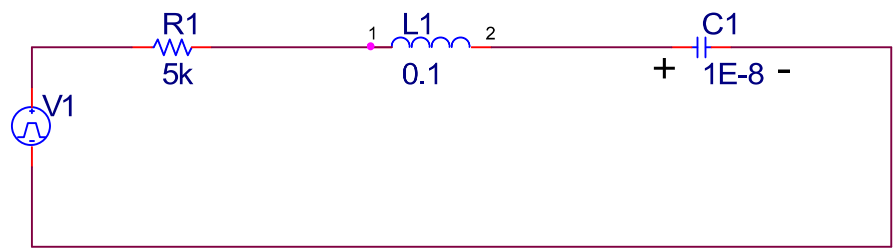
1) Initial Values, Final Values



At t = 0^-, the voltage across the capacitor is 5V (polarity shown), the current through the inductor is 2mA to the 'right' and the source is 10V. At t = 0^+, the voltage source becomes 5V and doesn't change for t > 0.

a. Determine the voltage across each component for t=0-. Determine the current through each component for t = 0-.

t=0- circuit, the inductor current is 2 mA and the capacitor voltage is 5V

$V_{1t0-} := 10V$ $\quad$ $I_{Lt0-} := 2mA$

$V_{Ct0-} := 5V$

$V_{Rt0-} := I_{Lt0-} \cdot 5 \cdot 10^3 \Omega$

$V_{Rt0-} = 10\,V$ $\quad$ Ohm's Law

To find VL use KVL

$V_{Lt0-} := -V_{Rt0-} - V_{Ct0-} + V_{1t0-}$

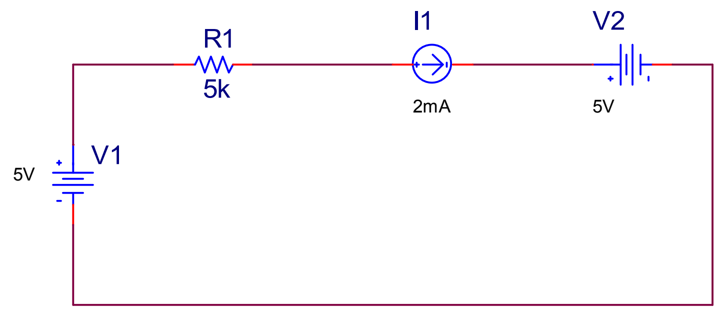
$V_{Lt0-} = -5\,V$

Current through each component must be the same because it is a series circuit

$I_{V1t0-} = I_{Rt0-} = I_{Lt0-} = I_{Ct0-} = 2mA$

b. Determine the voltage across each component for t = 0+. Determine the current across each component for t = 0+.

The t = 0+ circuit can then be considered (at that instant in time)



Remember continuity! $I_L(0\text{-}) = I_L(0+) = 2mA$ $V_C(0\text{-}) = V_C(0+) = 5V$

$I_{Lt0+} := I_{Lt0-}$

$V_{1t0+} := 5V$ from problem definition

$V_{Ct0+} := 5V$ from continuity

$V_{Rt0+} := I_{Lt0+} \cdot 5k\Omega$

$V_{Rt0+} = 10\,V$ Ohm's law

To find VL use KVL

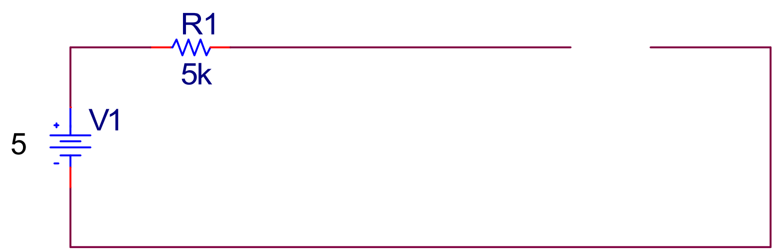
$V_{Lt0+} := -V_{Rt0+} - V_{Ct0+} + V_{1t0+}$

$V_{Lt0+} = -10\,V$

Current through each component must be the same because it is a series circuit

$I_{V1t0+} = I_{Rt0+} = I_{Lt0+} = I_{Ct0+} = 2mA$

c. Determine the voltage across each component for t goes to ∞. Determine the current across each component for to goes to ∞.



At steady state, the inductor is a short and the capacitor is an open circuit

$V_{1tinf} := 5V$ from defintion

$V_{Ctinf} := 5V$ Capacitor reach steady state...open circuit voltage

$V_{Rtinf} := 0mA \cdot 5k\Omega$ Inductor reach steady state...no current

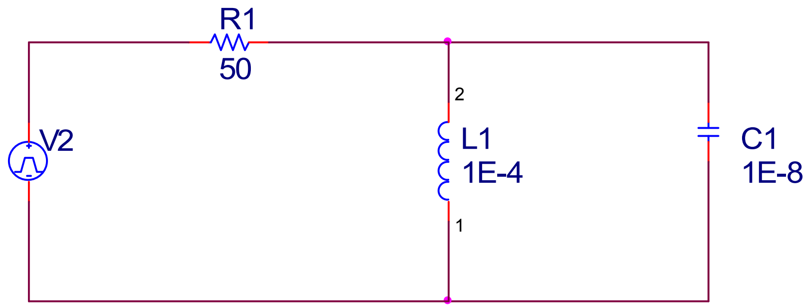
$V_{Rtinf} = 0$

To find VLinf use KVL

$V_{Ltinf} := -V_{Rtinf} - V_{Ctinf} + V_{1tinf}$

$V_{Ltinf} = 0\,V$

$I_{V1tinf} = I_{Rtinf} = I_{Ltinf} = I_{Ctinf} = 0mA$ Series circuit with an open component!



At t = 0^-, the voltage across the capacitor is 8V, the current through the inductor is 10mA 'downward' and the source is 10V. At t = 0^+, the voltage source becomes 3V and doesn't change for t > 0.

d. Determine the voltage across each component for t = 0-. Determine the current through each component for t = 0-. Determine the source voltage at t = 0-.

t = 0- the inductor current is 10 mA and the capacitor voltage is 8V

$V_{2t0-} := 10V$ given in the problem

$V_{C1t0-} := 8V$ given in the problem

$V_{L1t0-} := 8V$ in parallel with C1

$V_{R1t0-} := V_{2t0-} - V_{L1t0-}$ Nodal voltages at V2 and VL

$V_{R1t0-} = 2\,V$

$I_{L1t0-} := 10mA$ given in the problem

$$I_{R1t0-} := \frac{V_{R1t0-}}{50\Omega}$$

$I_{R1t0-} = 40 \cdot mA$

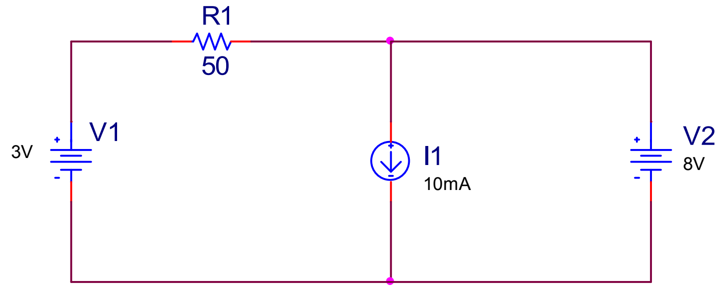
$I_{V2t0-} := 40mA$ In series with R1

$I_{C1t0-} := I_{R1t0-} - I_{L1t0-}$ KCL at node above L1

$I_{C1t0-} = 30 \cdot mA$

e. Determine the voltage across each component for t = 0+. Determine the current across each component the t = 0 +.

V1 is V2



t = 0+ circuit, the inductor current is 5 mA and the capacitor voltage is 10V

$V_{2t0+} := 3V$ given in the problem

$V_{C1t0+} := 8V$ continuity

$V_{L1t0+} := 8V$ in parallel with C1

$V_{R1t0+} := V_{2t0+} - V_{L1t0+}$ Nodal voltages

$V_{R1t0+} = -5\ V$

$I_{L1t0+} := 10mA$ continuity

$$I_{R1t0+} := \frac{V_{R1t0+}}{50\Omega}$$

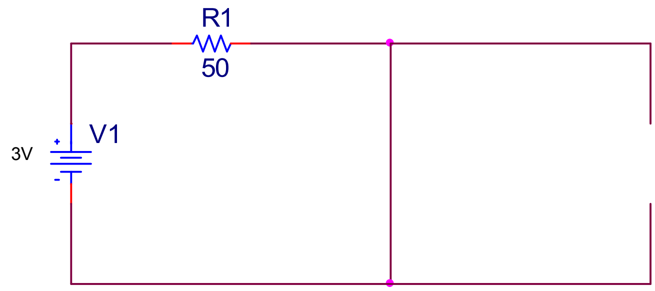
$I_{R1t0+} = -100 \cdot mA$

$I_{V2t0+} := -100mA$ In series with R1

$I_{C1t0+} := -I_{L1t0+} + I_{R1t0+}$

$I_{C1t0+} = -110 \cdot mA$ KCL upward current

f. Determine the voltage across each componenet for t goes to ∞. Determine the current across each component for t goes to ∞.



At steady state, the inductor is a short and the capacitor is an open circuit

$V_{2tinf} := 3V$ Given

$V_{C1tinf} := 0V$ in parallel with a short

$V_{L1inf} := 0V$ short circuit

$V_{R1inf} := V_{2tinf} - V_{L1inf}$ nodal voltages

$V_{R1inf} = 3\,V$

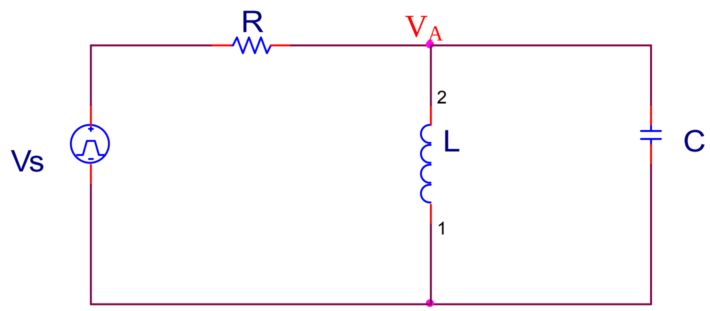
$$I_{R1inf} := \frac{V_{R1inf}}{50\Omega}$$

$I_{R1inf} = 60 \cdot mA$

$I_{L1inf} := 60mA$ In series with R1

$I_{C1inf} := 0mA$ Open circuit

2) Circuits and Differential Equations



a. In the above the circuit, find the differential equation for the voltage across the capacitor C, VC(t). The source is an arbitrary source.

Applying KCL at VA, I1 + I2 + I3 = 0, where

$$I_1 = \frac{(V_A - V_s)}{R}$$

$$I_2 = I_L = \frac{1}{L}\cdot\int V_L\,dt = \frac{1}{L}\cdot\int V_A\,dt$$

$$I_3 = C\cdot\frac{dV_C}{dt} = C\cdot\frac{dV_A}{dt}$$

Recognizing that VA=Vc and adding the terms

$$\frac{V_C}{R} + \frac{1}{L}\cdot\int V_C\,dt + C\cdot\frac{dV_c}{dt} = \frac{V_s}{R}$$

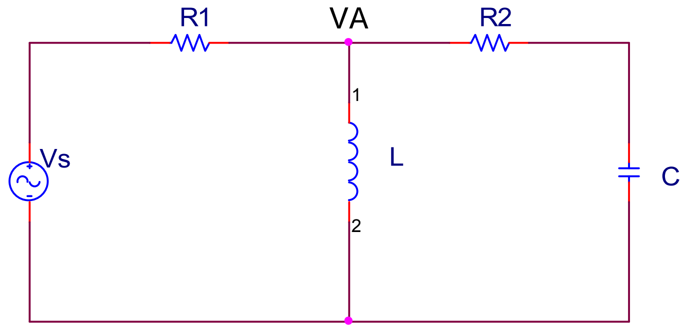
Get rid of the integral by differentiating, then rearrange

$$\frac{d^2V_C}{dt^2} + \frac{1}{RC}\cdot\frac{dV_C}{dt} + \frac{1}{LC}\cdot V_C = \frac{1}{RC}\cdot\frac{dV_s}{dt}$$

b. For the differential equation, determine the expression for the attenuation constant α, and the resonant frequency, ω_o.

$$\alpha = \frac{1}{2RC} \qquad \omega_o = \frac{1}{\sqrt{LC}}$$

c. In the circuit below, find a differential equation for the voltage across C, Vc(t). The source is an arbitrary source.



$$I_{R1} + I_L + I_{R2} = 0$$

$$I_{R1} = \frac{V_A - V_s}{R_1} \qquad I_L = \frac{1}{L}\cdot\int V_A\,dt \qquad I_{R2} = I_C = C\cdot\frac{dV_C}{dt}$$

substitute in

$$\frac{V_A}{R_1} + \frac{1}{L}\cdot\int V_A\,dt + C\cdot\frac{dV_C}{dt} = \frac{V_s}{R_1}$$

$$V_A = V_C + V_{R2} = V_C + I_{R2}\cdot R_2 = V_C + I_C\cdot R_2 = V_C + R_2\cdot C\cdot\frac{dV_C}{dt}$$

$$\frac{1}{R_1}\cdot\left(V_C + R_2\cdot C\cdot\frac{dV_C}{dt}\right) + \frac{1}{L}\cdot\int V_C + R_2\cdot C\cdot\frac{dV_C}{dt}\,dt + C\cdot\frac{dV_c}{dt} = \frac{V_s}{R_1}$$

Differentiating and rearranging

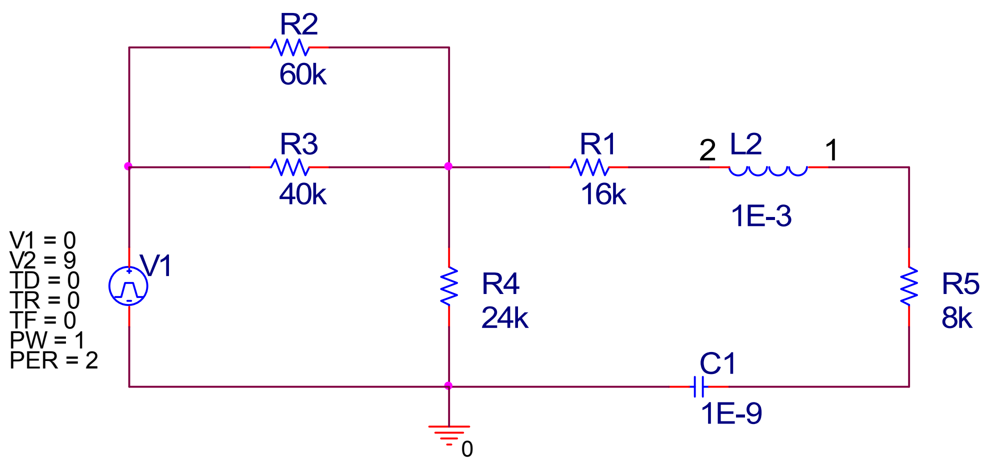
$$C\cdot\left(1 + \frac{R_2}{R_1}\right)\cdot\frac{d^2V_C}{dt^2} + \frac{1}{R_1}\cdot\frac{d\cdot V_C}{dt} + \frac{1}{L}\cdot V_C = \frac{1}{R_1}\cdot\frac{dV_s}{dt}$$

$$\frac{d^2V_C}{dt^2} + \frac{1}{C\cdot(R_1 + R_2)}\cdot\frac{dV_C}{dt} + \frac{R_1}{L\cdot C(R_1 + R_2)}\cdot V_C = \frac{1}{C(R_1 + R_2)}\cdot\frac{dV_s}{dt}$$

d. For the differential equation, determine symbolic expressions for the attenuation constant, α, and the resonant frequency, ω_o, in terms of R1, R2, L and C.

$$\alpha = \frac{1}{2\cdot C\cdot(R_1 + R_2)} \qquad \omega_o = \sqrt{\frac{R_1}{L\cdot C(R_1 + R_2)}}$$

3) RLC Series Circuits



In the above circuit, the initial conditions are zero and the source can be considered a step function, 9u(t).

a. Determine the simplified circuit schematic. (Hint: Thevenin equivalent with ***inductor and capacitor*** as a load...and yes, two components can be a load!).

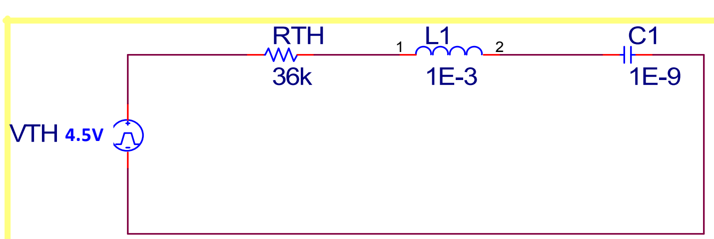
$R_{1p3} := 16k\Omega$ $R_{2p3} := 60k\Omega$ $R_{3p3} := 40k\Omega$ $R_{4p3} := 24k\Omega$ $R_{5p3} := 8k\Omega$

$$R_{TH} := \frac{\frac{R_{2p3} \cdot R_{3p3}}{R_{2p3} + R_{3p3}} \cdot R_{4p3}}{\frac{R_{2p3} \cdot R_{3p3}}{R_{2p3} + R_{3p3}} + R_{4p3}} + R_{1p3} + R_{5p3} = 36 \cdot k\Omega$$

$V_S := 9V$

$$V_{Th} := V_S \cdot \frac{R_{4p3}}{\left(\frac{R_{2p3} \cdot R_{3p3}}{R_{2p3} + R_{3p3}} + R_{4p3} \right)} = 4.5\,V$$

Redrawing the circuit



b. What is the intial (t= 0+) current through the capacitor? What is the initial (t= 0+) voltage through the capacitor?

Step function so intial conditions are defined as zero.

$V_C(0+) = 0V$ and

$\frac{dV_C}{dt}(0+) = 0$ so IC(0+)= 0mA

graders: give full credit even if dVc/dt is not given but please write on paper if it is not there

c. What is the DC steady state current though the capacitor as t approaches ∞?

The capacitor is an open circuit (inductor is a short circuit) and no current flows through the series RLC circuit.

$I_C(\infty) = 0A$

d. What is the differential equation defining the current through the capacitor?

$$\frac{d^2V_C}{dt^2} + \frac{R_{TH}}{L}\cdot\frac{dV_C}{dt} + \frac{1}{L\cdot C}\cdot V_C = \frac{1}{LC}\cdot V_{TH}$$

Thevenin resitance and voltage must be there

e. Based on the differential equation, determine the s-polynomial for the circuit.

$$s^2 + \frac{R_{TH}}{L}s + \frac{1}{LC} = 0$$

$L_1 := 1\,10^{-3}H$ $\quad C_1 := 1\cdot 10^{-9}F$

$$\frac{R_{TH}}{L_1} = 3.6\times 10^7\,\frac{1}{s}$$

$$s^2 + 3.6\cdot 10^7 s + 1\cdot 10^{12} = 0$$

$$\frac{1}{L_1\cdot C_1} = 1\times 10^{12}\,\frac{1}{s^2}$$

f. Determine the roots of the polynomial

$s_1 = -2.78\cdot 10^4$

$s_2 = -3.6\cdot 10^7$

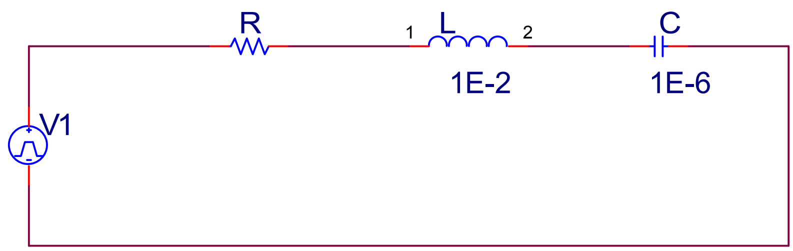
g. Is the system underdamped, overdamped or critically damped?

overdamped

h. Determine the general expression for the current through the capacitor (You do not need to determine the coefficients).

$$V_C(t) = A_1 \cdot e^{-2.78 \cdot 10^4 t} + A_2 \cdot e^{-3.6 \cdot 10^7 t} + A_3$$

Problem 4) RLC series circuits



In the above circuit, the source voltage is 5V for t<0 and 10 V for t>0

a. What is the intial (t=0+) voltage across the inductor? What is the intial(t=0+) current through the inductor?

At t=0-, the voltage across the capacitor is 5V. Enforcing continuity conditions at t=0+, the voltage across the capacitor is 5V.

$$V_{C3t0+} := 5V$$

The voltage across the resistor has to be zero since the current at t=0+ is zero from the inductor intial condition.

$$I_{L3}(0\text{-}) = I_{L3}(0+) = 0 \qquad V_{R3t0+} := 0V$$

Apply KVL, we obtain

$$V_{S3t0+} := 10V$$

$$V_{L3t0+} := V_{S3t0+} - V_{C3t0+} - V_{R3t0+}$$

$$V_{L3t0+} = 5\,V$$

b. What is the DC steady state current through the inductor at t approaches ∞.

In steady state, the capacitor is an open circuit and no current flows. Therefore, the current through the inductor as time approaches infinity is zero.

c. Symbollically (no values, just R, L, C etc.), what is the differential equation defining the voltage across the inductor?

$$\frac{d^2 \cdot V_L}{dt^2} + \frac{R}{L}\cdot\frac{dV_L}{dt} + \frac{1}{LC}\cdot V_L = \frac{d^2 V_s}{dt^2}$$

We can obtain an expression for IL as well. This expression is useful since circuit analysis gives us informaiton about the first derivative of the current through an inductor. On the other hand, we don't have a simple relationship for the first derivative of the voltage across an inductor.

$$\frac{d^2 I_L}{dt^2} + \frac{R}{L}\cdot\frac{dI_L}{dt} + \frac{1}{LC}\cdot I_L = \frac{1}{L}\cdot\frac{dV_s}{dt}$$

d. For R = 4kΩ, determine the voltage across the inductor as a function f time for t>0. (Hint: Use differential equation for current through the inductor. Then use the differential relationship between inductor current and inductor voltage.)

$R_3 := 4k\Omega$ $\qquad$ $L_3 := 1\cdot 10^{-2} H$ $\qquad$ $C_3 := 1\cdot 10^{-6} F$

In this case

$$\alpha_3 := \frac{R_3}{2\cdot L_3}$$

$$\alpha_3 = 2\times 10^5 \frac{1}{s}$$

$$\omega_{o3} := \frac{1}{\sqrt{L_3\cdot C_3}}$$

$$\omega_{o3} = 1\times 10^4 \cdot\frac{rad}{s}$$

$\alpha_3 > \omega_o$ $\qquad$ In this case, the system is overdamped and we have the general expression in the form

$I_L(t) = A_1 \cdot \exp(s_1 \cdot t) + A_2 \cdot \exp(s_2 \cdot t) + C$

$s_1 := -\alpha_3 + \sqrt{\alpha_3^{\ 2} - \omega_{o3}^{\ 2}}$ (-α1 is only written this way because of software)

$s_1 = -250.156 \frac{1}{s}$ rounded down to -250

$s_2 := -\alpha_3 - \sqrt{\alpha_3^{\ 2} - \omega_{o3}^{\ 2}}$

$s_2 = -3.997 \times 10^5 \frac{1}{s}$ rounded up to -4E5

The steady state condition for inductor current is zero, given C =0. (Remember IL(∞)=0)

The initial conditions at t=0+ give

$A_{1e}^{\ 0} + A_{2e}^{\ 0} = I_L(0+) = 0$

$A_1 + A_2 = 0$

$\frac{dI_L(0+)}{dt} \cdot L = V_L(0+)$ From the definition of the inductor we need dIL(0+)/dt as an inital condtion to solve this problem.

$\frac{dI_L(0+)}{dt} = \frac{V_L(0+)}{L}$ We'll substitue this in for the equaiton below which is the derivative for the circuit solution

$s_1 \cdot A_1 \cdot e^0 + s_2 \cdot A_2 \cdot e^0 = \frac{V_L(0+)}{L} = \frac{5}{1 \cdot 10^{-2}} = 500$

$-250 \cdot A_1 - 4 \cdot 10^5 \cdot A_2 = 500$ two equations two unknowns..substitute

$-250 \cdot (-A_2) - 4 \cdot 10^5 \cdot A_2 = 500$

$-4 \cdot 10^5 + 250 = -3.998 \times 10^5$

$-4 \cdot 10^5 \cdot A_2 = 500$

$A_2 = -0.00125$

$A_2 := \frac{500}{-4 \cdot 10^5} = -1.25 \times 10^{-3}$

$A_1 = 0.00125$

$$I_L(t) = 0.00125\,\exp(-250t) - 0.00125\cdot\exp\left(-4\cdot 10^{5}t\right)$$

Almost there! Now we can use the differential relationshp to find VL

$$V_L = L\cdot\frac{dI_L}{dt} \qquad -250\cdot 0.00125 = -0.313 \qquad -4\cdot 10^{5}\cdot -0.00125 = 500$$

$$V_L(t) = 1\cdot 10^{-2}\cdot\left(-0.313\cdot\exp(-250t) + 500\cdot\exp\left(-4\cdot 10^{5}t\right)\right)$$

$$1\cdot 10^{-2}\cdot -0.313 = -3.13\times 10^{-3}$$

$$V_L(t) = -0.00313\cdot\exp(-250t) + 5\cdot\exp\left(-4\cdot 10^{5}t\right)$$

$$1\cdot 10^{-2}\cdot 500 = 5$$

e. For R = 200Ω, determine the voltage across the inductor as a function of time for t >0.

$$R_{3d} := 200$$

In this case,

$$\alpha_{3d} := \frac{R_{3d}}{2\cdot L_3}$$

$$\alpha_{3d} = 1\times 10^{4}\,\frac{1}{H}$$

$$\omega_{o3} = 1\times 10^{4}\,\frac{1}{s}$$

$$\alpha_{3d} = \omega_{o3}$$

In this case, we are critcially damped. The solution is in the form.

$$I_L(t) = A_1\cdot\exp(-\alpha\cdot t) + A_2\cdot t\cdot\exp(-\alpha\cdot t) + C$$

where s1 and s2 are -α

The steady state condition for current is zero, giving C=0

$$A_1\cdot e^{0} + A_2\cdot 0 = A_1 = I_L(0+) = 0$$

$$A_2 = \frac{V_L(0+)}{L} = \frac{5}{1\cdot 10^{-2}} = 500$$

same initial condition using the derivative inductor equation substitution

$$I_L(t) = 500t\cdot\exp\left(-1\cdot 10^{4}t\right)$$

Using the differential realtionship, VL = L dIL/dt

$$L = 1\cdot 10^{-2}$$

$$V_L(t) = 5\cdot\exp\left(-1\cdot 10^{4}\cdot t\right) - 5\cdot 10^{4}\cdot t\cdot\exp\left(-1\cdot 10^{4}t\right)$$

$$500\cdot -1\cdot 10^{4}\cdot 1\cdot 10^{-2} = -5\times 10^{4}$$

f. For R = 50Ω, determine the voltage across the inductor as a functin of time for t>0.

In this case

$$R_{e3} := 50\Omega$$

$$\alpha_{3e} := \frac{R_{e3}}{2\cdot L_3}$$

$$\alpha_{3e} = 2.5\times 10^3\ \frac{1}{s} \qquad\qquad \omega_{o3} = 1\times 10^4\ \frac{1}{s}$$

The circuit is underdamped and the general expression is in the form

$$I_L(t) = A_1\cdot\exp[(-\alpha + j\beta)\cdot t] + A_2\cdot\exp[(-\alpha - j\beta)t]$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_o^2} = -\alpha + j\beta$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_o^2} = -\alpha - j\beta$$

$$\alpha_{3e} = 2.5\times 10^3\ \frac{1}{s}$$

$$\beta_{3e} := \sqrt{\omega_{o3}^2 - \alpha_{3e}^2}$$

notice that the parameters inside the square root switched...why....think about what the j represents!

$$\beta_{3e} = 9.682\times 10^3\ \frac{1}{s}$$

Easiest equation to work with

$$I_L(t) = e^{-\alpha t}\cdot\left[A_1\cdot\cos\cdot(\beta t) + A_2\cdot\sin(\beta t)\right] + C$$

The steady state conditon for current is zero giving C=0.

The inital condtions at t=0+ give

$$e^0\cdot\left(A_1\cdot\cos 0 + A_2\cdot\sin 0\right) = 0 \qquad\qquad A_1 = 0$$

also use the derivative and substitution

$$\beta\cdot A_2\cdot\cos 0 = \frac{V_L(0+)}{L} = 500$$

$$A_2 := \frac{500}{9682}$$

$A_2 = 0.052$

$I_L(t) = \exp(-2500t)\cdot -0.052\cdot\sin(9682t)$

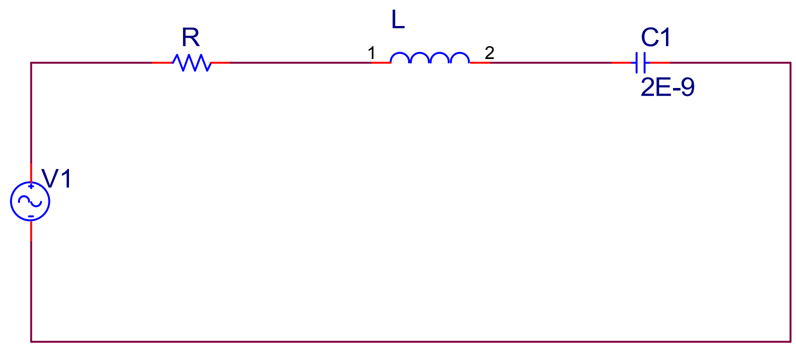
Take the derivative to get the voltage across the inductor, remember to multiply by L=1E-2

$V_L(t) = -25\cdot\exp(-2500t)\cdot\sin(9682t) - 5\cdot\exp(-2500t)\cdot\cos(9682t)$

$-0.052\cdot 9682\cdot 1\cdot 10^{-2} = -5.035$

$-2500\cdot 1\cdot 10^{-2} = -25$

4) RLC series design problem



The above circuit has a capacitor voltage defined as

$$V_c(t) = 2E7t\cdot\exp(-2E6t) + 10\cdot\exp(-2E6t) + 10$$

Determine the resistance, inductance, and source expression (the souce is a step function of some kind).

The circuit must be critically damped.

$$\alpha = \omega_o = \frac{1}{\sqrt{LC}} = \frac{R}{2L}$$

We know C so

$$L_4 := \frac{1}{2\cdot 10^{-9}\cdot\left(2\times 10^6\right)^2}$$

$L_4 = 1.25\times 10^{-4}$

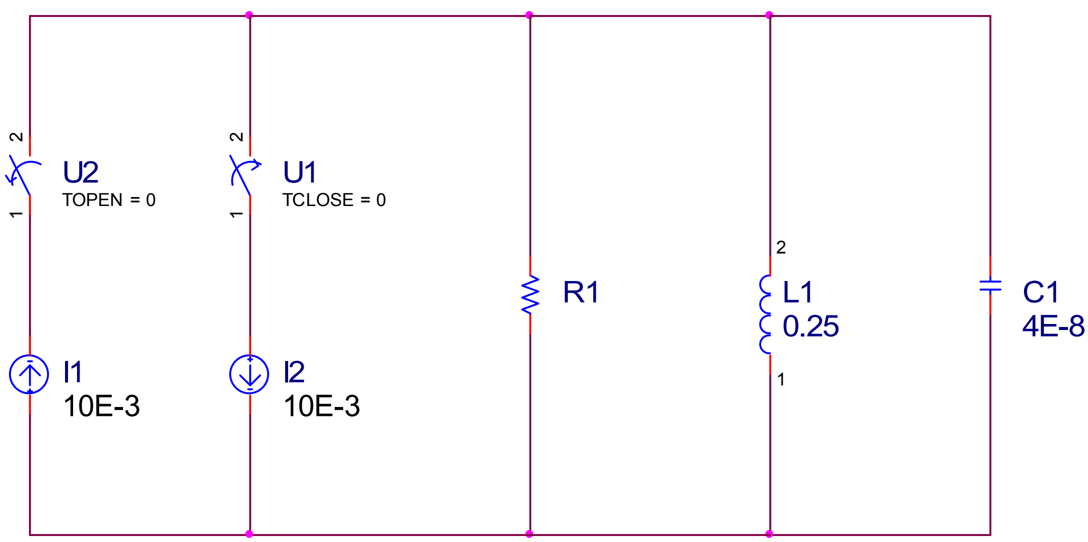
$R_4 := 2\cdot L_4\cdot 2\cdot 10^6$

$R_4 = 500$

Finally a steady state voltage of 10V exists as t approaches ∞

at t=0 the voltage source turns on with a voltage of 10V $10\,u(t)$

5) RLC Parallel Circuits



At t =0, U1 closes and U2 opens.

a. What is the intial (t=0+) current through the capacitor? What is the initial (t=0+) voltage across the capacitor?

The steady state voltage across the capacitor is 0V. Since the components are in parallel, the voltage across the resistor is also 0V. Therefore, at t = 0+, the resistor must have a zero current.

Applying KCL at t=0+, we obtain IC1 = -I2 - IR1 -IL1 (all downward)

$$I_{C1} := -10\text{mA} + 0 - 10\text{mA}$$

$$I_{C1} = -20\cdot\text{mA} \qquad \text{upward}$$

b. What is the DC steady state current though the capacitor as t goes to infinity?

A capacitor is an open circuit as time approaches infinity giving zero current

This part is not necessary for grading but as a point of reference.

RLC parallel circuit. The differential expression for voltage across the capacitor is given as

$$\frac{d^2V_C}{dt^2} + \frac{1}{RC}\cdot\frac{dV_C}{dt} + \frac{1}{LC}\cdot V_C = \frac{1}{C}\cdot\frac{dV_s}{dt}$$

Like in the previous problem, we have relationships for the initial conditions across the capacitor. Once the capacitor voltage is know we can determine the current by applying the current-voltage relationship for a capacitor

$$I_C = C \cdot \frac{dV_C}{dt}$$

c. Find the current through the **CAPACITOR** as a function of time for R = 12.5k. (***Hint: Find the voltage across the capacitor equation first then use the current-voltage relationship of a capacitor to get the current! The reason why is because we know the intial conditions for a capacitor voltage not for capacitor current which is necessary to solve the problem)***

In this case

$$R_5 := 12.5k\Omega \qquad C_5 := 4 \cdot 10^{-8} F \qquad L_5 := 0.25H$$

$$\alpha_5 := \frac{1}{2 \cdot R_5 \cdot C_5}$$

$$\alpha_5 = 1 \times 10^3 \frac{1}{s}$$

$$\omega_{o5} := \frac{1}{\sqrt{L_5 \cdot C_5}}$$

$$\omega_{o5} = 1 \times 10^4 \frac{1}{s} \qquad \alpha_5 < w_{o5}$$

In this case, the circuit is underdamped and we have the general expression in the form

$$V_C(t) = \exp(-\alpha t) \cdot \left(A_1 \cdot \cos(\beta t) + A_2 \cdot \sin(\beta t)\right) + C$$

$$\beta_5 := \sqrt{{\omega_{o5}}^2 - {\alpha_5}^2}$$

$$\beta_5 = 9.95 \times 10^3 \frac{1}{s}$$

The steady state condition for the voltage is zero, giving C = 0.

The intial conditions at t = 0+ give

$$0 = e^0 \cdot A_1 \cdot \cos 0$$

$$A_1 = 0$$

using the derivaite initial condition $I_C = C \cdot \frac{dV_c}{dt}$ therefore $\frac{I_C(0+)}{C} = \frac{dV_C}{dt}$

$\beta_5 \cdot A_2 \cdot \cos 0 = \frac{I_C(0+)}{C}$

$\frac{-20mA}{C_5} = -5 \times 10^5 \frac{m^2 \cdot kg}{A \cdot s^4}$

$A_2 := \frac{-5 \cdot 10^5}{9950}$

$A_2 = -50.251$

$4 \cdot 10^{-8} \cdot -50.25 \cdot -1000 = 2.01 \times 10^{-3}$

$\frac{dVc}{dt} \cdot C = Ic$

$9950 \cdot -50.25 \cdot 4 \cdot 10^{-8} = -0.02$

$V_c(t) = -50.25 \cdot \exp(-1000t) \cdot \sin(9950t)$

Now we must use the differential relationship IC=dVc/dt to obtain the current through the capacitor

$I_c(t) = 2.01 \cdot 10^{-3} \cdot \exp(-1000t) \cdot \sin(9950t) - 0.02 \cdot \exp(-1000t) \cdot \cos(9950t)$

d. Find the current through the **CAPACITOR** as a function of time for R = 0.25k.

$R_{5d} := 0.25k\Omega$

$\alpha_{5d} := \frac{1}{2 \cdot R_{5d} \cdot C_5}$

$\alpha_{5d} = 5 \times 10^4 \frac{1}{s}$

$\omega_{o5} = 1 \times 10^4 \frac{1}{s}$

$\alpha_{5d} > \omega_{o5}$

The circuit is overdamped and the general expression is

$V_C(t) = A_1 \cdot \exp(s_1 \cdot t) + A_2 \cdot \exp(s_2 \cdot t) + C$

$s_{1d} := -\alpha_{5d} + \sqrt{\alpha_{5d}^2 - \omega_{o5}^2}$

$s_{1d} = -1.01 \times 10^3 \frac{1}{s}$

$s_{2d} := -\alpha_{5d} - \sqrt{\alpha_{5d}^2 - \omega_{o5}^2}$

$s_{2d} = -9.899 \times 10^4 \frac{1}{s}$

The steady state conditions for current is zero, giving C = 0

The initial conditions at t = 0+ give

$$A_1 + A_2 = V_C(0+) = 0 \qquad\qquad A_1 = -A_2$$

using derivative initial condition and relationship

$$s_1 \cdot A_1 + s_2 \cdot A_2 = \frac{I_C(0+)}{C} \qquad\qquad -1010 \cdot A_1 - 9.899 \cdot 10^4 \cdot A_2 = -5 \cdot 10^5$$

subsituting in -A2 for A1

$$-1010 \cdot (-A_2) - 9.899 \cdot 10^4 \cdot A_2 = -5 \cdot 10^5$$

$$A_{2d} := 5.103 \qquad A_{1d} := -A_{2d} = -5.103$$

$$V_C(t) = -5.103 \cdot \exp(-1010t) + 5.103 \cdot \exp(-98989t)$$

$$V_C = V_L$$

Use the differential relationship, IC=C*dV/dt

$$I_C(t) = 0.0002 \cdot \exp(-1010t) - 0.02 \cdot \exp(98989t)$$

$$4 \cdot 10^{-8} \cdot -5.103 \cdot -1010 = 2.062 \times 10^{-4}$$

$$4 \cdot 10^{-8} \cdot 5.103 \cdot -98989 = -0.02$$