#### 1) General Current, Voltage, or Power

The plot below is the net positve charge flowing in a wire vs. time. Sketch the corresponding current during the same period of time.



check

Since current is dq/dt, The answer should be the derivative of each line which will be constants 10 from 0 to 2 -40 from 2 to 3 -5 from 3 to 5 50 from 5 to 6 Electric Circuits ECSE 2010

## 2) Source devices and Total Power



a. Determine the current through the voltage source, V1. Include the direction in your answer.

$$I_{V1} = I_1(upward) = 1 \text{ mA}(upward)$$
$$I_{V1} := 1 \text{ mA}$$

b. Determine the voltage across the current source, I1. Include the polarity in our answer

Apply KVL to the loop using polarities indicated.

$$-V_{V1} + V_{R1} + V_{I1} = 0$$
  
-4 + 1 \cdot 10^{-3} \cdot (1 \cdot 10^3) + V\_{I1} = 0  
$$V_{I1} = 3V \qquad V_{I1} := 3V$$

c. Determine the power supplied/consumed by each component and show they balance to 0.

$$V_1 := 4V$$
  $I_{R1} := 1mA$   $V_{R1} := 1k\Omega \cdot 1mA = 1V$   $I_1 := 1mA$   
 $-V_1 \cdot I_{V1} + I_{R1} \cdot V_{R1} + V_{11} \cdot I_1 = 0W$ 

### 3) Nodal voltages/voltage drops/currents



a. How many nodes are in the above circuit?

There are six junction/nodes where components meet.

b. Determine the voltage at every node

Step 1 (VF, VA, and VB):

$$V_F := 0$$

$$V_{A} := 5V$$

$$V_A := V_F + V_1 = 5V$$

$$I_{31} := 3A$$

$$R_{34} := 1k\Omega$$

 $\mathbf{V}_B := \mathbf{V}_A - \mathbf{I}_{31} \cdot \mathbf{R}_{34}$ 

 $V_B = -2.995 \cdot kV$ 

Step 2: (VD, VE, and VC)

 $v_{\rm D} \coloneqq 1v$  $v_{\rm E} \coloneqq v_{\rm D} - 2v$ 

 $V_E = -1 V$ 

 $V_C := V_D + 4V$ 

$$V_{C} = 5 V$$

c. Determine the current through R3, V2, and V3 (label or indicate current direction for full credit)

$$\begin{split} I_{R3} &\coloneqq \frac{V_E}{2\Omega} = -0.5 \, \text{A} & \text{right to left} \\ \\ \text{Using KCL} \\ I_{V2} &\coloneqq -3A + 2A = -1 \, \text{A} & \text{right to left} \\ \\ I_{V3} &\coloneqq I_{R3} = -0.5 \, \text{A} & \text{down} & \text{they are in series} \end{split}$$

## 4) KVL/KCL

### In this circuit,

a. Determine five linearly independent equations for the voltage across the resistors. You will have to use a combination of Ohm's law, KCL, and KVL.

Redraw the circuit with polarities for full credit.



Using "normal" polarity definitions (positive on the left and top for horization and vertical resistors, respectively)

$$v_{R1} = (1k) \cdot -10 \cdot 10^{-3}$$
 note: if positive on left side of R1 and negative on the right  
$$10 \cdot 10^{-3} + I_{R3} + I_{R4} - I_{R2} = 0$$
 
$$v_{R1} = -10v$$

KVL (top right)

$$-V_{R3} - V_{R2} - 4 = 0$$

KVL (bottom right)

$$-V_{R3} + V_{R4} - V_1 = 0$$

b. Set up these equations in matrix/vector form.

$$M_{1} := \begin{pmatrix} \frac{-1}{4 \cdot 10^{3}} & \frac{1}{5 \cdot 10^{3}} & \frac{1}{10 \cdot 10^{3}} \\ -1 & -1 & 0 \\ 0 & -1 & 1 \end{pmatrix} \qquad C_{1} := \begin{pmatrix} -10 \cdot 10^{-3} \\ 4 \\ 4 \end{pmatrix}$$

#### c. Solve for the voltages across each resistor.

Answer check: VR2 = 16.727V

$$Y := M_1^{-1} \cdot C_1$$

$$Y = \begin{pmatrix} 16.727 \\ -20.727 \\ -16.727 \end{pmatrix}$$
VR2
VR3
VR4

## 5) KVL/KCL



In the above circuit,

a. Determine five linearly independent equations for the voltage across the resistors. You will have to use a combination of Ohm's Law, KCL, and KVL.

Using "normal" polarity definitions (positive on the left and top for horization and vertical resistors, respectively)

KVL around loop 1 (left) gives

 $V_{R1} + V_{R2} - 10 = 0$ 

KCL at node 1, (left)

$$-I_{R1} + I_{R2} - I_1 + I_{R4} = 0$$

$$\frac{-V_{R1}}{2k} + \frac{V_{R2}}{6k} + \frac{V_{R4}}{4k} = 2 \cdot 10^{-3}$$

KCL at node 2, (middle)

$$-I_{R3} - I_1 = 0$$
  
 $\frac{-V_{R3}}{3k} = 2 \cdot 10^{-3}$ 

KVL around R4, R2, and R5 gives

KVL (right)

$$-V_{R2} + V_{R4} + V_{R5} = 0 \qquad -V_{R5} + 5 = 0$$

b. Set up these equations in matrix/vector form.

$$M_{2} \coloneqq \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ \frac{-1}{2 \cdot 10^{3}} & \frac{1}{6 \cdot 10^{3}} & 0 & \frac{1}{4 \cdot 10^{3}} & 0 \\ 0 & 0 & \frac{-1}{3 \cdot 10^{3}} & 0 & 0 \\ & & & 3 \cdot 10^{3} & & \\ 0 & -1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix} \qquad X_{2} = \begin{pmatrix} V_{R1} \\ V_{R2} \\ V_{R3} \\ V_{R4} \\ V_{R5} \end{pmatrix} \qquad C_{2} \coloneqq \begin{pmatrix} 10 \\ 2 \cdot 10^{-3} \\ 2 \cdot 10^{-3} \\ 0 \\ -5 \end{pmatrix}$$

c. Solve for the current through each resistor. Use some software like Maple or Matlab or online tools.

$$Y_{1} := M_{2}^{-1} \cdot C_{2}$$

$$Y_{1} = \begin{pmatrix} 1 \\ 9 \\ -6 \\ 4 \\ 5 \end{pmatrix} \quad VR1 \\ VR2 \\ VR3 \\ VR4 \\ VR5$$

Last step: convert to current using ohm's law.

$$t := 0, 1 ... 6$$

# $q(t) := if \big( t \le 2, 10 + 10 \cdot t, if \big( t \le 3, -5t + 5, if \big( t \le 4, -5t + 5, if \big( t \le 5, -5t + 5, if \big( t \le 6, 5t, 0 \big) \big) \big) \big)$