1) Equivalent impedances

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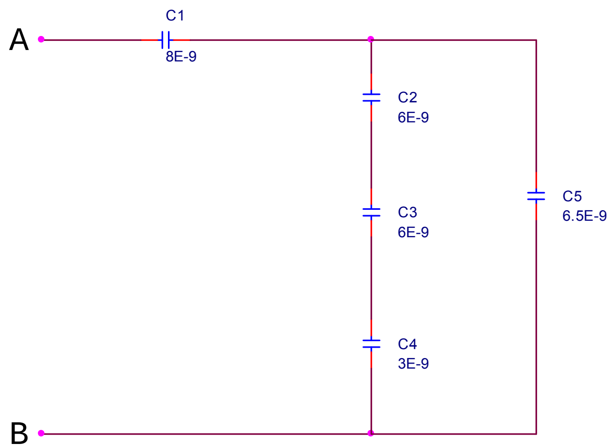


$C_1 := 8 \cdot 10^{-9} F$ $C_2 := 6 \cdot 10^{-9} F$ $C_3 := 6 10^{-9} F$ $C_4 := 3 \cdot 10^{-9} F$ $C_5 := 6.5 \cdot 10^{-9} F$

1.1: For the above circuit, determine the equivalent capacitance between A and B

Note: capacitors in series add like resistors in parallel

Long way:

$$C23 := \frac{C_2 \cdot (C_3)}{C_2 + C_3}$$

$$C23 = 3 \times 10^{-9} F$$

$$C234 := \frac{C23 \cdot C_4}{C23 + C_4}$$

$$C234 = 1.5 \times 10^{-9} F$$

$$C2345 := C234 + C_5$$

$$C2345 = 8 \times 10^{-9} F$$

$$CTotal := \frac{C2345 \cdot C_1}{C2345 + C_1}$$

$$CTotal = 4 \times 10^{-9} F$$

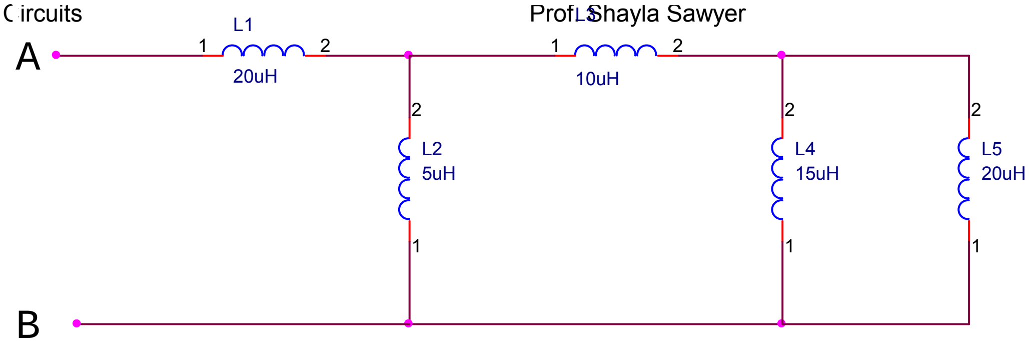
Short way:

$$TotC := \left[\left[\left(\frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} \right)^{-1} + C_5 \right]^{-1} + \frac{1}{C_1} \right]^{-1}$$

$$TotC = 4 \times 10^{-9} F$$

C2,C3,C4 in series + C5 in parallel which are all in series with C1

1.2:



For the above circuit, determine the equivalent inductance between A and B

$L_1 := 20\mu H$ $L_2 := 5\mu H$ $L_3 := 10\mu H$ $L_4 := 15\mu H$ $L_5 := 20\mu H$

Note: Inductors in parallel add like resistors in parallel....

Long way:

$$L45 := \frac{L_4 \cdot L_5}{L_4 + L_5}$$

$$L45 = 8.571 \times 10^{-6} H$$

$$L345 := L45 + L_3$$

$$L345 = 1.857 \times 10^{-5} H$$

$$L2345 := \frac{L_2 \cdot L345}{L_2 + L345}$$

$$L2345 = 3.939 \times 10^{-6} H$$

$$L_T := L2345 + L_1$$

$$L_T = 23.939 \cdot \mu H$$

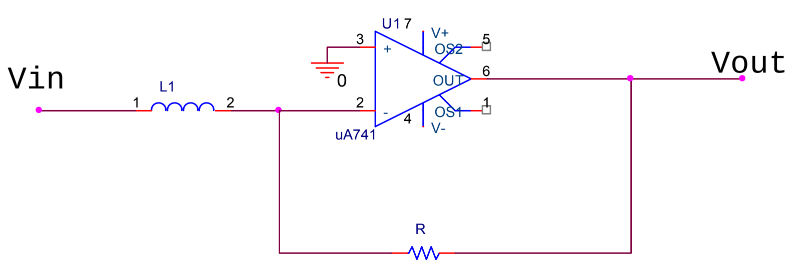
Short way:

$$Tot_L := \left[\left(\frac{L_4 \cdot L_5}{L_4 + L_5} + L_3\right)^{-1} + \frac{1}{L_2}\right]^{-1} + L_1$$

$$Tot_L = 23.939 \cdot \mu H$$

If students use sL, which is more accurate, or this answer mark correct.

2) Amplifier circuits



2.1: For the RL amplifier circuit, determine the relationship between Vout and Vin. As with RC amplifier circuits, KCL is a good starting point. (The power is taken out for simplicity but the op amp is powered).

Apply KCL at the negiative input node

$$I_1 + I_2 + I_3 = 0$$

$$I_1 = \frac{1}{L}\cdot\int V_{.L}\,dt = \frac{1}{L}\cdot\int (0 - Vin)\,dt = \frac{-1}{L}\cdot\int V_{in}\,dt$$

I1 is the current through the inductor. The negative terminal is also 0 since vp=vn and vp is grounded.
VL=0-Vin with current going away from the node.

$$I_2 = \frac{0 - Vout}{R} = \frac{-Vout}{R}$$

I2 is the current through the resistor

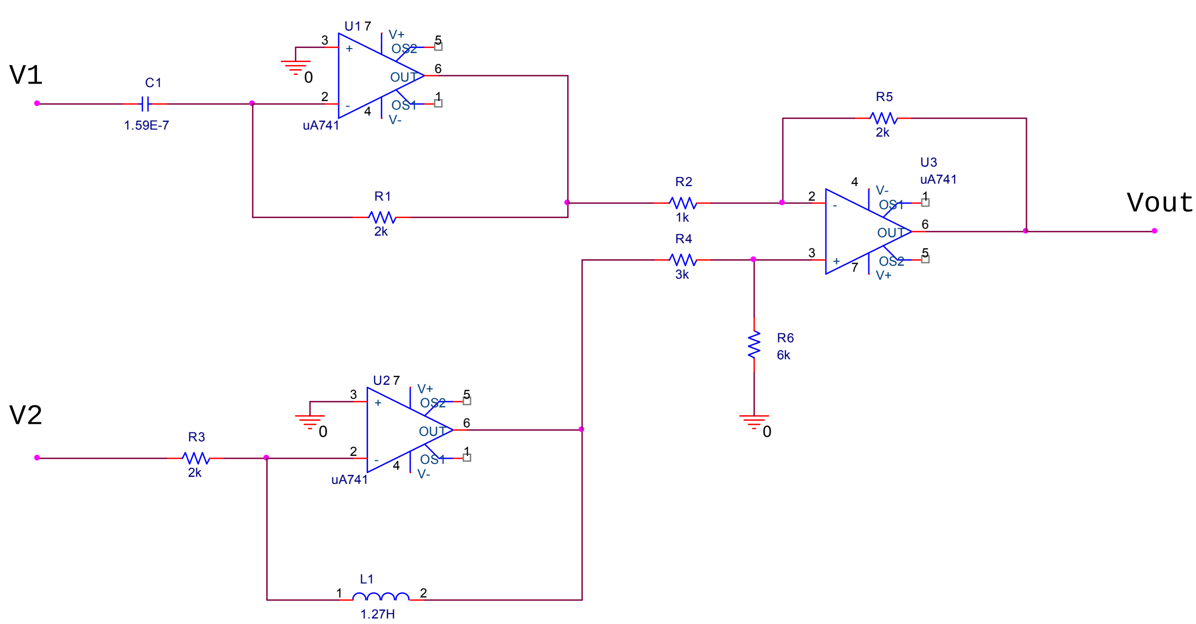
I3 is zero since there is now current draw in an ideal op amp.

$$\frac{-Vout}{R} - \frac{1}{L}\cdot\int V_{in}\,dt + 0 = 0$$

$$Vout = \frac{-R}{L}\cdot\int V_{in}\,dt$$

2.2: What type of op amp is this?

It is an integrator (the signal is also inverted, it is ok if students mention this).



2.3: In the above circuit V1=V2=1sin(2π f t) where the frequency is 1 kHz. Determine Vout.

Opamp U1 is a differentiator circuit

$$Vout_{U1} = -RC\cdot\frac{dV_{in}}{dt}$$

$$Vout_{U1} = -\left(2\cdot 10^{3}\right)\cdot\left(1.59\cdot 10^{-7}\right)\cdot(2000\pi)\cdot\cos(2000\pi t) = -2\cdot\cos(2000\pi t)$$ [v]

Opamp U2 is a differentiator circuit

$$Vout_{U2} = \frac{-L}{R}\cdot\frac{dV_{in}}{dt}$$

$$Vout_{U2} = \frac{-1.27}{2\cdot 10^{3}}\cdot[(2000\pi)\cdot\cos(2000\pi t)] = -4\cos(2000\pi t)$$ [v]

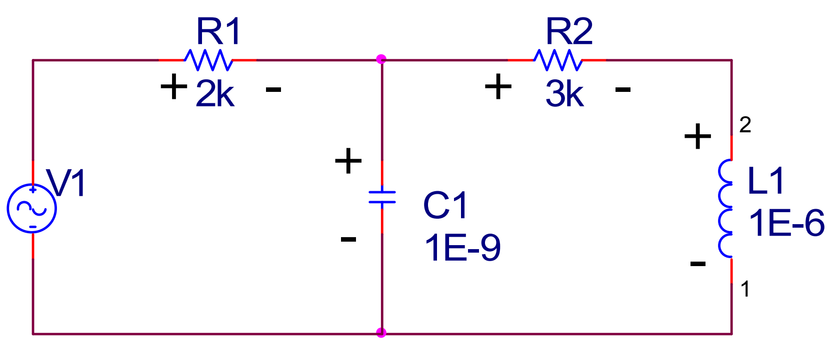
Opamp U3 is a difference amplifier

Note: R4/R2=R6/R5 so the equation can be simplified

$$Vout_{U3} = \frac{R_5}{R_2}\cdot[-4\cdot\cos(2000\pi t) - (-2\cdot\cos(2000\pi t))]$$

$$Vout_{U3} = 2\cdot -2\cdot\cos(2000\pi t) = -4\cdot\cos(2000\pi t)$$ [v]

3) Voltage/Current continuity



In the above circuit, the voltage is defined as follows:

$$V1 = \begin{cases} 5V & t < 0 \\ 10V & 0 < t \end{cases}$$ (the voltage source turns on at t = 0)

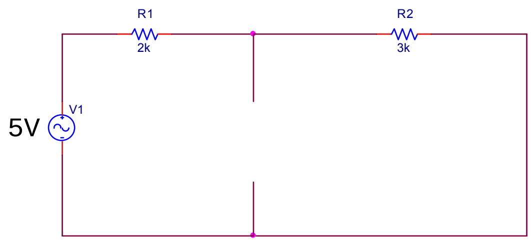
A piecewise solution with the step function can also be counted on this homework.

3.1: Determine a mathematical expression for the source.

$5 + 5u(t)$ When t is zero u(t) = 0 therefor 5 + 5*0 = 5
When t goes to infinity u(t) is 1 therefore 5+5*1= 10

3.2: At t =0- (just before the voltage changes), for the polarities indicated, determine the voltage across each component and the current through each component.

At t=0-, the souce is 5V. Since the intial conditions are V1=5V and dc stead state, the inductor is a short and the capacitor is an open circuit.



Voltage divider

$$V_{R1} := 5V \cdot \frac{2 \cdot 10^3}{2 \cdot 10^3 + 3 \cdot 10^3}$$

$$V_{R1} = 2V$$

$$V_{R2} := 5V \cdot \frac{3 \cdot 10^3}{2 \cdot 10^3 + 3 \cdot 10^3}$$

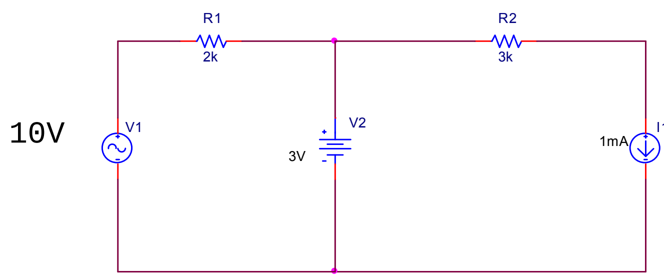
$$V_{R2} = 3V$$

Circuit analysis gives the following results

Component	Voltage	Current
R1	2 V	1mA
R2	3V	1mA
C1	3V	0
L1	0	1 mA

3.3: At $t=0^+$ (just after the voltage changes), determine the voltage across each component and the current through each component for the polarities indicated in the circuit.

Because of continuity IL(0-)=IL(0+) like a current source
VC(0-) = VC(0+) like a voltage source



Must use Unit 1 circuit analysis methods!

$$V_{R1b} := 10V - 3V$$

$$V_{R1b} = 7V$$

$$I_{R1b} := \frac{V_{R1b}}{2 \cdot 10^3 \Omega} = 3.5 \times 10^{-3} A$$

$$I_{R2b} := 1mA$$

$$V_{R2b} := 3 \cdot 10^3 \Omega \cdot 1mA$$

$$V_{R2b} = 3V$$

For IC1 use KCL

$$1mA + \left(\frac{3V - 10V}{2 \cdot 10^3 \Omega}\right) + I_{C1} = 0 \qquad 1mA + \left(\frac{3V - 10V}{2 \cdot 10^3 \Omega}\right) = -2.5 \cdot mA$$

$$I_{C1} := 2.5mA$$

For VI1 use KVL

CHECK

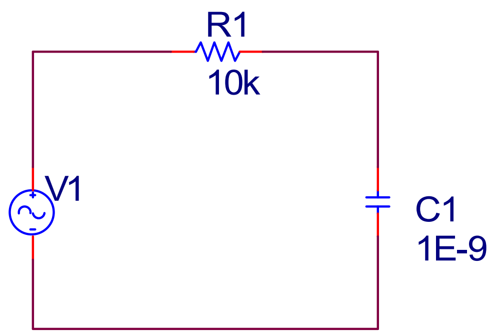
$$-3 + 3 + V_{I1} = 0$$

$$V_{I1} = 0$$

Circuit analysis gives the following results

Component	Voltage	Current
R1	7V	3.5mA
R2	3 V	1mA
C1	3 V (continuity)	2.5mA (KCL)
L1	0V (KVL)	1mA (continuity)

4) First order circuits



4.1: Determine the voltage as a function of time for the source voltage V1 = 10 u(t).

This is the familiar RC circuit with a step function source, which has a solution for t>0 of the form

$$V_c(t) = A_1 \cdot \exp\left(\frac{-t}{RC}\right) + A_2 = A_1 \cdot \exp\left(\frac{-t}{1 \cdot 10^{-5}}\right) + A_2$$

RC time constant

$$10 \cdot 10^3 \cdot 1 \cdot 10^{-9} = 1 \times 10^{-5}$$

The inital condition at t = 0 is Vc(t) = 0 = A1+A2
The steady state condition as t approaches ∞ (capacitor acts an a open circuit) is Vc= 10 = 0 + A2
The two conditions lead to A1 = -10 and A2 = 10

$$V_c(t) = -10 \cdot \exp\left(\frac{-t}{1 \cdot 10^{-5}}\right) + 10 \quad [V]$$

4.2: Determine the voltage as a function of time for the source voltage

$$V1 = \begin{cases} 0 & t < 0 \\ 10 & 0 < t < 0.001 \\ 0 & 0.001 < t \end{cases} \quad \text{[v]}$$

For 0<t<0.001 the solution for part a is the same expression

$$V_{c1}(t) = -10 \cdot \exp\left(\frac{-t}{1 \cdot 10^{-5}}\right) + 10$$

For 0.001<t, the solution takes the form

$$V_{c2}(t) = A_3 \cdot \exp\left[\frac{-(t - 0.001)}{1 \cdot 10^{-5}}\right] + A4 \quad \text{[v]}$$

For 0.001<t The DC steady state is zero since the source is off, 0 = 0 +A4, giving A4 = 0

For 0.001<t, when t = 0.001 we have continuity (and the circuit is in steady since since 0.001>>RC).

$V_c(0.001-) = V_c(0.001+) = 10V$

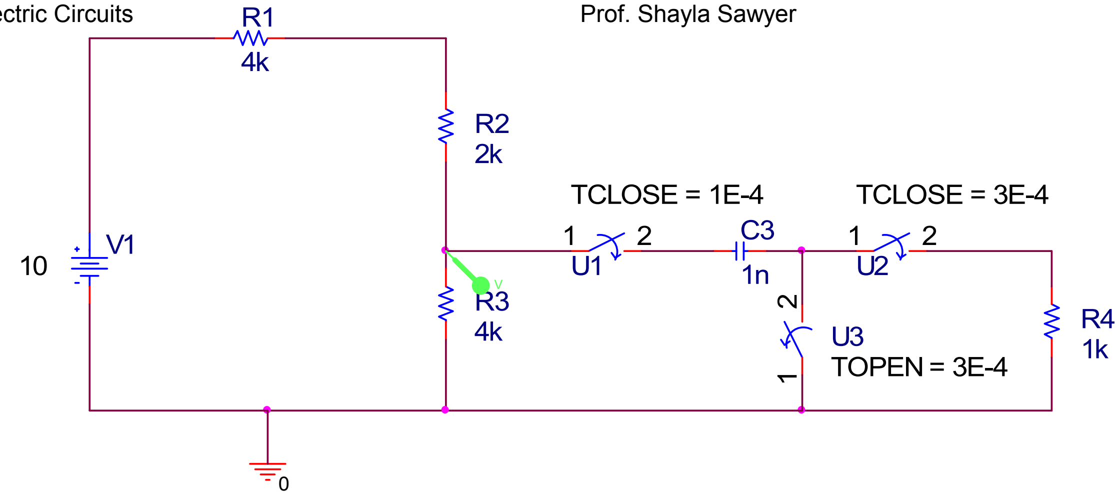
$10 = A3 + A4$

$10 = A_3 + 0$

$A_3 = 10$

$$v_c(t) = \begin{cases} V_{c1}(t) = -10 \cdot \exp\left(\frac{-t}{1 \cdot 10^{-5}}\right) + 10 \\ V_{c2}(t) = 10 \cdot \exp\left[\frac{-(t - 0.001)}{1 \cdot 10^{-5}}\right] \end{cases} \quad \text{[v]}$$

5. First order switching circuit



In the above circuit, the voltage source turns on at t = 0. Switch U1 closes at t = 0.1 ms. Switch U2 closes and switch U3 opens at t = 0.3 ms (effectively putting resistor R3 in series with C3 at t = 0.3 ms).

5.1: Determine the voltage across R3 as a function of time for t > 0.

Three regions of interest

$t < 0.1ms$

$0.1ms < t < 0.3ms$

$0.3ms < t$

For t<0.1ms, the circuit is a voltage divier with R1, R2 and R3

$$V_{R3} = V_1 \cdot \frac{R_3}{R1 + R_2 + R_3} = 10 \cdot \frac{4k}{10K} = 4V$$

For 0.1ms<t<0.3 ms, the circuit is an RC circuit. Use a Thevenin transformation

$$V_{TH} = 4V \qquad R_{TH} = 2.4k\Omega$$

The voltage across R3 is equal to the voltage acorss the capacitor. Including time delay

$$V_{R3}(t) = V_c(t) = -A_1 \cdot \exp\left[\frac{-(t - 0.0001)}{2.4 \cdot 10^{-6}}\right] + A_2$$

The initial condition at t = 0.1 ms is Vc(t) = 0 = A1 + A2

The steady state condition at t approaches ∞ is Vc(t) = 4 = 0+A2

Therefore A1 = -4 and A2 = 4

For 0.3ms<t, the capacitor reaches steady state voltage well before 0.3 ms. Essentially, the capacitor is an open circuit. Adding a resistor in seires with the capacitor at this point will not introduce change in the circuit Therefore...

$$V_{R3} = V_{cap} = 4V$$

$$V_{R3}(t) = \begin{cases} 4 & t < 0.1\text{mS} \\ -3\exp\left[\dfrac{-(t-0.001)}{2.4\cdot 10^{-6}}\right] + 4 & 0.1\text{ms} < t < 0.3\text{ms} \\ 4 & 0.3\text{ms} < t \end{cases}$$