eq}\right| = \text{MAXIMUM at } \omega = \omega_0$$

Series Resonance:

$$=> \left|Z_{eq}\right| = \text{MINIMUM at } \omega = \omega_0$$

Often the best way to determine type of resonance

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PARALLEL RESONANCE

KCL: $\underline{I}_L + \underline{I}_C + \underline{I}_R = \underline{I}_N$; At Resonance: $\underline{I}_L = -\underline{I}_C$

$$\underline{V} = \underline{I}_N Z_{eq} = \underline{I}_R R_T = \underline{I}_L (j\omega L) = \underline{I}_C \left(\frac{1}{j\omega C}\right)$$

At $\omega = \omega_0$; $Z_{eq} = R_T$

$$=> \frac{\underline{I}_L}{\underline{I}_N} = \frac{R_T}{j\omega_0 L} \quad => \quad \frac{\left|\underline{I}_L\right|}{\left|\underline{I}_N\right|} = \frac{R_T}{\omega_0 L}$$

$$=> \frac{\underline{I}_C}{\underline{I}_N} = R_T\ j\omega_0 C \quad => \quad \frac{\left|\underline{I}_C\right|}{\left|\underline{I}_N\right|} = \omega_0 R_T C$$

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PARALLEL RESONANCE

At Resonance: $\omega_0 C R_T = \dfrac{R_T}{\omega_0 L}$; $\omega_0 = \sqrt{\dfrac{1}{LC}}$;

$$=> Q_{parallel} = \frac{\left|\underline{I}_c\right|}{\left|\underline{I}_N\right|} = \frac{\left|\underline{I}_L\right|}{\left|\underline{I}_N\right|} = R_T \sqrt{\frac{C}{L}}\ ;$$

$= \text{"Current Gain" at } \omega = \omega_0$

Note: $Q_{parallel} = \dfrac{1}{Q_{series}}$

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PARALLEL RESONANCE

For $Q_{parallel} >> 1$:

Want "large" R_T, "small" L, "large"C

Just the opposite of what makes $Q_{series} >> 1$

=> A good Series Resonant circuit will be a lousy Parallel Resonant circuit

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SUMMARY

Series Resonance:	Parallel Resonance:
$X \to 0$ at $\omega = \omega_0$	$B \to 0$ at $\omega = \omega_0$
$\omega_0 = \sqrt{\frac{1}{LC}}$	$\omega_0 = \sqrt{\frac{1}{LC}}$
$Q_{series} = \frac{1}{R_T}\sqrt{\frac{L}{C}}$	$Q_{parallel} = R_T\sqrt{\frac{C}{L}}$
$\lvert Z_{eq}(\omega_0)\rvert = \text{MINIMUM}$	$\lvert Z_{eq}(\omega_0)\rvert = \text{MAXIMUM}$

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NOTES ON RESONANCE

- **In "real" circuits, often do not have a "pure" Series or "pure" Parallel situation:**
  - **Inductors always have R_w**
  - **Elements may not all be in series or parallel**
  - **Will still have Resonance, but at a slightly different Resonant Frequency**

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NOTES ON RESONANCE

- **In "real" circuits, often do not have a "pure" Series or "pure" Parallel situation:**
  - **May also have Multiple Resonances in same Circuit**
  - **Define Resonance as when X (series-like circuit) or B (parallel-like circuit) => 0**
  - **Let's do an Example**

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