## 1) General Current, Voltage, or Power

The plot below is the net positve charge flowing in a wire vs. time. Sketch the corresponding current during the same period of time.


> check

Since current is $\mathrm{dq} / \mathrm{dt}$, The answer should be the derivative of each line which will be constants 10 from 0 to 2
-40 from 2 to 3
-5 from 3 to 5
50 from 5 to 6

## 2) Source devices and Total Power


c. Determine the power supplied/consumed by each component and show they balance to 0 .

$$
\begin{aligned}
& \mathrm{V}_{1}:=4 \mathrm{~V} \quad \mathrm{I}_{\mathrm{R} 1}:=1 \mathrm{~mA} \quad \mathrm{~V}_{\mathrm{R} 1}:=1 \mathrm{k} \Omega \cdot 1 \mathrm{~mA}=1 \mathrm{~V} \quad \mathrm{I}_{1}:=1 \mathrm{~mA} \\
& -\mathrm{V}_{1} \cdot \mathrm{I}_{\mathrm{V} 1}+\mathrm{I}_{\mathrm{R} 1} \cdot \mathrm{~V}_{\mathrm{R} 1}+\mathrm{V}_{\mathrm{I} 1} \cdot \mathrm{I}_{1}=0 \mathrm{~W}
\end{aligned}
$$

## 3) Nodal voltages/voltage drops/currents


a. How many nodes are in the above circuit?

There are six junction/nodes where components meet.
b. Determine the voltage at every node

Step 1 (VF, VA, and VB):

$$
\begin{aligned}
& \mathrm{V}_{\mathrm{F}}:=0 \\
& \mathrm{~V}_{\mathrm{W}}:=5 \mathrm{~V} \\
& \mathrm{~V}_{\mathrm{A}}:=\mathrm{V}_{\mathrm{F}}+\mathrm{V}_{1}=5 \mathrm{~V} \\
& \qquad \mathrm{I}_{31}:=3 \mathrm{~A} \quad \mathrm{R}_{34}:=1 \mathrm{k} \Omega
\end{aligned}
$$

$\mathrm{V}_{\mathrm{B}}:=\mathrm{V}_{\mathrm{A}}-\mathrm{I}_{31} \cdot \mathrm{R}_{34}$

$$
\mathrm{V}_{\mathrm{B}}=-2.995 \cdot \mathrm{kV}
$$

Step 2: (VD, VE, and VC)

$$
\mathrm{V}_{\mathrm{D}}:=1 \mathrm{~V}
$$

$$
\mathrm{V}_{\mathrm{E}}:=\mathrm{V}_{\mathrm{D}}-2 \mathrm{~V}
$$

$$
\mathrm{V}_{\mathrm{E}}=-1 \mathrm{~V}
$$

$$
\mathrm{V}_{\mathrm{C}}:=\mathrm{V}_{\mathrm{D}}+4 \mathrm{~V}
$$

$$
\mathrm{V}_{\mathrm{C}}=5 \mathrm{~V}
$$

c. Determine the current through R3, V2, and V3 (label or indicate current direction for full credit)

$$
\mathrm{I}_{\mathrm{R} 3}:=\frac{\mathrm{V}_{\mathrm{E}}}{2 \Omega}=-0.5 \mathrm{~A} \quad \text { right to left }
$$

Using KCL

$$
\begin{array}{ll}
\mathrm{I}_{\mathrm{V} 2}:=-3 \mathrm{~A}+2 \mathrm{~A}=-1 \mathrm{~A} & \text { right to left } \\
\mathrm{I}_{\mathrm{V} 3}:=\mathrm{I}_{\mathrm{R} 3}=-0.5 \mathrm{~A} & \text { down } \quad \text { they are in series }
\end{array}
$$

## 4) KVLKCL

In this circuit,
a. Determine five linearly independent equations for the voltage across the resistors. You will have to use a combination of Ohm's law, KCL, and KVL.

Redraw the circuit with polarities for full credit.


Using "normal" polarity definitions (positive on the left and top for horization and vertical resistors, respectivelv)
$\mathrm{V}_{\mathrm{R} 1}=(1 \mathrm{k}) \cdot-10 \cdot 10^{-3}$ note: if positive on left side of R 1 and negative on the right
$10 \cdot 10^{-3}+\mathrm{I}_{\mathrm{R} 3}+\mathrm{I}_{\mathrm{R} 4}-\mathrm{I}_{\mathrm{R} 2}=0$

$$
\mathrm{V}_{\mathrm{R} 1}=-10 \mathrm{~V}
$$

KVL (top right)

$$
-V_{R 3}-V_{R 2}-4=0
$$

KVL (bottom right)

$$
-V_{R 3}+V_{R 4}-V_{1}=0
$$

b. Set up these equations in matrix/vector form.

$$
\mathrm{M}_{1}:=\left(\begin{array}{ccc}
\frac{-1}{4 \cdot 10^{3}} & \frac{1}{5 \cdot 10^{3}} & \frac{1}{10 \cdot 10^{3}} \\
-1 & -1 & 0 \\
0 & -1 & 1
\end{array}\right) \quad \mathrm{C}_{1}:=\left(\begin{array}{c}
-10 \cdot 10^{-3} \\
4 \\
4
\end{array}\right)
$$

c. Solve for the voltages across each resistor.

Answer check: VR2 $=16.727 \mathrm{~V}$

$$
\begin{aligned}
& \mathrm{Y}:=\mathrm{M}_{1}^{-1} \cdot \mathrm{C}_{1} \\
& \mathrm{Y}=\left(\begin{array}{c}
16.727 \\
-20.727 \\
-16.727
\end{array}\right) \quad \begin{array}{l}
\text { VR2 } \\
\text { VR3 } \\
\text { VR4 }
\end{array}
\end{aligned}
$$

## 5) $\mathrm{KV} / \mathrm{KCL}$



In the above circuit,
a. Determine five linearly independent equations for the voltage across the resistors. You will have to use a combination of Ohm's Law, KCL, and KVL.

Using "normal" polarity definitions (positive on the left and top for horization and vertical resistors, respectively)

KVL around loop 1 (left) gives

$$
\mathrm{V}_{\mathrm{R} 1}+\mathrm{V}_{\mathrm{R} 2}-10=0
$$

KCL at node 1, (left)

$$
\begin{aligned}
& -\mathrm{I}_{\mathrm{R} 1}+\mathrm{I}_{\mathrm{R} 2}-\mathrm{I}_{1}+\mathrm{I}_{\mathrm{R} 4}=0 \\
& \frac{-\mathrm{V}_{\mathrm{R} 1}}{2 \mathrm{k}}+\frac{\mathrm{V}_{\mathrm{R} 2}}{6 \mathrm{k}}+\frac{\mathrm{V}_{\mathrm{R} 4}}{4 \mathrm{k}}=2 \cdot 10^{-3}
\end{aligned}
$$

KCL at node 2, (middle)

$$
\begin{aligned}
& -\mathrm{I}_{\mathrm{R} 3}-\mathrm{I}_{1}=0 \\
& \frac{-\mathrm{V}_{\mathrm{R} 3}}{3 \mathrm{k}}=2 \cdot 10^{-3}
\end{aligned}
$$

$K V L$ around $R 4, R 2$, and $R 5$ gives

$$
-\mathrm{V}_{\mathrm{R} 2}+\mathrm{V}_{\mathrm{R} 4}+\mathrm{V}_{\mathrm{R} 5}=0
$$

$$
-\mathrm{V}_{\mathrm{R} 5}+5=0
$$

b. Set up these equations in matrix/vector form.

$$
\mathrm{M}_{2}:=\left(\begin{array}{ccccc}
1 & 1 & 0 & 0 & 0 \\
\frac{-1}{2 \cdot 10^{3}} & \frac{1}{6 \cdot 10^{3}} & 0 & \frac{1}{4 \cdot 10^{3}} & 0 \\
0 & 0 & \frac{-1}{3 \cdot 10^{3}} & 0 & 0 \\
0 & -1 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & -1
\end{array}\right) \quad \mathrm{X}_{2}=\left(\begin{array}{c} 
\\
\mathrm{V}_{\mathrm{R} 1} \\
\mathrm{~V}_{\mathrm{R} 2} \\
\mathrm{~V}_{\mathrm{R} 3} \\
\mathrm{~V}_{\mathrm{R} 4} \\
\mathrm{~V}_{\mathrm{R} 5}
\end{array}\right) \quad \mathrm{C}_{2}:=\left(\begin{array}{c}
10 \\
2 \cdot 10^{-3} \\
2 \cdot 10^{-3} \\
0 \\
-5
\end{array}\right)
$$

c. Solve for the current through each resistor. Use some software like Maple or Matlab or online tools.

\(\mathrm{Y}_{1}=\left(\begin{array}{c}1 <br>
9 <br>
-6 <br>
4 <br>

5\end{array}\right)\)| VR1 |
| :---: |
| VR2 |
| VR3 |
| VR4 |
| VR5 | Last step: convert to current using ohm's law.

$t:=0,1 . .6$
$\mathrm{q}(\mathrm{t}):=\operatorname{if}(\mathrm{t} \leq 2,10+10 \cdot \mathrm{t}, \mathrm{if}(\mathrm{t} \leq 3,-5 \mathrm{t}+5, \operatorname{if}(\mathrm{t} \leq 4,-5 \mathrm{t}+5, \operatorname{if}(\mathrm{t} \leq 5,-5 \mathrm{t}+5$, if $(\mathrm{t} \leq 6,5 \mathrm{t}, 0)))))$

