

In the above circuit determine:

a. The equivalent resistance seen by the voltage source

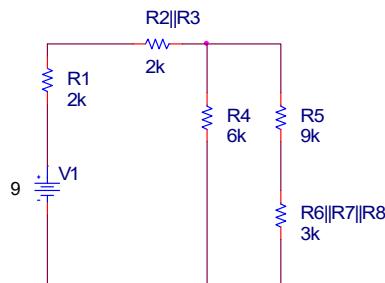
$$R_{1P1} := 2\text{k}\Omega \quad R_{4P1} := 6\text{k}\Omega \quad R_{7P1} := 8\text{k}\Omega \quad V_{1P1} := 9\text{V}$$

$$R_{2P1} := 4\text{k}\Omega \quad R_{5P1} := 9\text{k}\Omega \quad R_{8P1} := 12\text{k}\Omega$$

$$R_{3P1} := 4\text{k}\Omega \quad R_{6P1} := 8\text{k}\Omega$$

$$R_{23P1} := \frac{R_{2P1} \cdot R_{3P1}}{R_{2P1} + R_{3P1}} = 2 \times 10^3 \Omega$$

$$R_{678P1} := \frac{1}{\frac{1}{R_{8P1}} + \frac{1}{R_{7P1}} + \frac{1}{R_{6P1}}} = 3 \times 10^3 \Omega$$



$$R_{5678P1} := R_{678P1} + R_{5P1} = 1.2 \times 10^4 \Omega$$

$$R_{45678P1} := \frac{R_{4P1} \cdot R_{5678P1}}{R_{4P1} + R_{5678P1}} = 4 \times 10^3 \Omega$$

$$R_{123P1} := R_{1P1} + R_{23P1} = 4 \times 10^3 \Omega$$

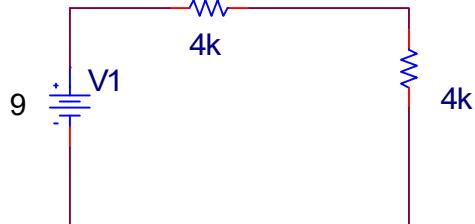
$$R_{EQP1} := R_{123P1} + R_{45678P1} = 8 \times 10^3 \Omega$$



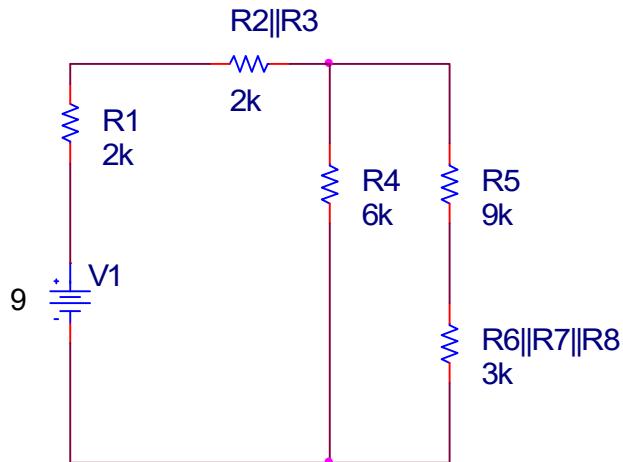
b. Find the current through the voltage source.

$$I_{V1P1} := \frac{V_{1P1}}{R_{EQP1}} = 1.125 \cdot \text{mA}$$

c. Find the voltage across R8.

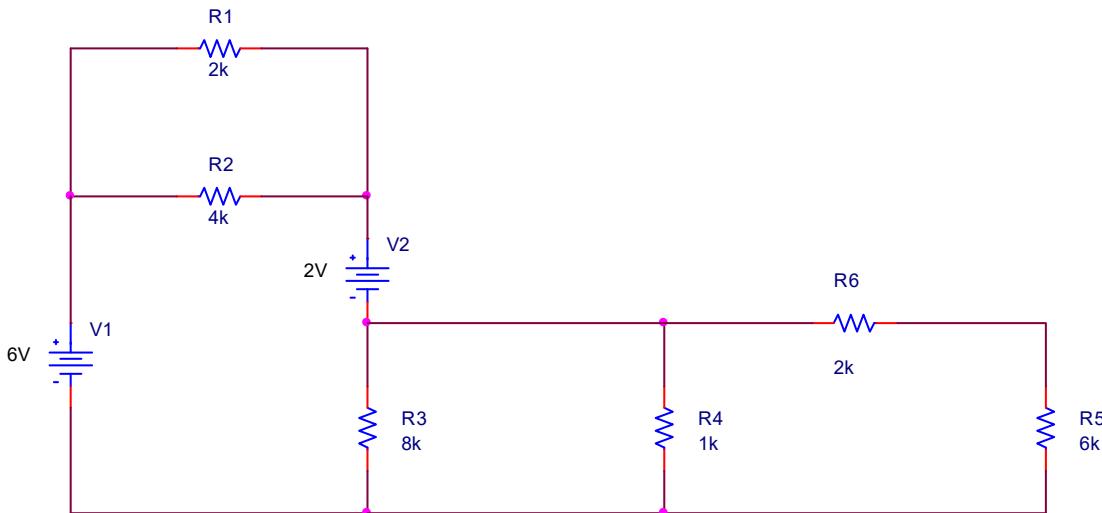


$$V_{R45678P1} := V_{1P1} \cdot \frac{R_{45678P1}}{R_{45678P1} + R_{123P1}} = 4.5 \text{ V}$$



$$V_{R45678P1} \cdot \frac{R_{678P1}}{R_{678P1} + R_{5P1}} = 1.125 \text{ V}$$

*Voltage across R6, R7 and R8 is the same.*

**2) Equivalent Circuit**

In the above circuit determine:

2.1: An equivalent source (a single voltage source)

V1 and V2 are effectively in series since the current through V1 must equal the current through V2.

$$V_1 := 6V \quad V_2 := 2V$$

$$V_{EQ} := V_1 - V_2 \quad \text{opposite polarities}$$

$$V_{EQ} = 4V$$

2.2: The equivalent resistance seen by the combined voltage source

$$R_1 := 2k\Omega \quad R_2 := 4k\Omega \quad R_3 := 8k\Omega \quad R_4 := 1k\Omega \quad R_5 := 6k\Omega \quad R_6 := 2k\Omega$$

$$R_{12} := \frac{R_1 \cdot R_2}{R_1 + R_2} = 1.333 \times 10^3 \Omega$$

$$R_{56} := R_5 + R_6 = 8 \times 10^3 \Omega$$

$$R_{3456} := \frac{1}{\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_{56}}}$$

$$R_{3456} = 0.8 \cdot k\Omega$$

$$R_{EQ} := R_{12} + R_{3456} \quad R_{EQ} = 2.133 \cdot k\Omega$$

2.3: The current through the voltage source, V1.

From part a, we know the current through VEQ is the same as the current through V1. Therefore,

$$I_{V1} := \frac{V_{EQ}}{R_{EQ}}$$

$$I_{V1} = 1.875 \cdot mA$$

2.4: The current through the resistor R6

We know the current from the source is divided between R3 in parallel with R4 and parallel with (R5+R6)

$$R_{34} := \frac{R_3 \cdot R_4}{R_3 + R_4} \quad R_{34} = 0.889 \cdot k\Omega$$

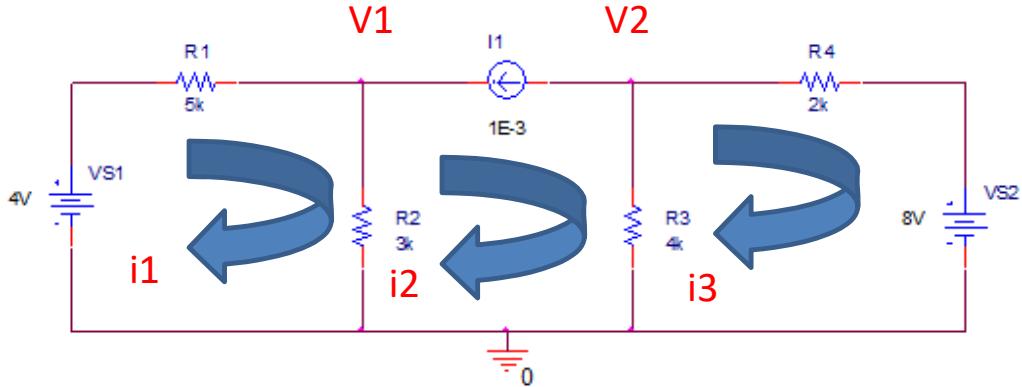
$$R_{56} = 8 \cdot k\Omega$$

Current divider equation

$$I_{R6} := \frac{R_{34}}{R_{34} + R_{56}} \cdot (I_{V1})$$

$$I_{R6} = 0.188 \cdot mA$$

## 3) Node/Mesh Analysis



3.1: Apply node analysis to determine V1 and V2

The ground node is given. Nodes at the end are constrained by voltage sources with  $V_3=4V$  and  $V_4=8V$ .

$$\text{KCL at } V_1: \frac{V_1 - V_{S1}}{5k\Omega} + \frac{V_1}{3k\Omega} - 1 \cdot 10^{-3} = 0 \quad V_{S1} := 4$$

$$\frac{V_1}{5k} - \frac{V_{S1}}{5k} + \frac{V_1}{3k} = 1 \cdot 10^{-3} \quad V_{S2} := 8$$

$$V_1 \left( \frac{1}{5k} + \frac{1}{3k} \right) = 1 \cdot 10^{-3} + \frac{V_{S1}}{5k}$$

$$V_{1b} := \frac{\left( 1 \cdot 10^{-3} + \frac{V_{S1}}{5 \cdot 10^3} \right)}{\left( \frac{1}{5 \cdot 10^3} + \frac{1}{3 \cdot 10^3} \right)}$$

$$V_{1b} = 3.375$$

KCL at  $V_2$ :

$$1 \cdot 10^{-3} + \frac{V_2}{4k} + \frac{V_2 - V_{S2}}{2k} = 0$$

$$V_2 \left( \frac{1}{4k} + \frac{1}{2k} \right) = -1 \cdot 10^{-3} + \frac{V_{S2}}{2 \cdot 10^3}$$

$$V_{2b} := \frac{-1 \cdot 10^{-3} + \frac{V_{S2}}{2 \cdot 10^3}}{\left( \frac{1}{4 \times 10^3} + \frac{1}{2 \times 10^3} \right)}$$

$$V_{2b} = 4$$

3.2: Apply mesh analysis to determine  $i_1$ ,  $i_2$ , and  $i_3$

The current in loop 2 is constrained by the current source,  $i_2 = -1 \cdot 10^{-3}$

$$i_{2b} := -1 \cdot 10^{-3} \text{ A}$$

since there is a current source that is not shared by any other loops

KVL on loop 1

$$\begin{aligned} i_1 \cdot R_1 + i_1 \cdot R_2 - i_{2b} \cdot R_2 - V_{S1} &= 0 \\ i_1 \cdot 5k + i_1 \cdot 3k + 1 \cdot 10^{-3} \cdot 3k - V_{S1} &= 0 \end{aligned}$$

$$i_{1b} := \frac{V_{S1} + -1 \cdot 10^{-3} \cdot 3 \cdot 10^3}{8 \times 10^3}$$

$$i_{1b} = 1.25 \times 10^{-4}$$

$$i_{1b} = 0.125 \text{ mA}$$

KVL on loop 3

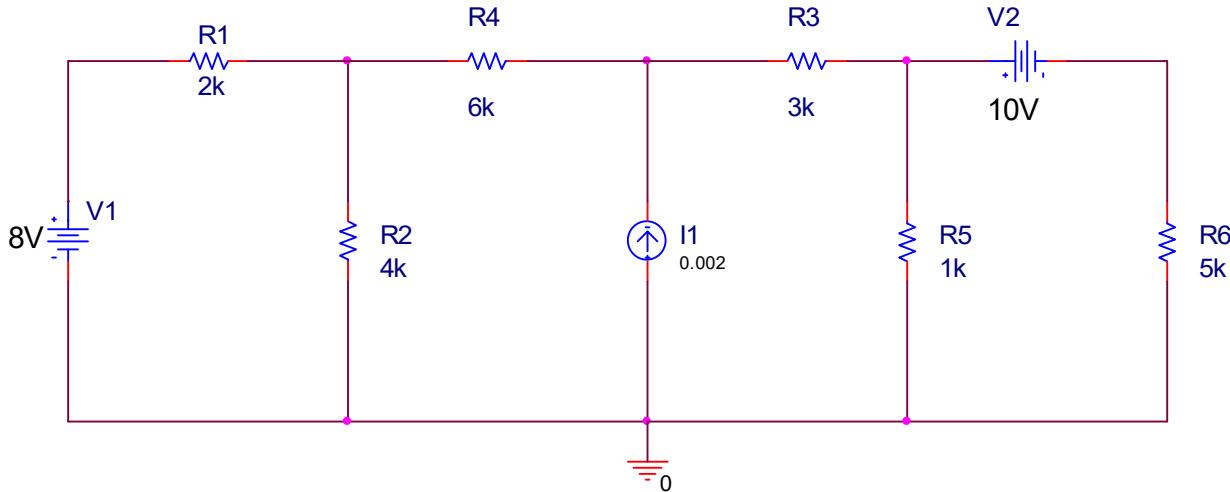
$$\begin{aligned} i_3 \cdot R_4 + V_{S2} + i_3 \cdot R_3 + i_2 \cdot R_3 &= 0 \\ i_3 \cdot 2k + V_{S2} + i_3 \cdot 4k + 1 \cdot 10^{-3} \cdot 4k &= 0 \end{aligned}$$

$$i_3 \cdot 6k = -V_{S2} + (1 \cdot 10^{-3} \cdot 4 \cdot 10^3) \quad \frac{V_{S2} + (1 \cdot 10^{-3} \cdot 4 \cdot 10^3)}{6 \cdot 10^3} = 2 \times 10^{-3}$$

$$i_3 := \frac{V_{S2} + (1 \cdot 10^{-3} \cdot 4 \cdot 10^3)}{6 \cdot 10^3}$$

$$i_3 = 2 \text{ mA}$$

## 4) Superposition



3.1: Use any method in the parentheses to determine the voltage across R2 (node, mesh, circuit reduction, source transformation)

$$\begin{aligned}
 V_{1p4} &:= 8 & R_{1p4} &:= 2 \cdot 10^3 & R_{2p4} &:= 4 \cdot 10^3 & R_{4p4} &:= 6 \cdot 10^3 \\
 I_{1p4} &:= 0.002 & R_{3p4} &:= 3 \cdot 10^3 & R_{5p4} &:= 1 \cdot 10^3 & V_{2p4} &:= 10 \\
 R_{6p4} &:= 5 \cdot 10^3
 \end{aligned}$$

Nodal analysis

$$\frac{V_A - V_{1p4}}{R_{1p4}} + \frac{V_A}{R_{2p4}} + \frac{V_A - V_B}{R_{4p4}} = 0$$

$$V_A \left( \frac{1}{R_{1p4}} + \frac{1}{R_{2p4}} + \frac{1}{R_{4p4}} \right) - V_B \left( \frac{1}{R_{4p4}} \right) = \frac{V_{1p4}}{R_{1p4}}$$

$$\frac{V_B - V_A}{R_{4p4}} - I_{1p4} + \frac{V_B - V_C}{R_{3p4}} = 0$$

$$-V_A \left( \frac{1}{R_{4p4}} \right) + V_B \left( \frac{1}{R_{4p4}} + \frac{1}{R_{3p4}} \right) - V_C \left( \frac{1}{R_{3p4}} \right) = I_{1p4}$$

$$\frac{V_C - V_B}{R_{3p4}} + \frac{V_C}{R_{5p4}} + \frac{V_C - V_{2p4}}{R_{6p4}} = 0$$

$$-V_B \left( \frac{1}{R_{3p4}} \right) + V_C \left( \frac{1}{R_{3p4}} + \frac{1}{R_{5p4}} + \frac{1}{R_{6p4}} \right) = \frac{V_{2p4}}{R_{6p4}}$$

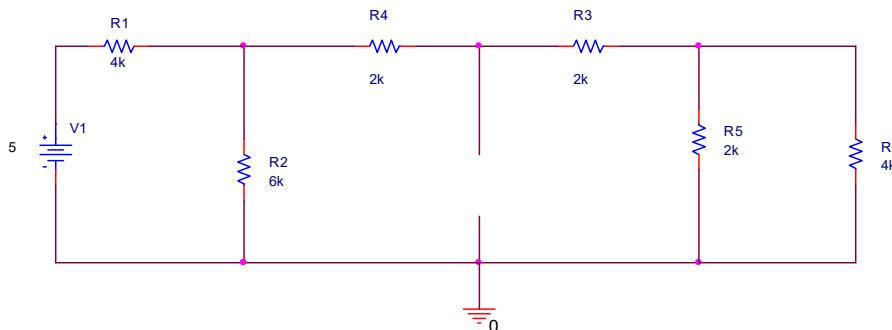
$$M_2 := \begin{bmatrix} \left( \frac{1}{R_{1p4}} + \frac{1}{R_{2p4}} + \frac{1}{R_{4p4}} \right) & -\frac{1}{R_{4p4}} & 0 \\ -\frac{1}{R_{4p4}} & \left( \frac{1}{R_{4p4}} + \frac{1}{R_{3p4}} \right) & -\frac{1}{R_{3p4}} \\ 0 & -\frac{1}{R_{3p4}} & \left( \frac{1}{R_{3p4}} + \frac{1}{R_{5p4}} + \frac{1}{R_{6p4}} \right) \end{bmatrix}$$

$$C_2 := \begin{pmatrix} \frac{V_{1p4}}{R_{1p4}} \\ I_{1p4} \\ \frac{V_{2p4}}{R_{6p4}} \end{pmatrix}$$

$$M_2^{-1} \cdot C_2 = \begin{pmatrix} 5.811 \\ 7.96 \\ 3.035 \end{pmatrix} \quad V_{R2} = V_A = 5.81V$$

3.2: Find VR2 using superposition. (For each source, draw the schematic).

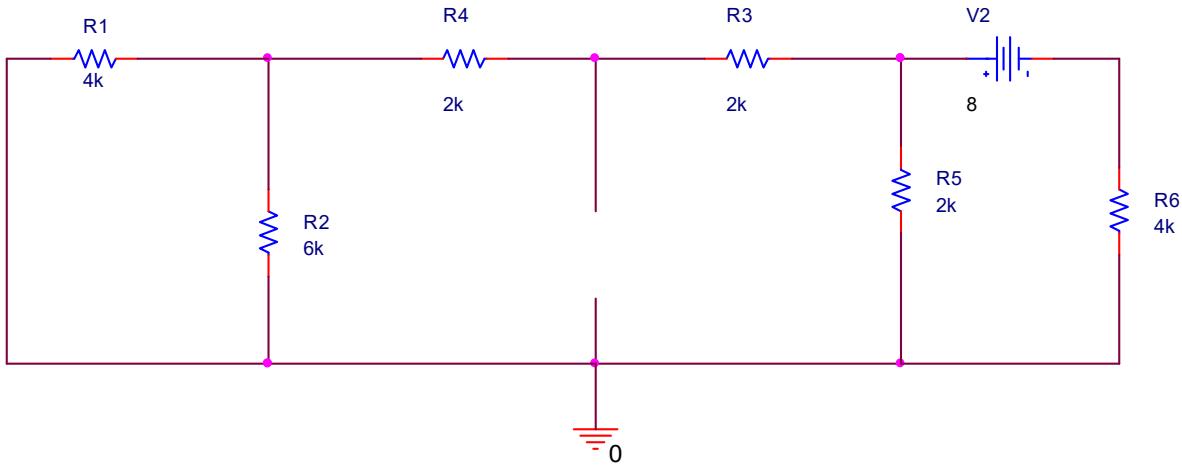
For V1:



$$R_{24356} := \frac{\left( \frac{R_{6p4} \cdot R_{5p4}}{R_{6p4} + R_{5p4}} + R_{3p4} + R_{4p4} \right) \cdot R_{2p4}}{\left( \frac{R_{6p4} \cdot R_{5p4}}{R_{6p4} + R_{5p4}} + R_{3p4} + R_{4p4} \right) + R_{2p4}} = 2.843 \times 10^3$$

$$V_{R2V1} := V_{1p4} \cdot \frac{R_{24356}}{R_{1p4} + R_{24356}} = 4.697$$

For V2:

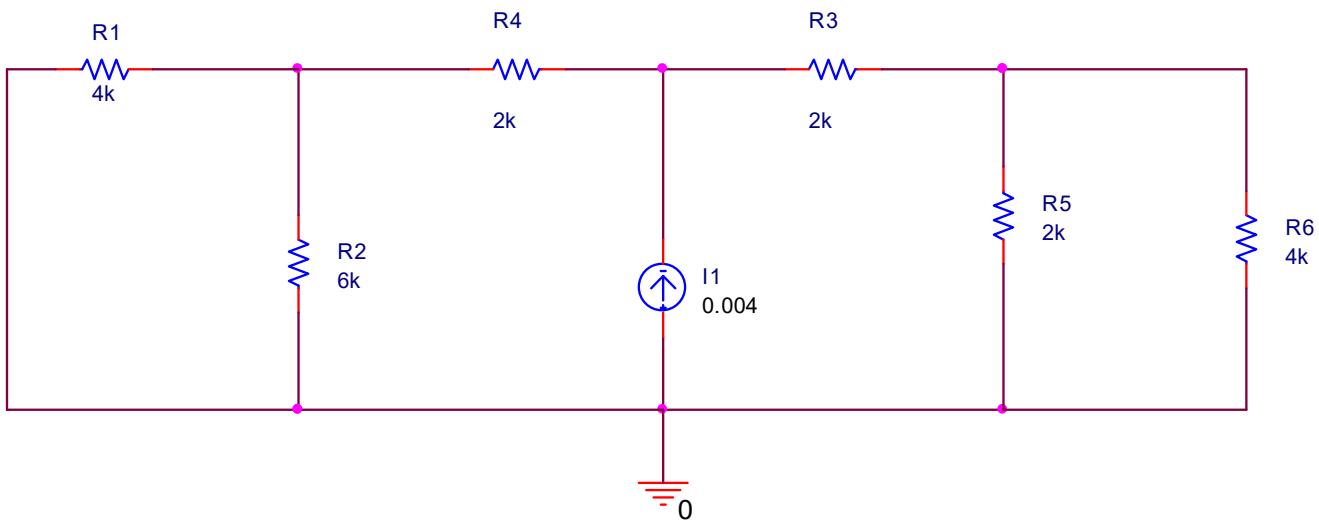


$$R_{12435} := \frac{\left( \frac{R_{1p4} \cdot R_{2p4}}{R_{1p4} + R_{2p4}} + R_{4p4} + R_{3p4} \right) \cdot R_{5p4}}{\left( \frac{R_{1p4} \cdot R_{2p4}}{R_{1p4} + R_{2p4}} + R_{4p4} + R_{3p4} \right) + R_{5p4}} = 911.765$$

double voltage divider

$$V_{R2V2} := \frac{R_{12435}}{R_{12435} + R_{6p4}} \cdot V_{2p4} \cdot \frac{\frac{R_{1p4} \cdot R_{2p4}}{R_{1p4} + R_{2p4}}}{\left( \frac{R_{1p4} \cdot R_{2p4}}{R_{1p4} + R_{2p4}} + R_{4p4} + R_{3p4} \right)} = 0.199$$

For I1:



double current divider

$$V_{R2I1} := \frac{\frac{R_{5p4} \cdot R_{6p4}}{R_{5p4} + R_{6p4}} + R_{3p4}}{\left( \frac{R_{1p4} \cdot R_{2p4}}{R_{1p4} + R_{2p4}} \right) + R_{4p4} + \left( \frac{R_{5p4} \cdot R_{6p4}}{R_{5p4} + R_{6p4}} + R_{3p4} \right)} \cdot I_{1p4} \cdot \frac{R_{1p4}}{R_{1p4} + R_{2p4}} \cdot R_{2p4} = 0.915$$

$$V_{R2V1} + V_{R2V2} + V_{R2I1} = 5.811 \text{ V}$$